University of Waterloo
Department of Electrical & Computer Engineering
E&CE 223 Digital Circuits and Systems
Midterm Examination
Instructor: M. Sachdev
October 23rd, 2007

Total Time = 90 Minutes, Total Marks = 50

Student Name:			**Student ID:**		
1 **/7**	**2** **/8**	**3** **/18**	**4** **/10**	**5** **/7**	**Total** **/50**

Attempt all problems. Show all work. If information appears to be missing make a reasonable assumption, state it, and proceed. Calculators are not needed and are not allowed.

Problem 1 [7 Marks]

(a) Convert $(853)_{10}$ to Excess-3 and 2421 code. [**2 Mark**]

$(1110\ 0101\ 0011)_{2421}$

$(1110\ 1011\ 1001)_{2421}$

(Note: There are multiple solutions to the 2421 code)

$(1011\ 1000\ 0110)_{Excess\text{-}3}$

(b) For BCD addition, explain with two examples when and why it is necessary to add 6 to obtain the correct answer. [**2 Marks**]

The addition of $6 = (0110)_2$ to the binary sum converts it to the correct digit and produces a carry as required 16-10=6.

4	0100	4	0100
5	0101	8	1000
9	1001	12	1100
			0110
			10010

(c) Convert $(10\ 0110\ 1000\ 0111\ 1001\ 0011.11011)_2$ to base 8 format. [**2 Marks**]

Organize the bits in group of three

$(11503623.66)_8$

(d) **True of False**: Even function can be used to implement odd parity checker and generator, and an XOR gate implements an even function. [**1 Mark**]

False: Even function is used to implement odd parity checker and generator. However, an XNOR gate implements an even function, not an XOR gate

Problem 2 [8 Marks]

(a) **Do not use K-Map!** Simplify the following expression to a minimum number of literals by algebraic means. [**3 Marks**]

$$
\begin{aligned}
F &= a \oplus b \oplus c + a'b'c' + a'bc + ab'c + abc' \\
&= a \oplus b \oplus c + (a'b' + ab)c' + (a'b + ab')c \\
&= a \oplus b \oplus c + (a \oplus b)'c' + (a \oplus b)c \\
&= a \oplus b \oplus c + (a \oplus b \oplus c)' \\
&= 1
\end{aligned}
$$

(b) Find $(-75)_{10} - (20)_{10}$ using 8-bit signed 2's complement form. Express you answer in signed 2's complement format. Show all your work! [**5 Marks**]

$(-75)_{10}$ = (10110101) in Signed and 2's complement

$(-20)_{10}$ = (11101100) in Signed and 2's complement

$$
\begin{array}{r}
10110101 \\
\underline{11101100} \\
110100001
\end{array}
$$

Discard the carry, the final answer is 10100001 in Signed and 2's complement format.

Problem 3 [18 Marks]

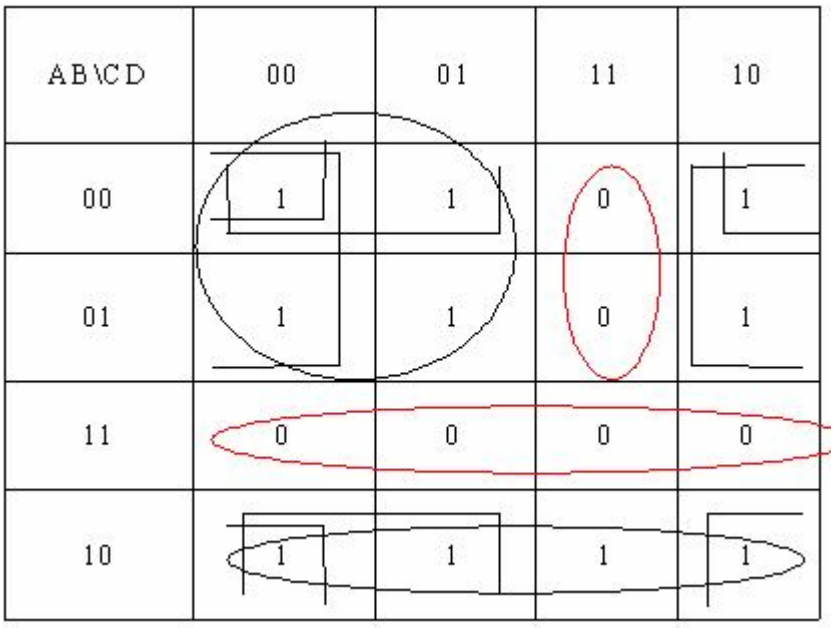
$$F(A, B, C, D) = \prod (3, 7, 12, 13, 14, 15)$$

(a) Express the complement of the above expression in both canonical *sum of products* and *product of sums* form. [**2 Marks**]

$$F'(A, B, C, D) = \sum (3, 7, 12, 13, 14, 15)$$

$$F'(A, B, C, D) = \prod (0, 1, 2, 4, 5, 6, 8, 9, 10, 11)$$

(b) Simplify the expression F in (i) sum of products and (ii) product of sums forms. [**6 Marks**]



$$F = AB' + A'C' + A'D'$$

$$F' = AB + A'CD$$

$$F = (A' + B')(A + C' + D')$$

(c) List all the prime implicants and essential prime implicants. Show all your work! [**6 Marks**]

	0	1	2	4	5	6	8	9	10	11
(0, 1, 8, 9) (B'C')	Δ	Δ					Δ	Δ		
(0, 2, 4, 6) (A'D')	Δ		Δ	Δ		Δ				
(0, 1, 4, 5) (A'C')	Δ	Δ		Δ	Δ					
(0, 2, 8, 10) (B'D')	Δ		Δ				Δ		Δ	
(8, 9, 10, 11) (AB')							Δ	Δ	Δ	Δ

Prime Implicants: (0, 1, 8, 9), (0, 2, 4, 6), (0, 1, 4, 5), (0, 2, 8, 10), (8, 9, 10, 11)

Essential Prime Implicants: (0, 1, 4, 5), (0, 2, 4, 6), (8, 9, 10, 11)

(d) P (A, B, C, D) = ∑ (0, 1, 2, 5, 8, 9, 10)

Based on the K-Map provided below, fill out the K-map provided below for expression P. Is it possible to simplify the expression in terms of product of sums directly from the K-Map without finding the complement of P? You may use the example in this question to explain your answer. [**4 Marks**]

AB\CD	11	10	00	01
11	0	0	0	0
10	0	1	1	1
00	0	1	1	1
01	0	0	0	1

Yes, it is possible.

- Combine all the zeroes
- Look at the corresponding variables.
- Write out the product of sums in terms the complement of the corresponding variables.
    - For example, combine all the zeroes in the row where AB is equal to "11"; the corresponding maxterm is then (A'+B')
    - Combine all the zeroes in the column where CD is equal to "11"; the corresponding maxterm is then (C'+D')

$P = (A'+B)(C'+D')(B'+D)$

Problem 4 [10 marks]

You are asked to design a digital system with the following characteristics and criteria. There are four inputs (A, B, C, D) and one output (Y) to the system. The output *Y* is true (i.e. logic "1") when ABCD is greater than 7 and the binary input value is odd. When the binary value of ABCD is less than 7, output *Y* follows the result of the expression $B \oplus C$. You can assume A is the most significant digit and D is the least significant digit.

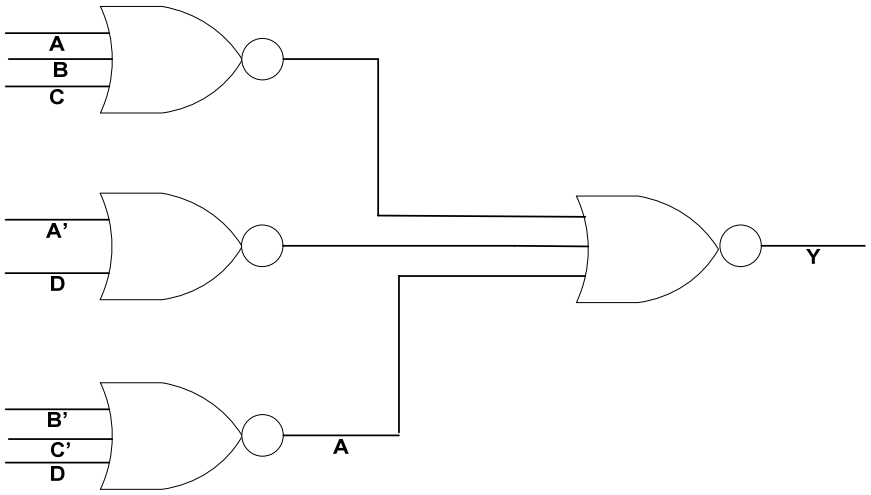
(a) Complete the truth table provided below [**4 Marks**]

A	B	C	D	Y
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	0
0	1	1	1	x
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	1

(b) Implement the digital system described above using two-level NOR gates. You may assume both the input and its complement form are available. [**6 Marks**]

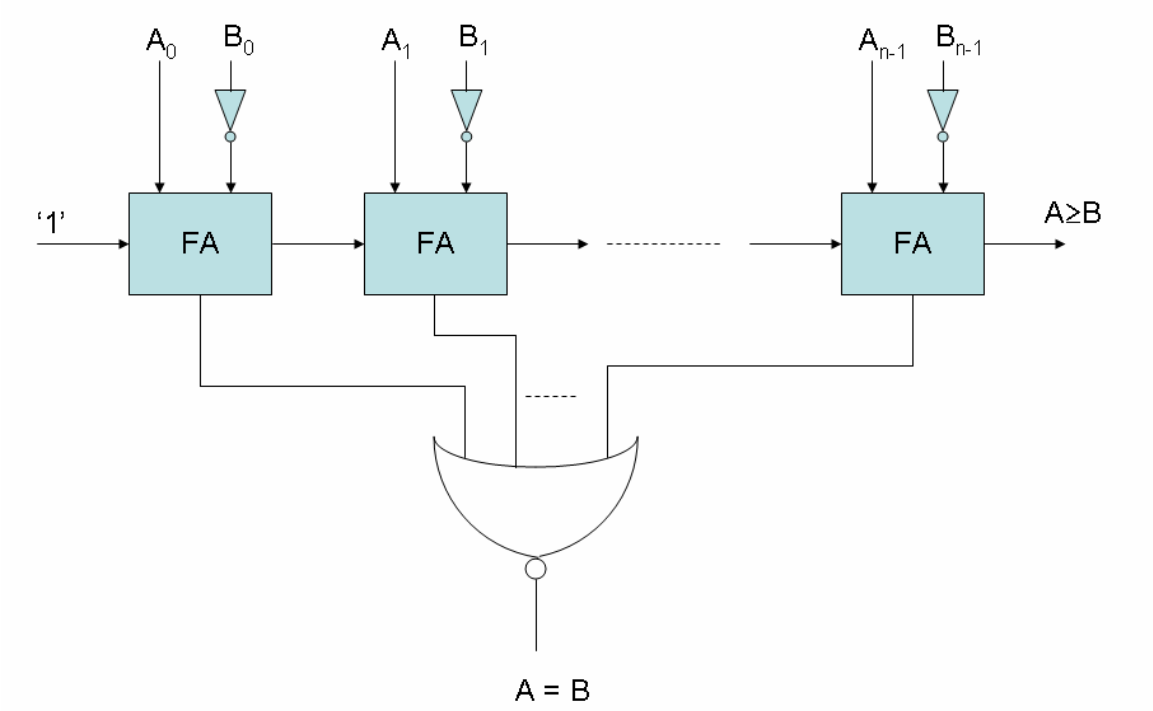
AB\CD	00	01	11	10
00	0	0	1	1
01	1	1	x	0
11	0	1	1	0
10	0	1	1	0

$$Y = (A + B + C)(A'+D)(B'+C'+D)$$
$$= [(A + B + C)'+(A'+D)'+(B'+C'+D)']'$$



Problem 5 [7 Marks]

(a) With the addition of few logic gates, an *n*-bit ripple carry adder can be used to compare the magnitude of two n-bit unsigned numbers. For vectors $A=a_3a_2a_1a_0$ and $B=b_3b_2b_1b_0$, design a 4-bit digital circuit with appropriate logic gates and adder block diagrams with output *X* such that *X* is true when $A = B$. You may assume both A and B are unsigned numbers. Specify the value of any required control signals. [**5 Marks**]



(b) Modify your design slightly to include another output *Y* such that *Y* is true when $A \geq B$. [**2 Marks**]

See above diagram