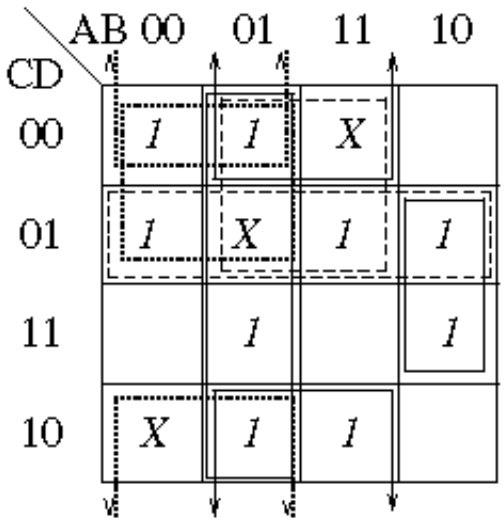
Assignment #3 E & CE 223

E&CE 223
Assignment 3 - Solutions

1 Find the minimum sum of products (fewest gates) for F defined below. Indicate the essential prime implicants.



There are three solutions:

$F = C'A' + C'D + A'B + D'B + DAB'$

$F = D'A' + C'D + A'B + D'B + DAB'$

$F = C'B + C'A' + A'B + D'B + DAB'$

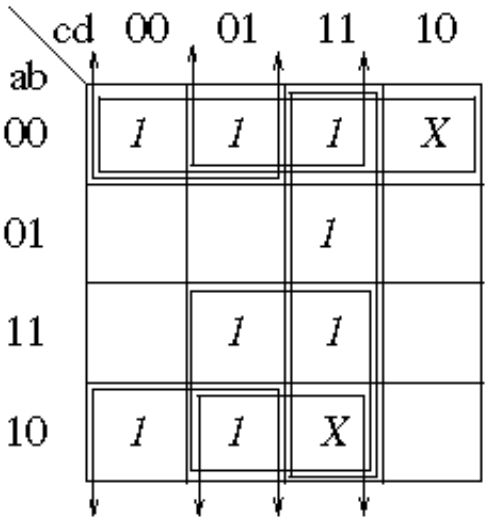
Essential prime implicants: $A'B$, $D'B$, DAB'

2 Given the function F where

$$F(a,b,c,d) = \sum(0, 1, 3, 7, 8, 9, 13, 15) + d(2, 11)$$

(a) Find all the prime implicants

(b) Find all the essential prime implicants and indicate why each one is essential



prime implicants	essential	reason
$a'b'$		
cd	yes	only cover for $a'bcd$
ad	yes	only cover for $abc'd$
$b'd$		
$b'c'$	yes	only cover for $ab'c'd'$

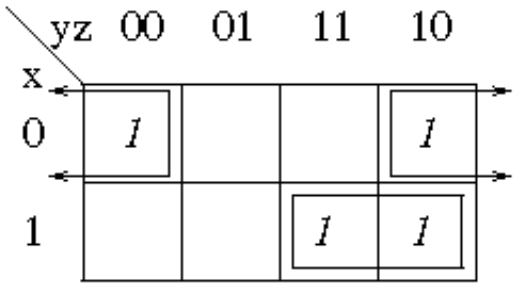
(c) find a minimum number of gates sum of products expression for F

