Assignment #2 E & CE 223

E&CE 223
Assignment 2 - Solutions

Mano 2.2:

(a) $x'y' + xy + x'y = x'(y' + y) + xy = x' + xy = (x' + x)(x' + y)$

$= x' + y$

(b) $(x + y)(x + y') = x + yy' = x$

(c) $x'y + xy' + xy + x'y' = x'(y + y') + x(y' + y) = x' + x = 1$

(d) $x' + xy + xz' + xy'z' = x' + xy + xz'(1 + y') = x' + xy + xz'$

$= x' + x(y + z') = (x' + x)(x' + y + z')$

$= x' + y + z'$

(e) The Consensus Theorem states:

$xy + yz + zx' = xy + zx'$

Hence

$xy' + y'z' + z'x' = xy' + z'x'$

by the concensus theorem

The algebraic proof is as follows:

$xy' + y'z' + z'x' = xy' + (x + x')y'z' + z'x' = xy' + xy'z + x'y'z' + z'x'$

$= xy'(1 + z) + z'x'(y' + 1) = xy' + z'x'$

Mano 2.3:

(a) $ABC + A'B + ABC' = B[AC + A' + AC']$

$= B[A' + A(C + C')] = B[A' + A] = B$

(b) $x'yz + xz = (x'y + x)z = (x' + x)(y + x)z = (y + x)z$

(c) $(x+y)'(x'+y') = x'y'(x'+y') = x'y'$

(d) $xy + x(wz + wz') = xy + xw(z + z') = xy + xw = x(y + w)$

(e) $(BC' + A'D)(AB' + CD') = BC'AB' + BC'CD' + A'DAB' + A'DCD'$

$$= 0 + 0 + 0 + 0$$

$$= 0$$

Mano 2.6:

(a) $[xy' + x'y]' = (x' + y)(x + y') = x'x + x'y' + yx + yy'$

$$= xy + x'y'$$

(b) $[(AB' + C)D' + E]' = [(A' + B)C' + D]E'$ (answer)

$$= (A' + B + D)(C' + D)E'$$ (product-of-sums form)

(c) $[AB(C'D + CD') + A'B'(C' + D)(C + D')]'$

$$= [A' + B' + (C + D')(C' + D)][A + B + CD' + C'D]$$ (Mano answer)

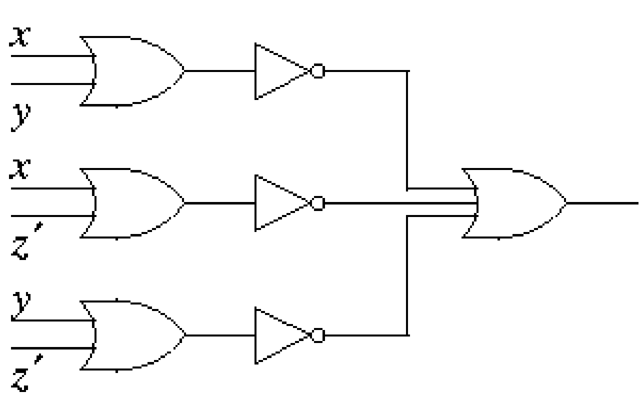
$$= [A' + B' + CD + C'D'][A + B + CD' + C'D]$$ (somewhat simpler)

(d) $[(x + y' + z)(x' + z')(x + y)]' = x'yz' + xz + x'y'$

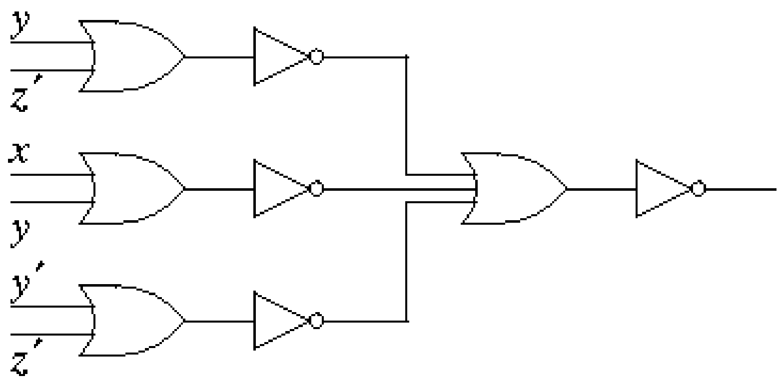
Mano 2.7:

(a) $F = x'y' + x'z + y'z$

$$= (x + y)' + (x + z')' + (y + z')'$$



(b) $F = (y + z')(x + y)(y' + z) = [(y + z')' + (x + y)' + (y' + z)]'$



Mano 2.9:

(a) $F = (xy + z)(y + xz) = xy + zy + xyz + xz$

$= xy(z+z') + (x+x')yz + xyz + x(y+y')z$

$= xyz + xyz' + xyz + x'yz + xyz + xyz + xy'z$

$= xyz + xyz' + x'yz + xy'z$

xyz	F
000	0
001	0
010	0
011	1
100	0
101	1
110	1
111	1

$F = \sum(3,5,6,7) = \prod(0,1,2,4)$

(b) $F = (A' + B)(B' + C) = A'B' + BB' + A'C + BC$

$= A'B'(C+C') + A'(B+B')C + (A+A')BC$

$= A'B'C + A'B'C' + A'BC + A'B'C + ABC + A'BC$

$= A'B'C + A'B'C' + A'BC + ABC$

ABC	F
000	1
001	1
010	0
011	1
100	0
101	0
110	0
111	1

$$F = \sum(0,1,3,7) = \prod(2,4,5,6)$$

(c) $F = y'z + wxy' + wxz' + w'x'z$

$$= (w+w')(x+x')y'z + wxy'(z+z') + wx(y+y')z' + w'x'(y+y')z$$

$$= wxy'z + wx'y'z + w'xy'z + w'x'y'z + wxy'z + wxy'z'$$

$$+ wxyz' + wxy'z' + w'x'yz + w'x'y'z$$

$$= wxy'z + wx'y'z + w'xy'z + w'x'y'z + wxy'z' + wxyz' + w'x'yz$$

wxyz	F
0000	0
0001	1
0010	0
0011	1
0100	0
0101	1
0110	0
0111	0
1000	0
1001	1
1010	0
1011	0
1100	1
1101	1
1110	1
1111	0

$$F = \sum(1,3,5,9,12,13,14)$$

$$= \prod(0,2,4,6,7,8,10,11,15)$$

Mano 2.11:

