University of Waterloo
Department of Electrical and Computer Engineering
ECE 223 Digital Circuits and Systems

Solutions

Midterm Examination   Instructor: M. Sachdev   Date Feb 16, 2000

Total Marks = 100

Name:...				Student id:..			
1.		2.	3.	4.		5.	Total:

Attempt all problems. If information appears to be missing make a reasonable assumption, state it and proceed. Calculators are not needed and are not allowed

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Problem 1

(A): Convert following number from one radix to another [**12**]

(i) $(212.785)_{10}$ to radix 7

Integral part

Integer	**Remainder**	**Coefficient**
212		
30	**2**	**a0**
4	**2**	**a1**
0	**4**	**a2**

Fractional part

		Integer	**Fraction**	**Coefficient**
0.785x7	**=**	**5**	**0.495**	**a-1**
0.495x7	**=**	**3**	**0.465**	**a-2**
0.465x7	**=**	**3**	**0.255**	**a-3**
0.255x7	**=**	**1**	**0.785**	**a-4**

the number is $\mathbf{(422.5331)_7}$

(ii) $(0110\ 1111)_2$ to radix 8

the binary number is first converted into radix 10

$$0x2^7 + 1x2^6 + 1x2^5 + 0x2^4 + 1x2^3 + 1x2^2 + 1x2^1 + 1x2^0 = (111)_{10}$$

Integer	**Remainder**	**Coefficient**	
111			
13	**7**	**a0**	
1	**5**	**a1**	
0	**1**	**a2**	**hence, the number is** $\mathbf{(157)_8}$

Name:.. Student id:..

(B): How many different ways signed numbers can be represented in computers? Name them. Explain why in modern computers signed binary numbers are represented in “2’s complement” format? [**8**]

Signed binary numbers can be represented in three forms in digital computers

(a) Sign and magnitude
(b) Signed 1’s complement
(c) Signed 2’s complement

The sign and magnitude as well as 1’s complement forms have two representations for 0 which is not desirable. The signed 2’s complement has only 1 representation for 0. Therefore, it is commonly used.
Both complement methods can use adders for performing subtractions which simplify and speedup the arithmetic operations compared to sign and magnitude representation. However, in 1’s complement subtraction, end-round carry is required which makes subtraction slower. Therefore, 2’s complement is preferred representation.

Name:... Student id:..

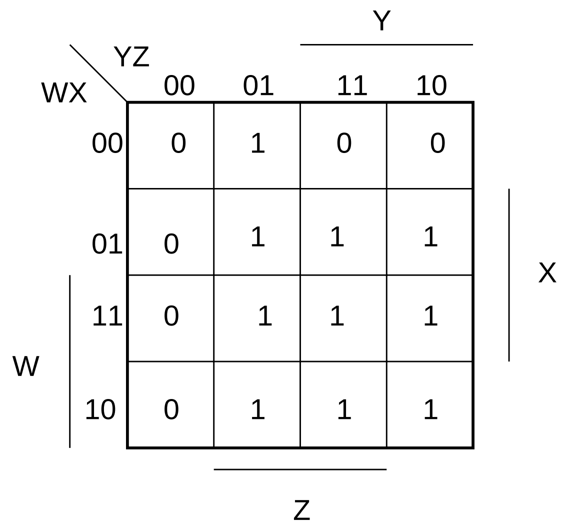
Problem 2

Given the following Boolean function, F = xy’z + x’y’z + w’xy + wx’y + wxy

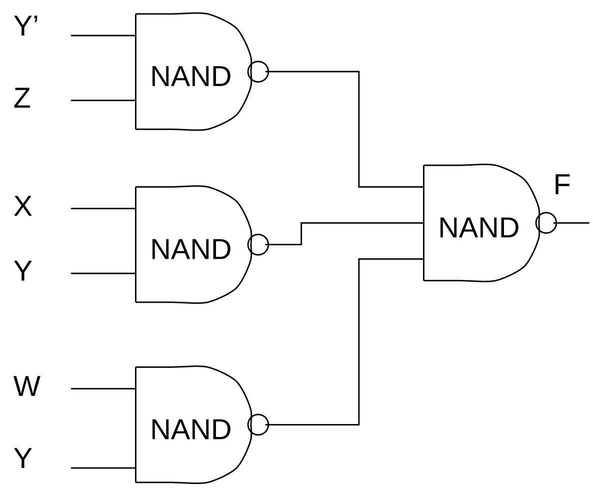
(i) Draw a corresponding Karnaugh map of the function