$F = cd + ad + b'c'$

3 Do Mano 3.2, 3.3, 3.9, 3.10, 3.12, 2.23, 3.27

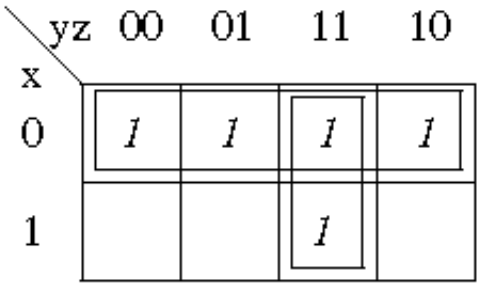
Mano 3.2

(a) $F = xy + x'y'z' + x'yz'$



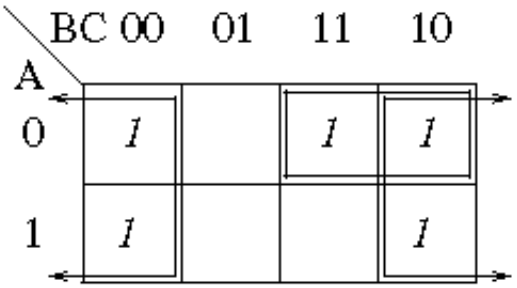
$F = x'z' + xy$

(b) $F = x'y' + yz + x'yz'$



$F = x' + yz$

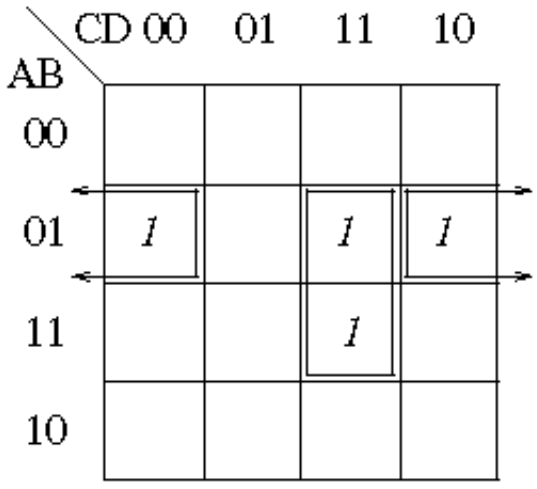
(c) $F = A'B + BC' + B'C'$



$F = C' + A'B$

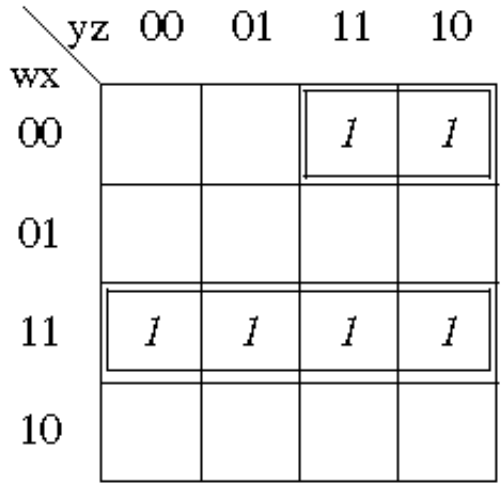
Mano 3.3

(a) $F(A,B,C,D) = \sum(4, 6, 7, 15)$



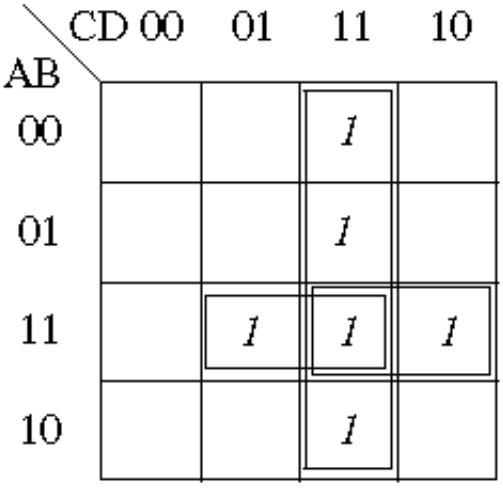
$$F(A,B,C,D) = A'BD' + BCD$$

(b) $F(w,x,y,z) = \sum(2, 3, 12, 13, 14, 15)$



$$F(w,x,y,z) = wx + w'x'y$$

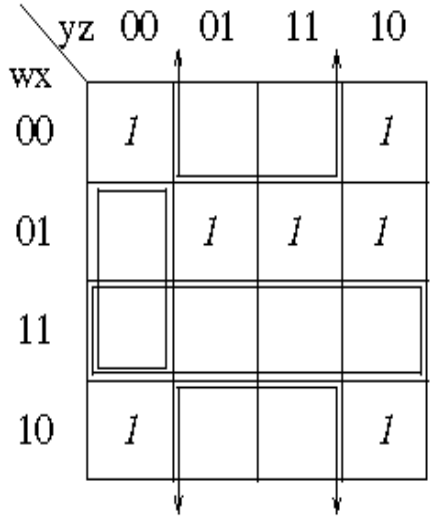
(c) $F(A,B,C,D) = \sum(3, 7, 11, 13, 14, 15)$



