Assignment #4 E & CE 223

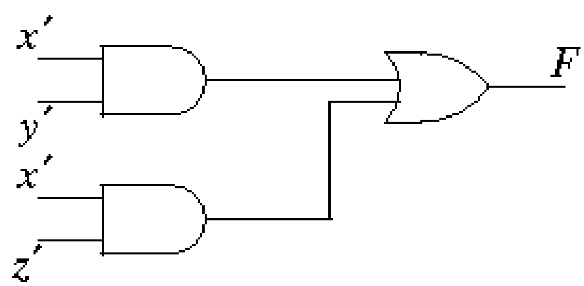
E&CE 223
Assignment 4 - Solutions

1. Mano 4.1, 4.19

Mano 4.1

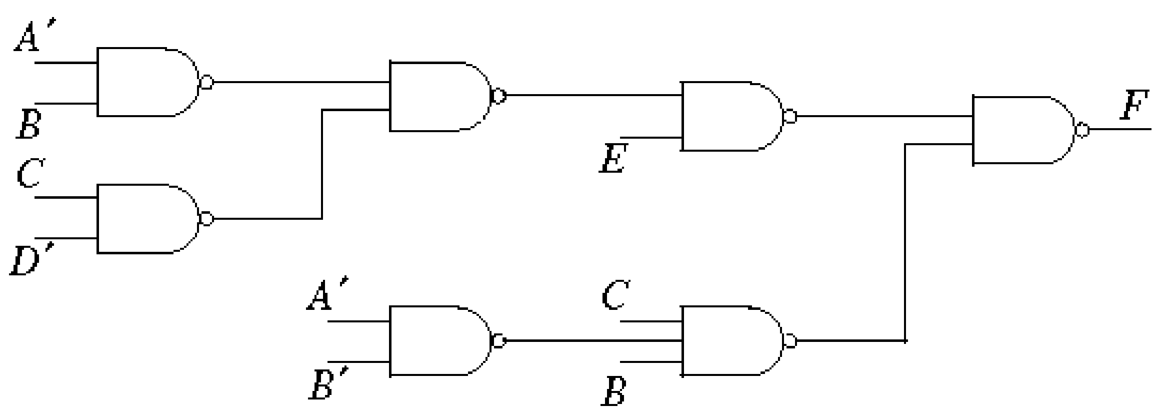
xyz	F
000	1
001	1
010	1
011	0
100	0
101	0
110	0
111	0