(a) $F = xy'z + x'y'z + w'xy + wx'y + wxy$

$= (w+w')xy'z + (w+w')x'y'z + w'xy(z+z') + wx'y(z+z') + wxy(z+z')$

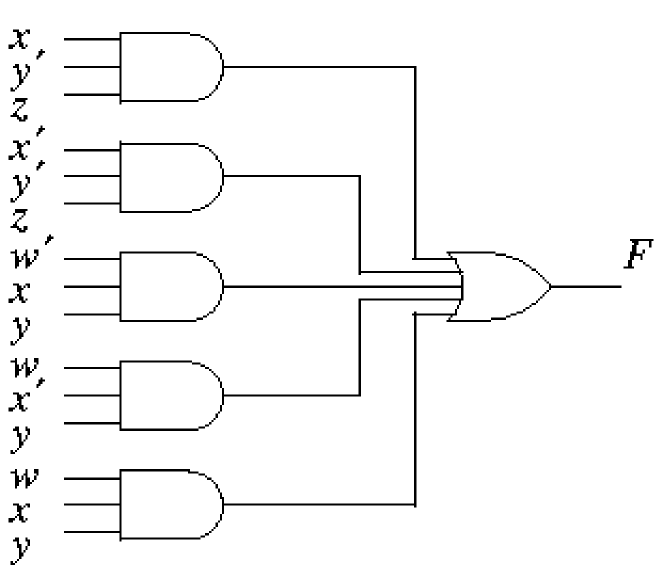
$= wxy'z + w'xy'z + wx'y'z + w'x'y'z + w'xyz + w'xyz'$

$\quad + wx'yz + wx'yz' + wxyz + wxyz'$

wxyz	F
0000	0
0001	1
0010	0
0011	0
0100	0
0101	1
0110	1
0111	1
1000	0
1001	1
1010	1
1011	1
1100	0
1101	1
1110	1
1111	1

$F = \sum(1,5,6,7,9,10,11,13,14,15)$

(b)



(c) $F = xy'z + x'y'z + w'xy + wx'y + wxy$

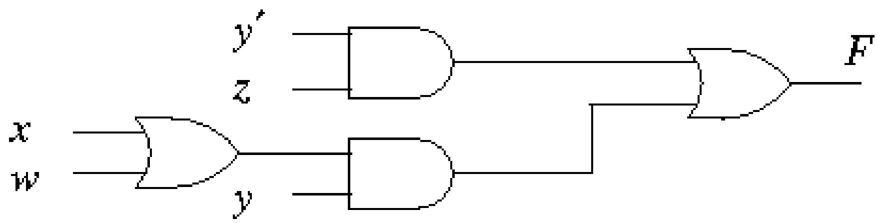
$= (x + x')y'z + (w' + w)xy + (x' + x)wy$

[note: wxy used twice, $A = A + A$]

$= y'z + xy + wy = y'z + y(x + w)$

(d) truth table the same as (a)

(e)



(four 2-input gates) vs (five 3-input gates and one 5-input gate)

Mano 2.13:

(a) $F(A,B,C,D) = \sum(0,2,6,11,13,14)$

$F'(A,B,C,D) = \sum(\text{minterms not in } F)$

$= \sum(1,3,4,5,7,8,9,10,12,15)$

(b) $F(x,y,z) = \prod(0,3,6,7)$

$F'(x,y,z) = [\prod(0,3,6,7)]'$

$= \sum(0,3,6,7)$

Mano 2.14:

(a) $F(x,y,x) = \sum(1,3,7)$

$= \prod(0,2,4,5,6)$

(b) $F(A,B,C,D) = \prod(0,1,2,3,4,6,12)$

$= \sum(5,7,8,9,10,11,13,14,15)$

Mano 2.19(b):

$$F_1 = a \oplus b = ab' + a'b \quad \text{(definition)}$$

$$F_2 = b \oplus a = ba' + b'a \quad \text{(from definition)}$$

$$= ab' + a'b$$

$$= F_1 \quad \text{therefore XOR is commutative}$$

$$F_3 = a \oplus (b \oplus c)$$

$$= a \oplus (bc' + b'c)$$

$$= a(bc' + b'c)' + a'(bc' + bc')$$

$$= a(b' + c)(b + c') + a'(bc' + bc')$$

$$= a(b'b + b'c' + cb + cc') + a'bc' + a'bc'$$

$$= ab'c' + abc + a'bc' + a'bc'$$

$$F_4 = (a \oplus b) \oplus c$$

$$= (ab' + a'b) \oplus c$$

$$= (ab' + a'b)c' + (ab' + a'b)'c$$

$$= ab'c' + a'bc' + (a' + b)(a + b')c$$

$$= ab'c' + a'bc' + (a'a + a'b' + ba + bb')c$$

$$= ab'c' + a'bc' + a'b'c + abc$$

$$= F_3 \quad \text{therefore, XOR is associative}$$