$$F(A,B,C,D) = CD + ABD + ABC$$

Mano 3.9

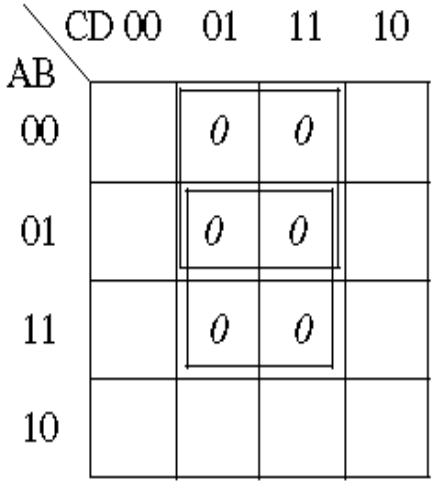
(a) $F(w,x,y,z) = \sum(0, 2, 5, 6, 7, 8, 10)$



$$F'(w,x,y,z) = xy'z' + wx + x'z$$

$$F(w,x,y,z) = (x'+y+z)(w'+x')(x+z')$$

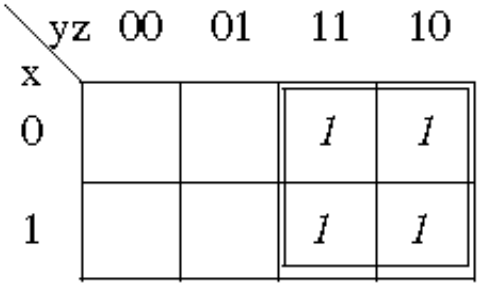
(b) $F(A,B,C,D) = \prod(1, 3, 5, 7, 13, 15)$



$$F'(A,B,C,D) = A'D + BD$$

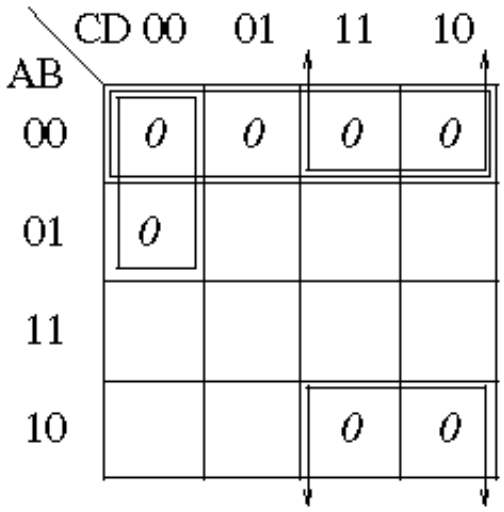
$$F(A,B,C,D) = (A+D')(B'+D')$$

(c) $F(x,y,z) = \sum(2, 3, 6, 7)$



$F(x,y,x) = y$

(d) $F(A,B,C,D) = \prod(0, 1, 2, 3, 4, 10, 11)$

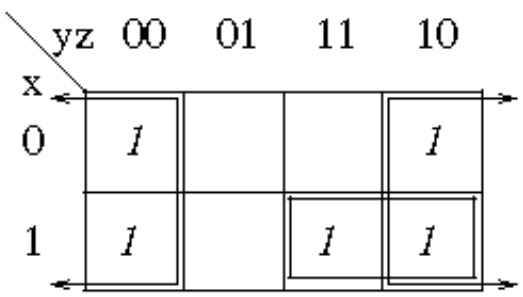


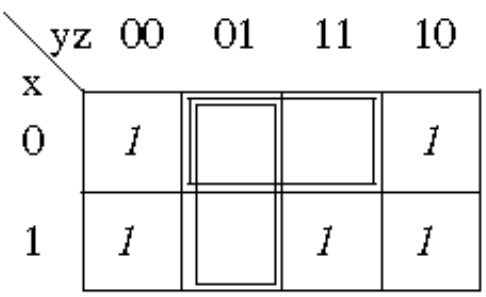
$F'(A,B,C,D) = A'C'D' + A'B' + B'C$

$F(A,B,C,D) = (A+C+D)(A+B)(B+C')$

Mano 3.10

(a) $F = x'z' + y'z' + yz' + xy$



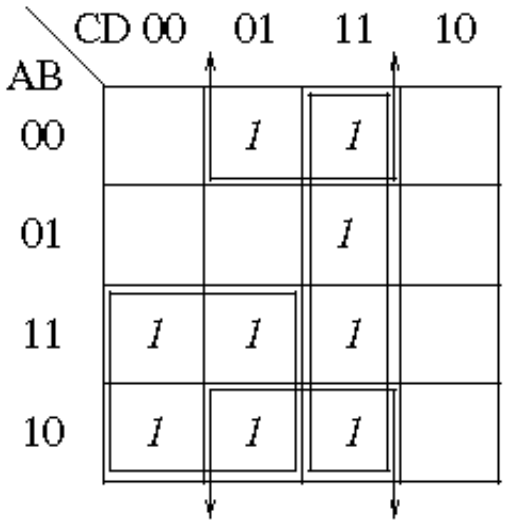


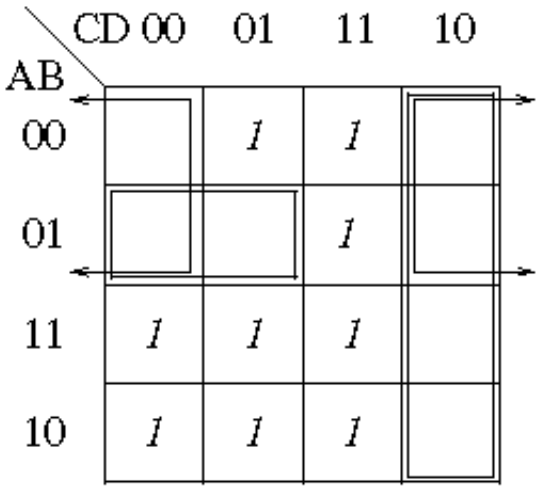
(i) $F = z' + xy$

(ii) $F' = x'z + y'z$

$F = (x+z')(y+z')$

(b) $F = AC' + B'D + A'CD + ABCD$





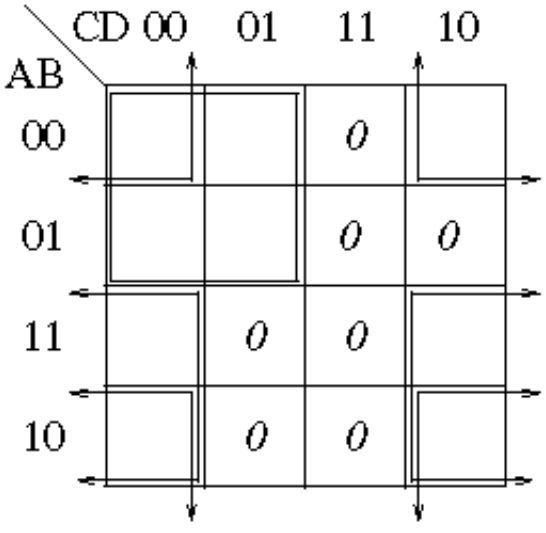
(i) $F = AC' + CD + B'D$

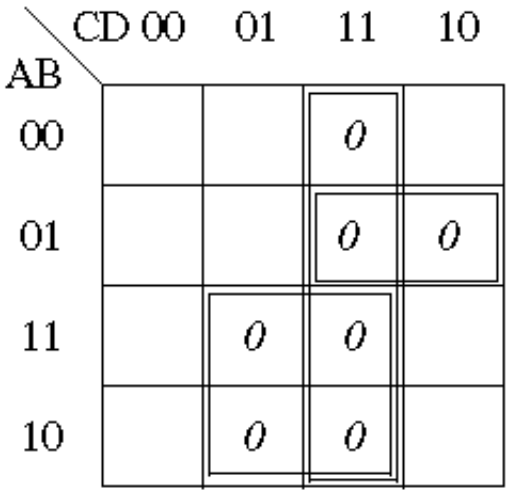
(ii) $F' = A'D' + A'BC' + CD'$

$F = (A+D)(A+B'+C)(C'+D)$

(c) $F = (A'+B'+D')(A+B'+C')(A'+B+D')(B+C'+D')$

$F' = ABD + A'BC + AB'D + B'CD$





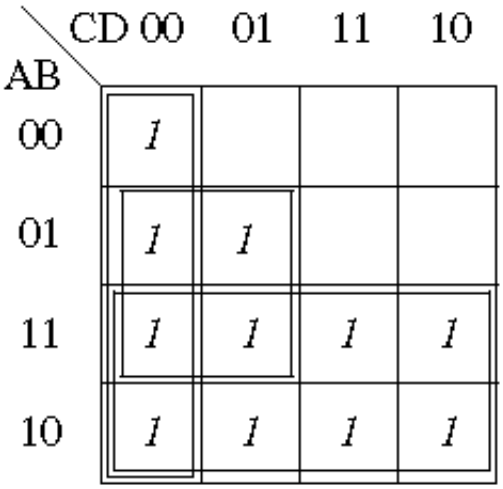
(i) $F = A'C' + B'D' + AD'$

(ii) $F' = AD + CD + A'BC$

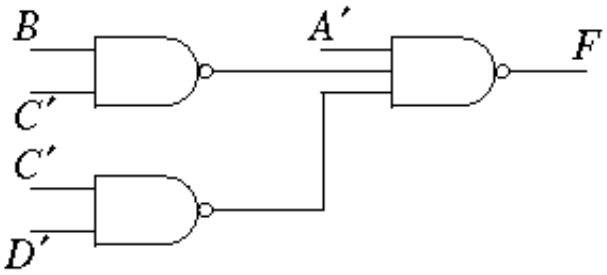
$F = (A'+D')(C'+D')(A+B'+C')$

Mano 3.12

(a) $F = AB' + ABD + ABD' + A'C'D' + A'BC'$



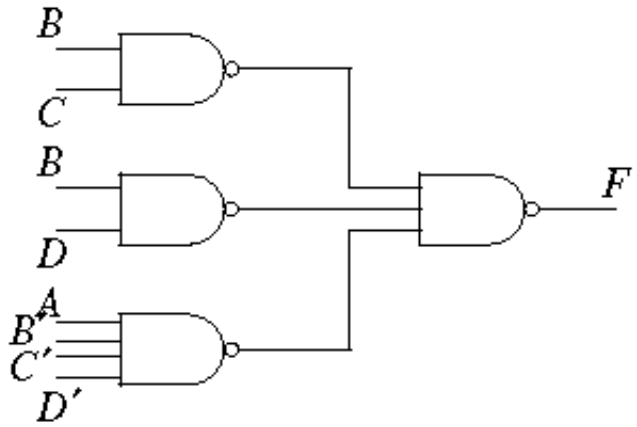
$F = A + BC' + C'D'$



(b) $F = BD + BCD' + AB'C'D'$

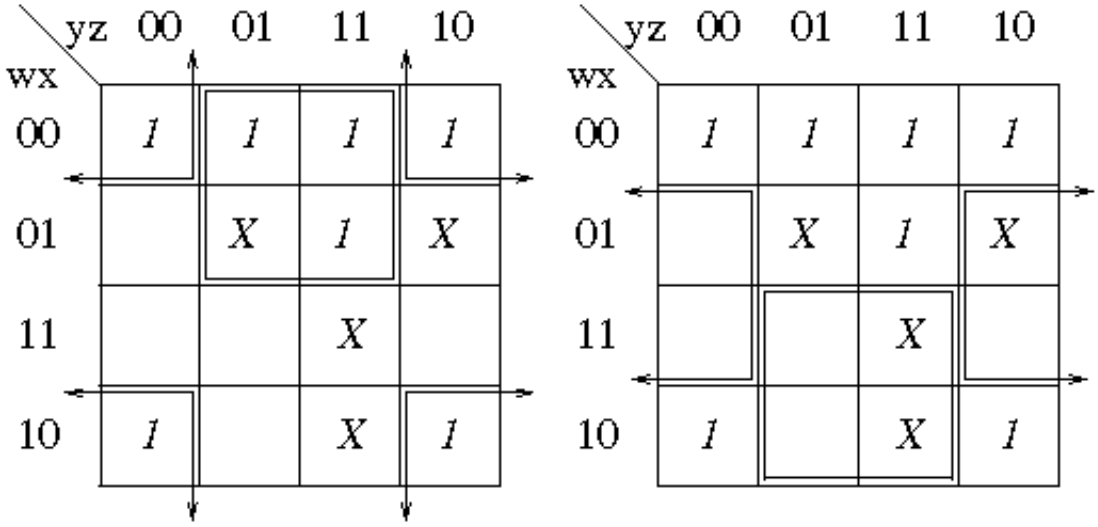
AB \ CD	00	01	11	10
00				
01		1	1	1
11		1	1	1
10	1			

$F = BC + BD + AB'C'D'$



Mano 3.23

(a) $F(w,x,y,z) = \sum(0, 1, 2, 3, 7, 8, 10) + d(5, 6, 11, 15)$

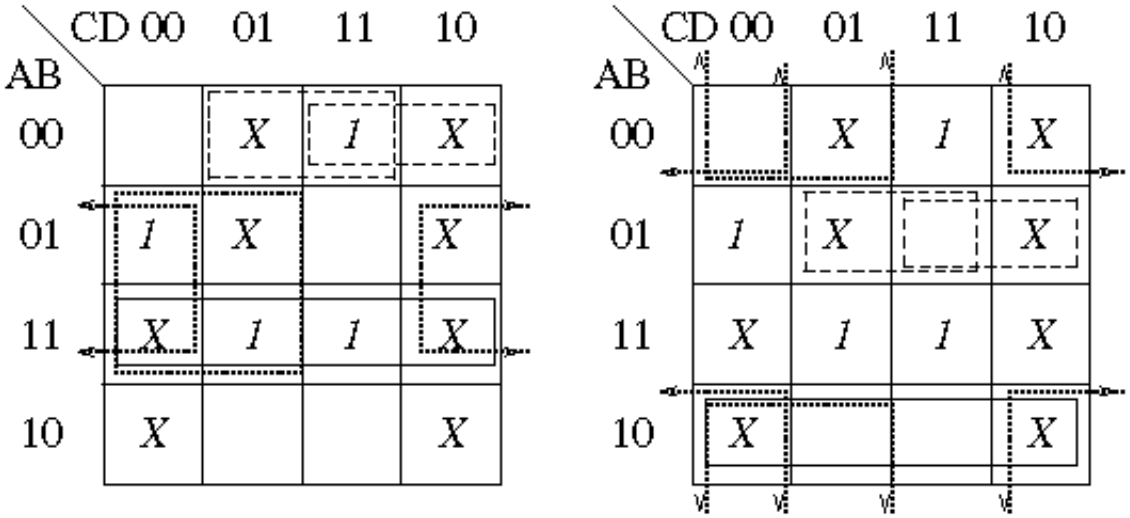


(i) $F = w'z + x'z'$

(ii) $F' = xz' + wz$

$F = (x'+z)(w'+z')$

(b) $F(A,B,C,D) = \sum(3, 4, 13, 15)+d(1, 2, 5, 6, 8, 10, 12, 14)$



(i)

$$F = AB + \begin{matrix} BC' \\ \text{or} \\ BD' \end{matrix} + \begin{matrix} A'B'C \\ \text{or} \\ A'B'D \end{matrix}$$

(four equally simple solutions)

(ii)

$$F' = AB' + \begin{matrix} B'D' \\ \text{or} \\ B'C' \end{matrix} + \begin{matrix} A'BC \\ \text{or} \\ A'BD \end{matrix}$$

$$F = (A'+B)(\begin{matrix} B+D \\ \text{or} \\ B+C \end{matrix})(\begin{matrix} A+B'+C' \\ \text{or} \\ A+B'+D' \end{matrix})$$

Mano 3.27

(a) $P(A,B,C,D,E,F,G) = \sum(20, 28, 52, 60)$

	ABCDEFG	ABCDEFG		ABCDEFG	
(20)	0010100*	001-100*	[20,28(8)]	0-1-100	[20,28,52,60(8,32)]
	———	0-10100*	[20,52(32)]		
(28)	0011100*	———			
(52)	0110100*	0-11100*	[28,60(32)]		
	———	011-100*	[52,60(8)]		
(60)	0111100*				

$$P = 0\text{–}1\text{–}100 = A'CEF'G'$$

(b) $P(A,B,C,D,E,F,G) = \sum(20, 28, 38, 39, 52, 60, 102, 103, 127)$

	ABCDEFG			
20	0010100	20*	20,28(8)*	20,28,52, 60(8,32)
28	0011100	———	20,52(32)*	———
38	0100110	28*	———	38,39,102,103(1,64)
39	0100111	38*	28,60(32)*	
52	0110100	52*	38,39(1)*	
60	0111100	———	38,102(64)*	
102	1100110	39*	52, 60(8)*	
103	1100111	60*	———	
127	1111111	102*	39,103(64)*	
		———	102,103(1)*	
		103*		
		———		
		127		

Prime implicants	ABCDEFG	
127	1111111	*ABCDEFG*
20,28,52,60(8,32)	0-1-100	*A′CEF′G′*
38,39,102,103(1,64)	-10011-	*BC′D′EF*

Prime implicant table:

	20	28	38	39	52	60	102	103	127
127									x
20,28,52,60(8,32)	x	x			x	x			
38,39,102,103(1,64)			x	x			x	x	

All prime implicants are essential.

$P = ABCDEFG + A'CEF'G' + BC'D'EF$

(c) $P(A,B,C,D,E,F) = \sum(6, 9, 13, 18, 19, 25, 27, 29, 41, 45, 57, 61)$

6	000110	6	9,13(4)*	9,13,25,29(4,16)*	9,13,25,29,41,45,
9	001001	9*	9,25(16)*	9,13,41,45(4,32)*	57,61(4,16,32)
13	001101	18*	9,41(32)*	9,25,41,57(16,32)*	
18	010010	——	18,19(1)	————	
19	010011	13*	————	13,29,45,61(16,32)*	
25	011001	19*	13,29(16)*	25,29,57,61(4,32)*	
27	011011	25*	13,45(32)*	41,45,57,61(4,16)*	
29	011101	41*	19,27(8)		
41	101001	——	25,27(2)		
45	101101	27*	25,29(4)*		
57	111001	29*	25,57(32)*		
61	111101	45*	41,45(4)*		
		57*	41,57(16)*		
		61*	29,61(32)*		
			45,61(16)*		
			57,61(4)*		

Prime Implicants	ABCDEF		
6	000110	$A'B'C'DEF'$	= a
18,19(1)	01001-	$A'BC'D'E$	= b
19,27(8)	01-011	$A'BD'EF$	= c
25,27(4)	0110-1	$A'BCD'F$	= d
9,13,25,29,41,45,57,61(4,16,32)	--1-01	$CE'F$	= e

Prime implicant table

	6	9	13	18	19	25	27	29	41	45	57	61
a 6	x											
b 18,19				x	x							
c 19,27					x		x					
d 25,27						x	x					
e 9,13,25,29,41,45,57,61		x	x			x		x	x	x	x	x
covered by essential	+	+	+	+	+	+		+	+	+	+	+

a, b, e are essential. Also require c or d.

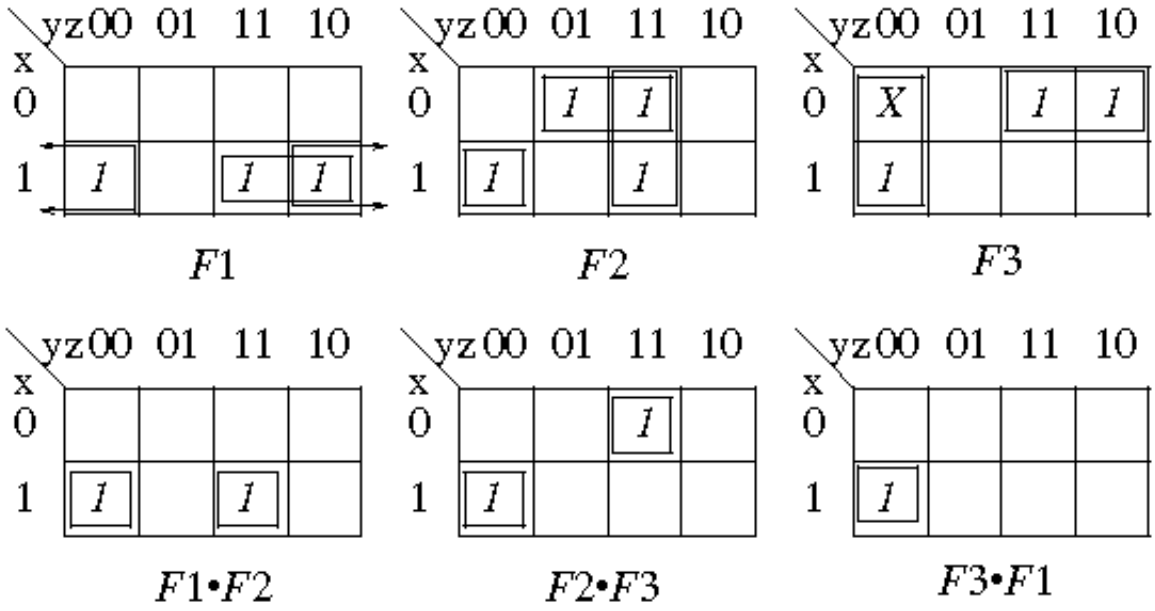
$$P = A'B'C'DEF' + A'BC'D'E + CE'F + \begin{matrix} A'BD'EF \\ \text{or} \\ A'BCD'F \end{matrix}$$

4 Develop a two-level NAND circuit that generates $F1$, $F2$ and $F3$ using the minimum number of gates. Assume the true and complemented values of all variables are available.

$F1 = \sum(4, 6, 7)$

$F2 = \sum(1, 3, 4, 7)$

$F3 = \sum(2, 3, 4) + d(0)$



Prime implicants:

$F1: xy, xz'$ $F2: xy'z', x'z, yz$ $F3: y'z', x'y$

Shared implicants:

$F1 \bullet F2: xy'z', xyz$ $F2 \bullet F3: xy'z', x'yz$ $F3 \bullet F1: xy'z'$

The shared implicant $xy'z'$ is common to all three functions.

Prime implicant table (chart):

		F1			F2				F3			
		4	6	7	1	3	4	7	2	3	4	
all	$xy'z'$	x					x				x	essential
$F1 \bullet F2$	xyz			x				x				A
$F2 \bullet F3$	$x'yz$					x				x		B
F1	xy		x	x								C
	xz'	x	x									D
F2	$x'z$				x	x						essential
	yz					x		x				E
F3	$y'z'$										x	F
	$x'y$								x	x		essential
		+			+	+	+		+	+	+	covered
require		$(C+D)$					$(A+E)$					
		$(A+C)$										

Required

$$= (C+D)(A+C)(A+E) = (C+AD)(A+E)$$

$$= CA + CE + AD + ADE = AC + CE + AD$$

AC - 3-input gate, 2-input gate
AD - 3-input gate, 2-input gate
CE - two 2-input gates → simplest

$$F1 = xy'z' + xy$$

$$F2 = xy'z' + x'z + yz$$

$$F3 = xy'z' + x'y$$