x \ yz	00	01	11	10
0	1	1		1
1				

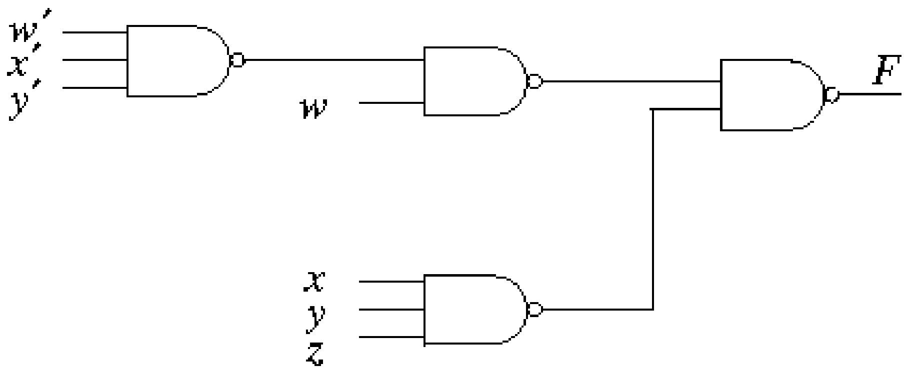


Mano 4.19

(a) $F = (A'B+CD')E + BC(A+B)$



(b) $F = w(x+y+z) + xyz$



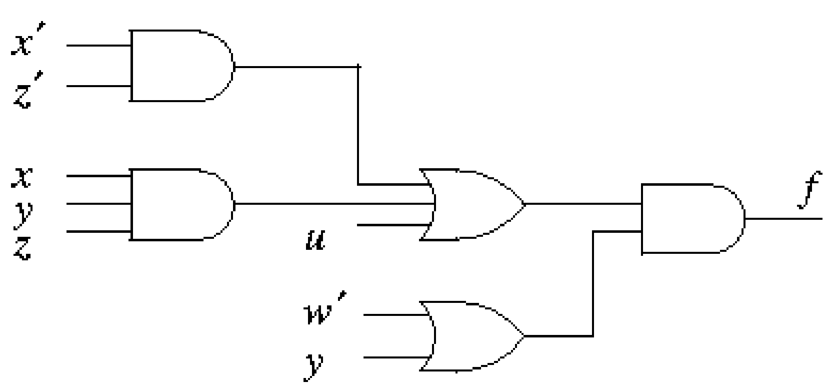
2. Using AND and OR gates find the circuit that minimizes the number of gate inputs for the function:

$$f(u,w,x,y,z) = uy + uw' + xyz + x'yz' + w'x'z'$$

[2 levels, 6 gates, 18 inputs]

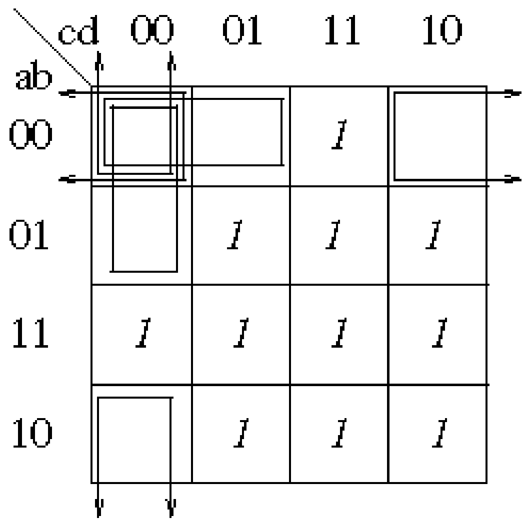
$$\begin{aligned} f(u,w,x,y,z) &= w'(u + x'z') + y(u+x'z') + xyz \\ &= w'(u+x'z') + y(u+x'z') + xy(y+w')z \\ &= w'(u + x'z'+xyz) + y(u+x'z'+xyz) \\ &= (w'+y)(u+x'z'+xyz) \end{aligned}$$

[3 levels, 5 gates 12 inputs]



3. A function $F(a,b,c,d)$ is 1 if more that one of its inputs is 1.

(a) Write the maxterm expression for $F(a,b,c,d)$



$F(a,b,c,d) = \prod(0, 1, 2, 4, 8)$

(b) Find the minimum gate three level (AND-OR-AND) circuit.

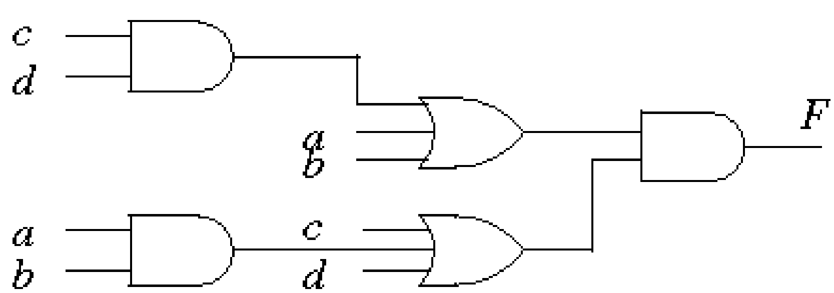
Start with product-of-sums

$F' = a'b'c' + a'b'd' + a'c'd' + b'c'd'$

$F = (a+b+c)(a+b+d)(a+c+d)(b+c+d)$

$= ([a+b]+c)([a+b]+d)(a+[c+d])(b+[c+d])$

$= ([a+b]+cd)(ab+[c+d])$



[3 levels, 5 gates, 12 inputs]