(ii) Give minterm and maxterm expressions

(iii) Simplify the function and implement it by NAND gates only [**20**]



(i)



(ii)

(ii) Minterms, or sum of products

F = w’x’y’z
+ w’xy’z + w’xyz + w’xyz’
+ wxy’z + wxyz + wxyz’
+ wx’y’z + wx’yz + wx’yz’

Maxterms, of product of sums

F = (w+x+y+z).(w+x+y’+z’).(w+x+y’+z)
.(w+x’+y+z)
.(w’+x’+y+z)
.(w’+x+y+z)

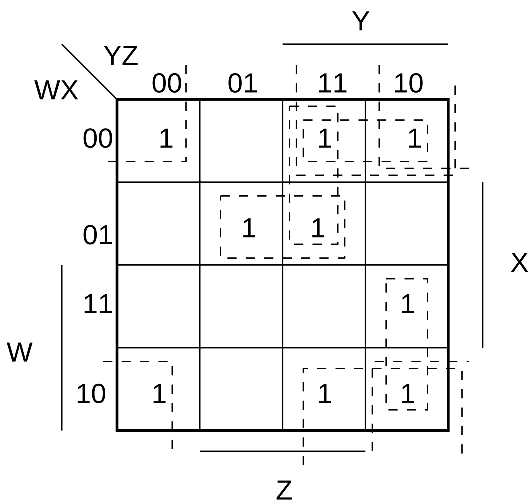
(iii) F = xy’z + x’y’z + w’xy + wx’y + wxy
= y’z(x+x’) + xy(w+w’) + wy(x+x’)
= y’z + xy + wy
= {(y’z)’.(xy)’.(wy)’}’

Name:.. Student id:..

Problem 3

Given the function F (w, x, y, z) = Σ(0, 2, 3, 5, 7, 8, 10, 11, 14) [**20**]

(i) Find all prime implicants
(ii) Find all essential prime implicants
(iii) Represent the function in the simplest “sum of products” form



prime imp.	0	2	3	5	7	8	10	11	14
0,2,8,10	x	x				x	x		
2,3,10,11		x	x				x	x	
3,7			x		x				
5,7				x	x				
10,14							x		x

From the chart, the prime implicants are
(0,2,8,10), (2,3,10,11), (3,7), (5,7), and (10,14)

The essential prime implicants are
0,2,8,10), (2,3,10,11), (5,7), and (10,14)

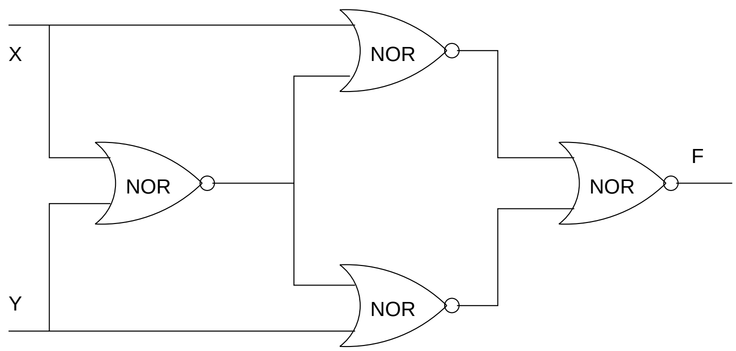
From the chart it is evident that all essential prime implicants are sufficient to represent the function. Therefore, the simplified function can be represented in the sum of product, as

F = X’Z’ + X’Y + WYZ’ + W’XZ

Name:.. Student id:..

Problem 4

(A): Verify algebraically that the circuit shown below generates the "exclusive-NOR" function **[10]**



F = (A+B)' = A'.B'

A = (X + (X+Y)')' hence A' = X + (X+Y)'

and B = (Y + (X+Y)')' hence B' = Y + (X+Y)'

therefore, F = {X + (X+Y)'}.{Y + (X+Y)'}

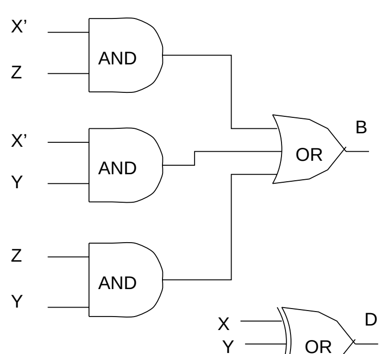
= (X+Y)' + XY = X'Y' + XY = exclusive NOR

(B): A "full subtractor" performs [(x-y) - z] operation. [**10**]

(i) Generate the truth table of the full subtractor

(ii) Implement the function with logic gates

x	y	z	B	D
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	1	0
1	0	0	0	1
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1



(ii)

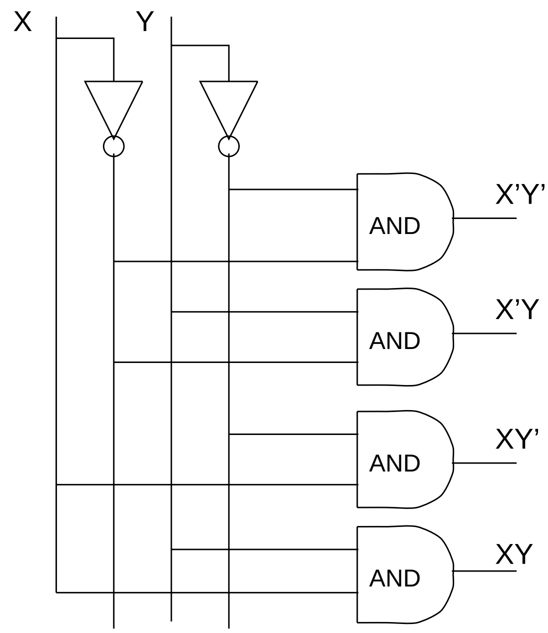
D = X'Y'Z + X'YZ' + XY'Z' + XYZ = X exOR Y exOR Z

B = X'Y'Z + X'YZ' + X'YZ + XYZ = X'Z + X'Y + YZ

Name:.. Student id:..

Problem 5

(A): Draw a gate level diagram of a 2-to-4 decoder **[8]**



(B): Redefine the carry propagate and carry generate as follows: **[12]**

$$P_i = A_i + B_i \quad \textbf{and} \quad G_i = A_i.B_i$$

Show that the output carry and output sum of a full adder becomes

$$C_{i+1} = (C_i'G_i' + P_i')' = G_i + P_i.C_i$$

$$S_i = (P_i G'_i) \oplus C_i$$

Let $F = (C_i'G_i' + P_i')' = (C_i'G_i')'.P_i' = (C_i + G_i)P_i = G_i P_i + P_i.C_i$

$G_i P_i = A_i.B_i(A_i + B_i) = A_i.B_i = G_i$

Therefore, $F = G_i + P_i.C_i$

$= A_i.B_i + C_i.(A_i+B_i) = A_i.B_i + A_i.C_i + B_i.C_i = C_{i+1}$

and $Si = (A_i + B_i).(A_i.B_i)' = (A_i + B_i).(A_i'+B_i') = A_i.B_i' + A_i'.B_i$

$= A_i \text{ xor } B_i$

Therefore, $(P_i G'_i) \oplus C_i = A_i \oplus B_i \oplus C_i = S_i$