4. Realize the following functions using only 2-input NAND gates. Repeat for 2-input NOR gates.

NAND

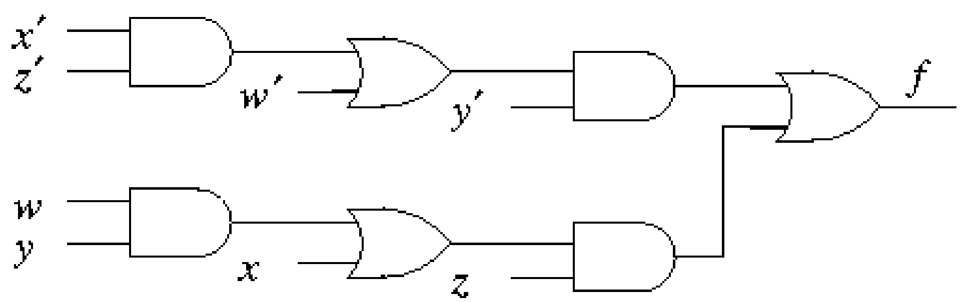
(a) $f(w,x,y,z) = w'y' + wyz + x'y'z' + xz$

$$= y'(w'+x'z') + z(wy+x)$$

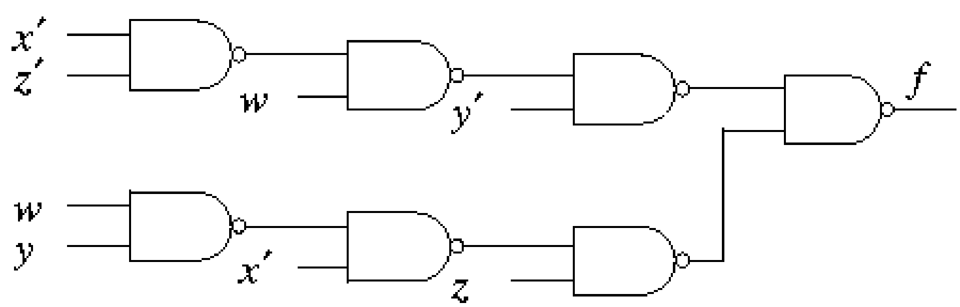
[now using DeMorgan's theorem]

$$= (\{y'(w[x'z']')'\}'\{z([wy]'x')'\}')'$$

from which the NAND circuit below can be drawn or (more simply) draw AND-OR circuit



insert inverter bubbles and replace with NANDs

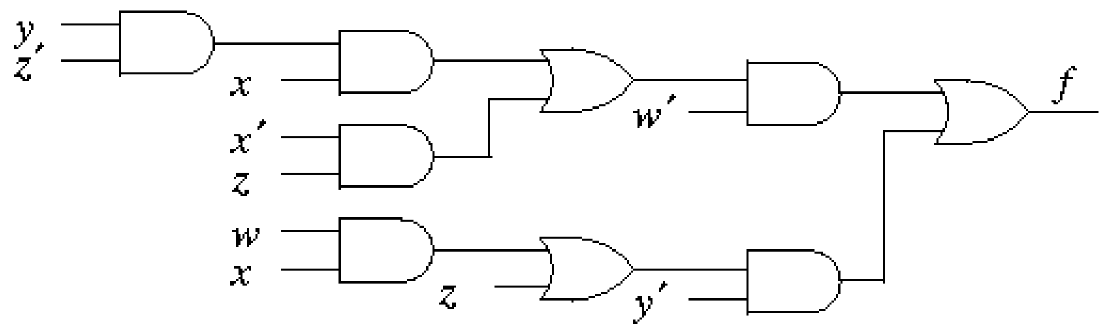


NB: Note inputs to third level have been complemented compared to AND-OR-AND-OR circuit.

(b) A straight forward factoring would proceed as follows:

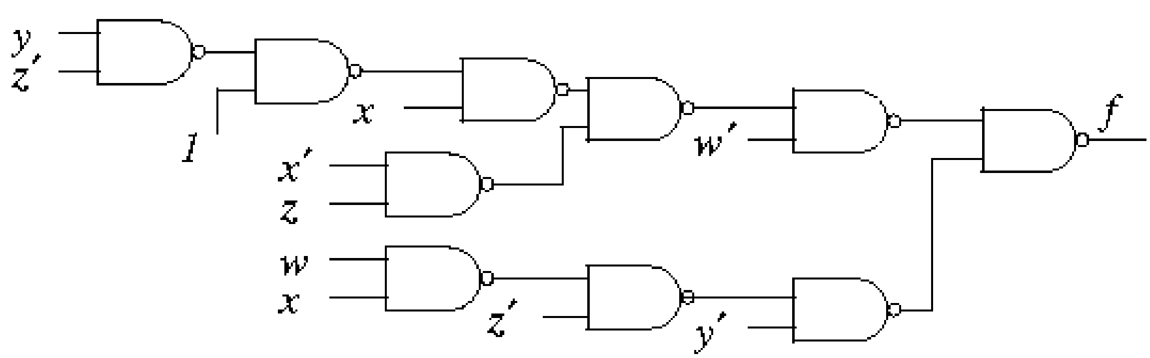
$$f(w,x,y,z) = w'x'z + w'xyz' + wxy' + y'z$$
$$= w'[x'z + xyz'] + y'[wx+z]$$
$$= w'[x'z+x(yz')] + y'[wx+z]$$

Note: 3-input AND was split into two 2-input ANDS



Nine 2-input ANDs required (5 levels).

Insert inverter bubbles and replace gates with NAND. An extra gate is required in the AND-AND sequence.



Ten 2-input NAND gates required (6 levels).

A better factoring is suggested by inspecting the Karnaugh map.

wx \ yz	00	01	11	10
00		*1*	*1*	
01		*1*		*1*
11	*1*	*1*		
10		*1*		

There are four minterms in XOR patterns. The following algebra obtains those XOR expressions.

$$
\begin{aligned}
f(w,x,y,z) &= w'x'z + w'xyz' + wxy' + y'z \\
&= w'x'z(y+y') + w'xyz' + wxy'(z+z') + y'z \\
&= w'x'yz + w'xyz' + wxy'z' + wx'y'z + y'z \\
&= w'y(x'z+xz') + wy'(xz'+x'z) + y'z \\
&= (w'y+wy')(xz'+x'z) + y'z
\end{aligned}
$$

The above factoring can be visualized by "factoring" the Karnaugh map:

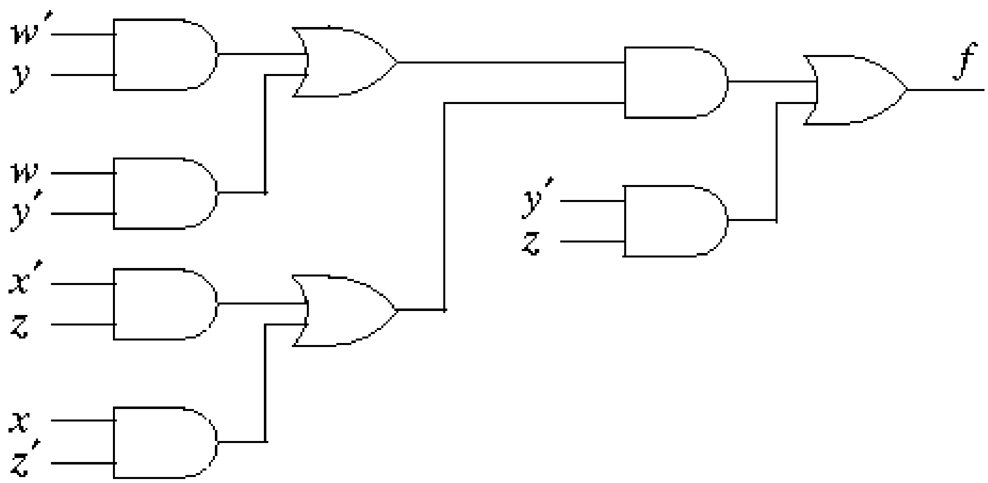
		1	*1*
		1	*1*
1	*1*		
1	*1*		

•

	1	*1*	
1			*1*
1			*1*
	1	*1*	

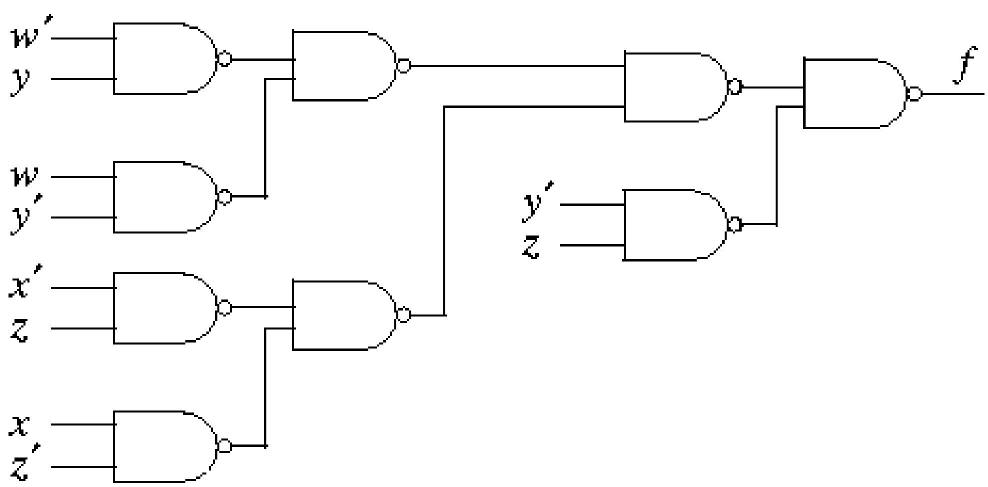
+

	1		
	1		
	1		
	1		



Nine 2-input gates required (4 levels).

Inserting inverter bubbles and converting to NAND gates yields:



Nine 2-input NAND gates required.

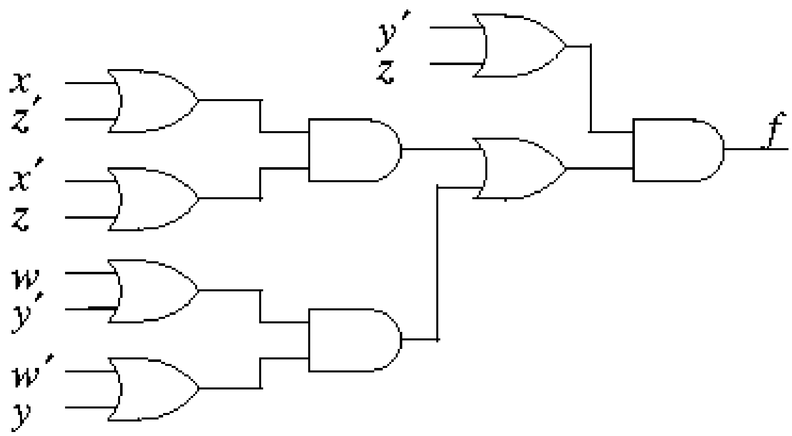
NOR

(a) $f(w,x,y,z) = w'y' + wyz + x'y'z' + xz$

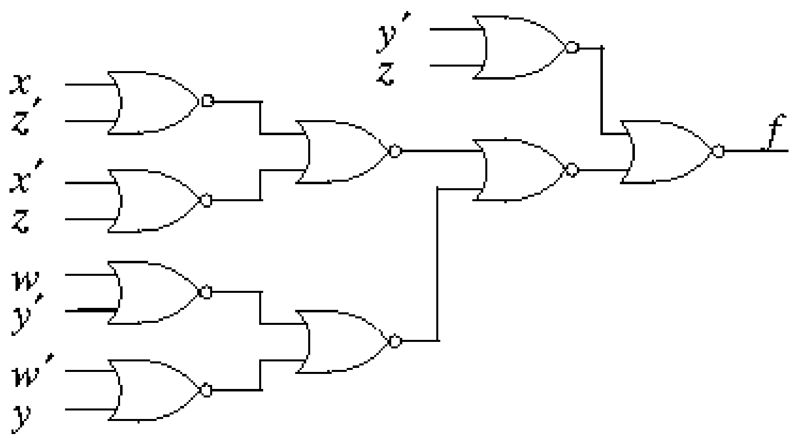
wx \ yz	00	01	11	10
00	1	1		
01	1	1	1	
11		1	1	
10	1		1	

$$f'(w,x,y,z) = yz' + w'x'y + wxz' + wx'y'z$$

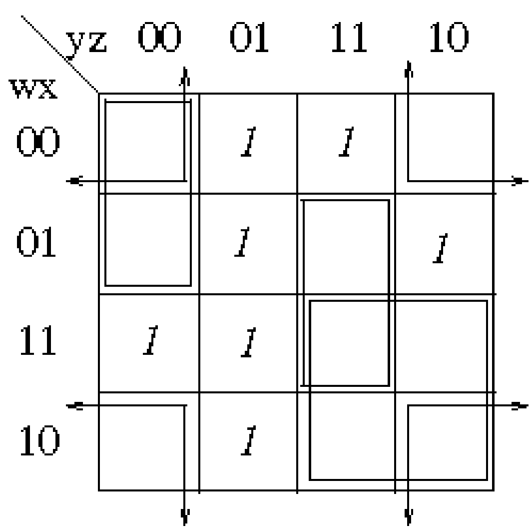
$$\begin{aligned}
f(w,x,y,z) &= (y'+z)(w+x+y')(w'+x'+z)(w'+x+y+z') \\
&= (y'+z)(w+x+y'+z')(w+x+y'+z) \\
&\quad\ (w'+x'+z+y')(w'+x'+z+y)(w'+x+y+z') \\
&= (y'+z)(w+x+y'+z')(w'+x'+z+y)(w'+x+y+z') \\
&= (y'+z)(x+z'+[w+y'][w'+y])(w'+x'+z+y) \\
&= (y'+z)(x+z'+[w+y'][w'+y])(w'+x'+z+y)(w+x'+z+y') \\
&= (y'+z)(x+z'+[w+y'][w'+y])(x'+z+[w+y'][w'+y]) \\
&= (y'+z)([x+z'][x'+z]+[w+y'][w'+y])
\end{aligned}$$



Insert inverter bubbles and convert to NOR gates:



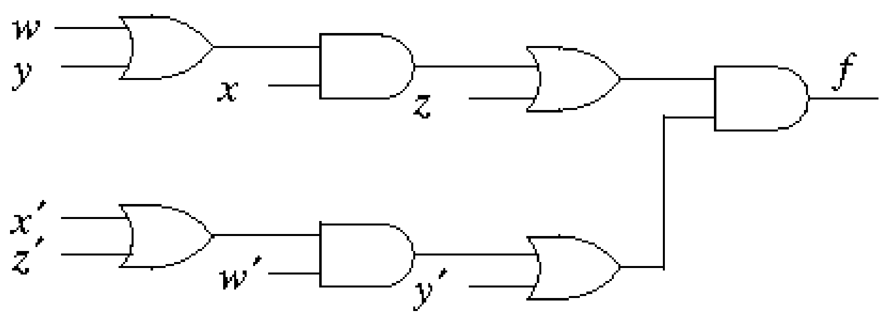
(b) $f(w,x,y,z) = w'x'z + w'xyz' + wxy' + y'z$



$$f'(w,x,y,z) = wy + x'z' + w'y'z' + xyz$$

$$f(w,x,y,z) = (w'+y')(x+z)(w+y+z)(x'+y'+z')$$

$$= [w'(x'+z')+y'][x(w+y)+z]$$



Insert inverter bubbles and convert to NOR gates: