Performance tradeoffs offered by beamforming in cognitive radio systems: an analytic approach

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***Abstract*—This paper studies the design of beamforming weights for a multi-antenna secondary transmitter in an underlay cognitive setting that simultaneously maximizes the secondary received-power while limiting the primary interference to some threshold ϵ. With perfect channel state information (CSI), a closed-form expression for the maximum secondary received-power is found. Under imperfect CSI and when the beamforming weights are computed using the channel estimates, the actual secondary received-power, G, and the actual primary interference-power, I, are derived. We show that the mean $\mathsf{E}[G]$ has a term that grows linearly with the number of secondary antennas, N, and additional terms dependent on ϵ. Consequently, we obtain tradeoffs between $\mathsf{E}[G]$ and ϵ. Under perfect CSI, we show that small increases in ϵ from zero lead to moderate enhancements in $\mathsf{E}[G]$ for small N. However, increasing N reduces the enhancements. Under imperfect CSI, the gain in $\mathsf{E}[G]$ is less compared to the perfect CSI case. Furthermore, we show that the dominant parts of $\mathsf{E}[I]$ are independent of N. Thus, we conclude that there is no significant loss for the secondary to perform null-steering beamforming instead. Moreover, it can employ additional antennas to improve $\mathsf{E}[G]$ without generating significant extra interference on the primary.**

***Index Terms*—Underlay cognitive radio, Multiple-input single-output, Beamforming, Null-steering, Estimation error.**

I. Introduction

Radio spectrum occupancy measurements indicate that fixed spectrum licensing policy has failed to accommodate wireless services in an efficient manner, and has lead to an under-utilization of frequency bands [1], [2]. On the other hand, unlicensed wireless users and applications have a rapidly growing demand for bandwidth. This inspires the idea of cognitive radio systems, which was first introduced in [3]. A cognitive radio system allows the unlicensed (secondary) users to access the licensed bands under the condition that the interference on the licensed (primary) system is controlled and thus its quality of service (QoS) remains satisfactory.

Spectrum underlay, overlay, and interweave are three basic operation models for cognitive radio systems that have been under significant developments throughout the recent decade [4]-[8]. Spectrum underlay focuses on the scenario in which both secondary and primary systems operate in the same frequency bands simultaneously [4]. The secondary system aims to maximize its own received-power while imposing minimal co-channel interference on the primary receivers.

A widely recognized approach to enhance the secondary system's performance in underlay cognitive systems is to exploit spatial diversity by using multiple antennas at the secondary transmitter [9]-[19]. If the secondary receiver has a single antenna, this leads to a multiple-input single-output (MISO) channel. In such systems, with a secondary transmit power constraint, the transmit covariance matrix at the secondary transmitter can be chosen to satisfy the primary interference-power constraints while maximizing the secondary received-power. As [9], [10], and [11] show, the optimal covariance matrix in a MISO channel is rank-one which implies that beamforming is optimal. Therefore, taking advantage of small-scale channel fading, the secondary transmitter should set the beamforming weights such that the received signals from different transmit antennas combine destructively at the primary receiver and constructively at the secondary receiver.

Beamforming has been broadly investigated in the literature as a technique to provide performance enhancements in MISO cognitive settings. Under perfect channel state information (CSI), [11] and [12] derive optimal solutions to their associated optimization problems and subsequently evaluate the performance of primary and secondary systems analytically, whereas [13] takes a numerical approach to the same problem. In the realistic case of imperfect CSI, where the channel gains are not perfectly known at the beamformer, [14] and [15] consider probabilistic interference constraints for the primary system and subsequently study robust beamforming numerically. In [16] and [17], even though the actual channel gains are unknown at the beamformer, the knowledge of some uncertainty regions containing the actual gains is assumed to be available. Under such assumptions, [16] solves the associated problem numerically while [17] solves it analytically. In this paper, we take a different approach to performance analysis under imperfect CSI. Unlike [16] and [17], we do not assume that uncertainty regions containing the actual gains are known at the beamformer. Thus, an optimal beamforming vector is found merely based on the estimates of the channel gains.

In cognitive settings, obtaining closed-form expressions for performance metrics such as maximum secondary received-power and primary interference-power are undoubtedly crucial, specifically in the realistic case of imperfect CSI. Such expressions can provide insightful performance tradeoffs and knowledge on the impact of different parameters (such as the estimation error's variance) on the performance of secondary and primary systems. This paper aims to obtain such closed-form results in both cases of perfect and imperfect CSI. A key contribution of this paper is to introduce an alternative analytic approach to optimal beamforming in the perfect CSI case which is also applicable to the case of imperfect CSI.

In our previous study in [18], null-steering beamforming is addressed for a MISO cognitive setting. Null-steering refers to the case when the primary system tolerates no interference from the secondary and thus the secondary system must nullify its co-channel interference at the primary receivers. Subsequently, under this condition, the maximum secondary received-power is derived in a closed-form expression and its mean value is found to grow linearly with the number of secondary antennas.

Intuitively, as the primary interference constraint is relaxed and a small nonzero interference on the primary receiver is permitted, a higher secondary received-power is anticipated. In this paper, we are interested in finding such a tradeoff between the primary interference-power and the mean secondary received-power. We consider a MISO underlay cognitive setting with P as the maximum secondary transmit power and a small threshold $\epsilon = \alpha P$ ($\alpha \geq 0$) as the maximum interference tolerable at the primary receiver. First, we examine the case of perfect CSI and find the maximum secondary received-power while the primary interference-power is limited to the threshold ϵ. In the realistic case of imperfect CSI, we assume that the secondary transmitter ignores or is ignorant of the existence of estimation errors. Therefore, beamforming wights are computed merely by using the channel estimates. Subsequently, the actual secondary received-power, G, and the actual primary interference-power, I, are derived. Our results in this paper indicate that:

- $\mathsf{E}[G]$ consists of a term independent of α which is referred to as the *null-steering result* in the sequel. This term grows linearly with the number of secondary transmit antennas, N, (due to spatial diversity). Therefore, employing more transmit antennas leads to a better performance for the secondary system.
- $\mathsf{E}[G]$ has additional terms dependent on α with a dominant term that grows as $\sqrt{\alpha}$. Therefore, we can obtain the following tradeoffs between $\mathsf{E}[G]$ and α:
  - Under perfect CSI, an small increase in α can result in a moderate improvement in $\mathsf{E}[G]$ when N is small. Particularly, increasing α from zero to 0.1 leads to an increase of 31% for $\mathsf{E}[G]$ when $N = 2$. However, the amount of increase reduces as N grows larger.
  - Under imperfect CSI, the enhancement in $\mathsf{E}[G]$ is less compared to the perfect CSI case.
- Under imperfect CSI, the dominant parts of $\mathsf{E}[I]$ are independent of N. This implies that while employing additional antennas at the secondary transmitter can benefit the secondary link by increasing $\mathsf{E}[G]$, it results in little extra interference on the primary receiver. Furthermore, since the primary's allowance for a small nonzero interference threshold results in no significant enhancement to the secondary's performance compared to when the interference threshold is zero ($\alpha = 0$), there is no significant loss for the secondary system to perform null-steering beamforming instead.

The rest of this paper is organized as follows. Our system setting is introduced in Section II. The case of perfect CSI is studied in Section III. In Section IV, we consider the case of imperfect CSI. Finally, we conclude this paper in Section V. Detailed derivations of $\mathsf{E}[I]$ and $\mathsf{E}[G]$ in the case of imperfect CSI are provided in Appendix A and Appendix B respectively.

In the sequel, boldface uppercase and lowercase letters denote matrices and vectors, respectively. Notations $(.)^*$, $(.)^T$, and $(.)^\dagger$ respectively refer to complex conjugate, transpose, and conjugate transpose of a vector or a matrix. For a complex number x, $\arg(x)$ denotes its phase. For a vector $\mathbf{y} = (y_1, y_2, \ldots, y_N)$, we denote $\mathbf{y_{-1}} = (y_2, y_3, \ldots, y_N)$. The notation $=^d$ refers to equality in distribution and $\|.\|$ is the Euclidean norm of a vector. Distribution of a circularly symmetric complex Gaussian (CSCG) vector with mean $\boldsymbol{\mu}$ and covariance matric $\boldsymbol{\Sigma}$ is written as $\mathcal{CN}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. $\mathsf{Tr}(\mathbf{S})$ is the trace of a square matrix $\mathbf{S}$ and $\mathbf{S} \succeq 0$ means that $\mathbf{S}$ is positive semi-definite. For functions $p(x)$ and $q(x)$ defined on some subset of real numbers, we have $p(x) = O(q(x))$ as $x \to 0$ if and only if there exist positive numbers δ and M such that $|p(x)| \leq M|q(x)|$ for $|x| < \delta$.

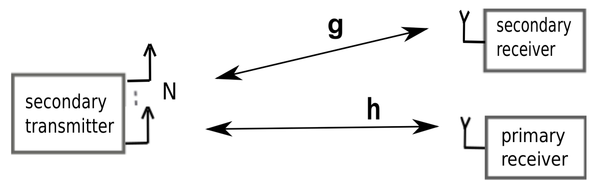

Fig. 1. A secondary system with a multi-antenna transmitter and a single-antenna receiver coexists with a primary system with one single-antenna receiver.

II. System Model

As depicted in Fig. 1, we consider a system setting that consists of a secondary transmitter equipped with N antennas and one single-antenna secondary receiver. This secondary system shares the same frequency band concurrently with a primary system with one single-antenna receiver (underlay cognitive setting) and aims to satisfy a primary interference constraint. Note that we only consider one-way communication from the secondary transmitter to the secondary receiver.

The channel gain between the secondary transmitter and the secondary receiver is denoted by $\mathbf{g} \in \mathbb{C}^{N\times 1}$ where $\mathbf{g} \sim \mathcal{CN}(\mathbf{0}, 2\sigma^2\mathbf{I})$. The channel gain between the secondary transmitter and the primary receiver is denoted by $\mathbf{h} \in \mathbb{C}^{N\times 1}$ where $\mathbf{h} \sim \mathcal{CN}(\mathbf{0}, 2\sigma_I^2\mathbf{I})$. Moreover, the entries of $\mathbf{g}$ are independent from the entries of $\mathbf{h}$. The vectors $\mathbf{g}$ and $\mathbf{h}$ are interchangeably referred to as the secondary channel gain and the primary channel gain respectively throughout the paper. Without loss of generality, we assume $\sigma_I^2 = \sigma^2$.

III. Perfect CSI

In this section, we assume that perfect knowledge of the channel gains is available at the secondary transmitter. We formulate an optimization problem based on our system setting. Then we solve the problem to find the optimal beamforming vector and the maximum secondary received-power.

A. Problem Formulation

We denote the secondary transmitted signal at the discrete time instant n by $\mathbf{s}[n]$. Therefore, the received signal at the secondary (resp. primary) receiver at time instant n can be expressed as $r_s[n] = \mathbf{g}^T\mathbf{s}[n] + w[n]$ (resp. $r_p[n] = \mathbf{h}^T\mathbf{s}[n] + v[n]$), where $w[n]$ (resp. $v[n]$) is the additive noise at the secondary (resp. primary) receiver. Denoting $\mathbf{S} = \mathsf{E}[\mathbf{s}[n]\mathbf{s}^\dagger[n]]$

as the secondary transmitter's covariance matrix, the main optimization problem ($\mathbf{P0}$) can be formulated as

$$G = \text{maximize} \quad \mathbf{g}^T\mathbf{S}\mathbf{g}^* \tag{1}$$
$$\text{subject to:} \quad \mathbf{h}^T\mathbf{S}\mathbf{h}^* \le \epsilon, \tag{2}$$
$$\mathsf{Tr}(\mathbf{S}) \le P, \quad \mathbf{S} \succeq 0. \tag{3}$$

In $\mathbf{P0}$, the optimization (1) is over $\mathbf{S}$, while $\mathbf{g}^T\mathbf{S}\mathbf{g}^*$ is the secondary received-power, $\mathbf{h}^T\mathbf{S}\mathbf{h}^*$ is the primary interference-power, and the secondary transmitted-power is limited to P according to (3).

As shown in [9], [10], and [11], beamforming is optimal to maximize the secondary received-power in MISO channels. Therefore, the optimal solution, $\mathbf{S}$, to the problem $\mathbf{P0}$ is rank-one and we can express $\mathbf{S}$ as $\mathbf{S} = \mathbf{x}\mathbf{x}^\dagger$, where $\mathbf{x} \in \mathbb{C}^{\mathbf{N}\times\mathbf{1}}$ is the beamforming vector. Consequently, by replacing $\mathbf{S} = \mathbf{x}\mathbf{x}^\dagger$ in $\mathbf{P0}$ we obtain the equivalent optimization problem ($\mathbf{P1}$) as

$$G = \text{maximize} \quad |\mathbf{x}^T\mathbf{g}|^2 \tag{4}$$
$$\text{subject to:} \quad |\mathbf{x}^T\mathbf{h}|^2 \le \epsilon, \tag{5}$$
$$\|\mathbf{x}\|^2 \le P. \tag{6}$$

Therefore, we focus on beamforming at the secondary transmitter which aims to make the primary interference-power less than or equal to the threshold ϵ and at the same time maximize the secondary received-power.

It is worth noting that in this paper, beamforming is performed at each discrete time instant n based on the channel knowledge at that instant. Consequently, the interference power constraint (5) is satisfied at each time instant n and thus is characterized as the peak interference power constraint [20]. Alternatively, the average interference power constraint is a long term constraint for the average interference power over all the fading states of the channel [20].

B. Solving the Optimization Problem

To solve $\mathbf{P1}$, we apply a rotation matrix $\mathbf{U} \in \mathbb{C}^{N\times N}$ that rotates the vector $\mathbf{h}$ to $\|\mathbf{h}\|\mathbf{e_1}$, where $\mathbf{e_1}$ is the unit vector in the direction of the first coordinate of $\mathbf{x}$. Since $\mathbf{U}^{-1}\mathbf{U} = \mathbf{I}$, we can express the constraint $|\mathbf{x}^T\mathbf{h}|^2 \le \epsilon$, as $|\mathbf{x}^T\mathbf{h}|^2 = |(\mathbf{x}^T\mathbf{U}^{-1})(\mathbf{U}\mathbf{h})|^2 \le \epsilon$. By Defining $\mathbf{x}^T\mathbf{U}^{-1} = \mathbf{y}^T$, where $\mathbf{y} = (y_1, \dots, y_N)^T$ is a new coordinate system, and knowing that $\mathbf{U}\mathbf{h} = (\|\mathbf{h}\|, 0, \dots, 0)$, we obtain $|\mathbf{x}^T\mathbf{h}|^2 = \|\mathbf{h}\|^2|y_1|^2 \le \epsilon$. Consequently, applying a change of coordination according to the rotation $\mathbf{U}$, we have a new formulation for $\mathbf{P1}$ in terms of $\mathbf{y} = (y_1, y_2, \dots, y_N)^T$ expressed as ($\mathbf{P2}$)

$$G = \text{maximize} \quad f(\mathbf{y}) = |\mathbf{y}^T\tilde{\mathbf{g}}|^2 \tag{7}$$
$$\text{subject to:} \quad \|\mathbf{h}\|^2|y_1|^2 \le \epsilon, \tag{8}$$
$$\|\mathbf{y}\|^2 \le P. \tag{9}$$

In $\mathbf{P2}$, the vector $\tilde{\mathbf{g}} = \mathbf{U}\mathbf{g}$ is a rotated vector. Since a rotation matrix is a unitary matrix and the distribution of the CSCG-distributed vector $\mathbf{g}$ is invariant under rotation, we have $\tilde{\mathbf{g}} \sim \mathcal{CN}(\mathbf{0}, 2\sigma^2\mathbf{I})$. Furthermore, the entries of $\tilde{\mathbf{g}}$ are independent from the entries of $\mathbf{h}$.

$\mathbf{P2}$ implies that only y_1 contributes to the interference-power at the primary receiver. Using $|y_1|$ as a slack variable, we can convert $\mathbf{P2}$ to the equivalent problem ($\mathbf{P3}$)

$$G = \text{maximize} \quad f(\mathbf{y}) = |\mathbf{y}^T\tilde{\mathbf{g}}|^2 \tag{10}$$
$$\text{subject to:} \quad 0 \le |y_1| \le \frac{\sqrt{\epsilon}}{\|\mathbf{h}\|}, \tag{11}$$
$$\|\mathbf{y_{-1}}\|^2 \le P - |y_1|^2. \tag{12}$$

Maximization of the objective $f(\mathbf{y}) = |\mathbf{y}^T\tilde{\mathbf{g}}|^2$ over $\mathbf{y}$ yields $\arg(y_i^{\mathsf{opt}}) = -\arg(\tilde{g}_i)$ for $i = 1, \dots, N$. Therefore, making (12) tight in the constraint, we obtain $\mathbf{y_{-1}}^{\mathsf{opt}} = \frac{\tilde{\mathbf{g}}^*_{\mathbf{-1}}}{\|\tilde{\mathbf{g}}_{\mathbf{-1}}\|}\sqrt{P - |y_1|^2}$. Consequently, the objective function is $f(\mathbf{y}) = \left(|y_1||\tilde{g}_1| + \sqrt{P - |y_1|^2}\,\|\tilde{\mathbf{g}}_{\mathbf{-1}}\|\right)^2$. Maximizing $f(\mathbf{y})$ over $0 \le |y_1| \le \frac{\sqrt{\epsilon}}{\|\mathbf{h}\|}$ yields the following two cases:

If $\|\mathbf{h}\|^2 \le \frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\tilde{g}_1|^2}$; then $|y_1^{\mathsf{opt}}| = \frac{|\tilde{g}_1|}{\|\tilde{\mathbf{g}}\|}\sqrt{P}$. Thus, we obtain the optimal solution as $\mathbf{y}^{\mathsf{opt}} = \frac{\tilde{\mathbf{g}}^*}{\|\tilde{\mathbf{g}}\|}\sqrt{P}$ and the optimal secondary received-power is $G = P\|\tilde{\mathbf{g}}\|^2$.

If $\|\mathbf{h}\|^2 > \frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\tilde{g}_1|^2}$; then we obtain $|y_1^{\mathsf{opt}}| = \frac{\sqrt{\epsilon}}{\|\mathbf{h}\|}$, and thus $\mathbf{y_{-1}}^{\mathsf{opt}} = \frac{\tilde{\mathbf{g}}^*_{\mathbf{-1}}}{\|\tilde{\mathbf{g}}_{\mathbf{-1}}\|}\sqrt{P - \frac{\epsilon}{\|\mathbf{h}\|^2}}$. Therefore, the optimal secondary received-power is $G = \left(\frac{\sqrt{\epsilon}|\tilde{g}_1|}{\|\mathbf{h}\|} + \sqrt{P - \frac{\epsilon}{\|\mathbf{h}\|^2}}\,\|\tilde{\mathbf{g}}_{\mathbf{-1}}\|\right)^2$.

It is worth noting that, in order to convert $\mathbf{y}^{\mathsf{opt}}$ back to the original coordinate system $\mathbf{x}$, and thus find the solution to $\mathbf{P1}$, we can use the fact that $\mathbf{x} = \mathbf{U}^T\mathbf{y}$ and obtain $\mathbf{x}^{\mathsf{opt}} = \mathbf{U}^T\mathbf{y}^{\mathsf{opt}}$.

C. Maximum Secondary Received-power

Having the two different cases for the optimal solution as derived in Section III-B, the maximum secondary received-power can be expressed compactly as

$$G = P\|\tilde{\mathbf{g}}\|^2 \cdot \mathbf{1}_{\left\{\|\mathbf{h}\|^2 \le \frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\tilde{g}_1|^2}\right\}} + \left(\frac{|\tilde{g}_1|\sqrt{\epsilon}}{\|\mathbf{h}\|} + \sqrt{P - \frac{\epsilon}{\|\mathbf{h}\|^2}}\|\tilde{\mathbf{g}}_{\mathbf{-1}}\|\right)^2 \cdot \mathbf{1}_{\left\{\|\mathbf{h}\|^2 > \frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\tilde{g}_1|^2}\right\}}. \tag{13}$$

Therefore, using $\mathsf{E}[G] = \mathsf{E}_{\tilde{\mathbf{g}}}\left[\mathsf{E}_{\mathbf{h}}\left[G|\tilde{\mathbf{g}}\right]\right]$, the expected value of G is

$$\mathsf{E}[G] = P\mathsf{E}_{\tilde{\mathbf{g}}}\left[\mathsf{E}\left[\|\tilde{\mathbf{g}}\|^2 \cdot \mathbf{1}_{\left\{\|\mathbf{h}\|^2 \le \frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\tilde{g}_1|^2}\right\}}|\tilde{\mathbf{g}}\right]\right] + \mathsf{E}_{\tilde{\mathbf{g}}}\left[\mathsf{E}\left[\left(\frac{|\tilde{g}_1|\sqrt{\epsilon}}{\|\mathbf{h}\|} + \sqrt{P - \frac{\epsilon}{\|\mathbf{h}\|^2}}\|\tilde{\mathbf{g}}_{-1}\|\right)^2 \cdot \mathbf{1}_{\left\{\|\mathbf{h}\|^2 > \frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\tilde{g}_1|^2}\right\}}|\tilde{\mathbf{g}}\right]\right]. \tag{14}$$

Since $\mathbf{1}_{\left\{\|\mathbf{h}\|^2 > \frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\tilde{g}_1|^2}\right\}} \neq 0$ implies that $\epsilon < P\|\mathbf{h}\|^2$, we have

$$\left(\sqrt{P - \frac{\epsilon}{\|\mathbf{h}\|^2}}\right) \cdot \mathbf{1}_{\left\{\|\mathbf{h}\|^2 > \frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\tilde{g}_1|^2}\right\}} = \sqrt{P}\left(1 - \frac{\epsilon}{2\|\mathbf{h}\|^2 P} - O\left(\left(\frac{\epsilon}{P}\right)^2\right)\right) \cdot \mathbf{1}_{\left\{\|\mathbf{h}\|^2 > \frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\tilde{g}_1|^2}\right\}}. \tag{15}$$

Thus, since P is a fixed constant and $\epsilon \geq 0$ is varied, we can write

$$\left(\frac{|\tilde{g_1}|\sqrt{\epsilon}}{\|\mathbf{h}\|} + \sqrt{P - \frac{\epsilon}{\|\mathbf{h}\|^2}}\|\tilde{\mathbf{g}}_{-\mathbf{1}}\|\right)^2 \cdot \mathbf{1}_{\left\{\|\mathbf{h}\|^2 > \frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\tilde{g_1}|^2}\right\}}$$
$$= \left(P\|\tilde{\mathbf{g}}_{-\mathbf{1}}\|^2 + \sqrt{\epsilon}\left(\frac{2|\tilde{g_1}|\|\tilde{\mathbf{g}}_{-\mathbf{1}}\|}{\|\mathbf{h}\|}\sqrt{P}\right) - \epsilon\left(\frac{\|\tilde{\mathbf{g}}_{-\mathbf{1}}\|^2 - |\tilde{g_1}|^2}{\|\mathbf{h}\|^2}\right)\right.$$
$$\left.-O\left(\left(\frac{\epsilon}{P}\right)^{\frac{3}{2}}\right)\right) \cdot \mathbf{1}_{\left\{\|\mathbf{h}\|^2 > \frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\tilde{g_1}|^2}\right\}}. \quad (16)$$

Therefore, since $\tilde{\mathbf{g}}$ and $\mathbf{h}$ are independent, we obtain

$$\mathsf{E}[G] = P\mathsf{E}_{\tilde{\mathbf{g}}}\left[\|\tilde{\mathbf{g}}\|^2 \mathsf{Pr}\left\{\|\mathbf{h}\|^2 \leq \frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\tilde{g_1}|^2}|\tilde{\mathbf{g}}\right\}\right]$$
$$+ P\mathsf{E}_{\tilde{\mathbf{g}}}\left[\|\tilde{\mathbf{g}}_{-\mathbf{1}}\|^2 \mathsf{Pr}\left\{\|\mathbf{h}\|^2 > \frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\tilde{g_1}|^2}|\tilde{\mathbf{g}}\right\}\right]$$
$$+ 2\sqrt{P}\sqrt{\epsilon}\mathsf{E}_{\tilde{\mathbf{g}}}\left[|\tilde{g_1}|\|\tilde{\mathbf{g}}_{-\mathbf{1}}\|\mathsf{E}\left[\frac{\mathbf{1}_{\{\|\mathbf{h}\|^2 > \frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\tilde{g_1}|^2}\}}}{\|\mathbf{h}\|}|\tilde{\mathbf{g}}\right]\right]$$
$$- \epsilon\mathsf{E}_{\tilde{\mathbf{g}}}\left[\left(\|\tilde{\mathbf{g}}_{-\mathbf{1}}\|^2 - |\tilde{g_1}|^2\right)\mathsf{E}\left[\frac{\mathbf{1}_{\{\|\mathbf{h}\|^2 > \frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\tilde{g_1}|^2}\}}}{\|\mathbf{h}\|^2}|\tilde{\mathbf{g}}\right]\right]$$
$$- O\left(\left(\frac{\epsilon}{P}\right)^{\frac{3}{2}}\right). \quad (17)$$

The random variable $\frac{\|\mathbf{h}\|^2}{\sigma^2}$ is chi-square distributed with $2N$ degrees of freedom (see [21]). Therefore,

$$\mathsf{Pr}\left\{\|\mathbf{h}\|^2 > u\right\} = \left(1 + \frac{u}{2\sigma^2} + \frac{u^2}{(2\sigma^2)^2 2!} + \cdots\right.$$
$$\left.+ \frac{u^{N-1}}{(2\sigma^2)^{N-1}(N-1)!}\right)e^{-\frac{u}{2\sigma^2}}, \quad (18)$$

where u is a positive real number (see [21]), and thus using the series expansion of $e^{-\frac{u}{2\sigma^2}}$ with $u = \frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\tilde{g_1}|^2}$ for a fixed $\tilde{\mathbf{g}}$, we find

$$\mathsf{Pr}\left\{\|\mathbf{h}\|^2 > \frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\tilde{g_1}|^2}|\tilde{\mathbf{g}}\right\} = 1 - \frac{1}{N!}\left(\frac{\|\tilde{\mathbf{g}}\|^2}{|\tilde{g_1}|^2}\right)^N\left(\frac{\epsilon}{2P\sigma^2}\right)^N$$
$$+ O\left(\left(\frac{\epsilon}{P}\right)^{N+1}\right). \quad (19)$$

Furthermore, for a fixed $\tilde{\mathbf{g}}$ we can write

$$\mathsf{E}\left[\frac{\mathbf{1}_{\{\|\mathbf{h}\|^2 > \frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\tilde{g_1}|^2}\}}}{\|\mathbf{h}\|}|\tilde{\mathbf{g}}\right] = \mathsf{E}\left[\frac{1}{\|\mathbf{h}\|}\right]$$
$$- \frac{2^{-N}}{\sigma\Gamma(N)}\int_0^{\frac{\epsilon}{P\sigma^2}\frac{\|\tilde{\mathbf{g}}\|^2}{|\tilde{g_1}|^2}} x^{N-\frac{3}{2}}e^{-\frac{x}{2}}dx = \mathsf{E}\left[\frac{1}{\|\mathbf{h}\|}\right]$$
$$- \frac{2^{-N}}{\sigma\Gamma(N)}\int_0^{\frac{\epsilon}{P\sigma^2}\frac{\|\tilde{\mathbf{g}}\|^2}{|\tilde{g_1}|^2}} x^{N-\frac{3}{2}}\left(1 + \sum_{n=1}^{\infty}\frac{(-1)^n(x^n)}{2^n}\right)dx, \quad (20)$$

where $\Gamma(.)$ is the Gamma function.
Since $\mathsf{E}\left[\frac{1}{\|\mathbf{h}\|}\right] = \frac{\sqrt{2}\,\Gamma(N+\frac{1}{2})}{\sigma\Gamma(N)(2N-1)}$ (see [21]), we obtain

$$\mathsf{E}_{\tilde{\mathbf{g}}}\left[\mathsf{E}\left[\frac{\mathbf{1}_{\{\|\mathbf{h}\|^2 > \frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\tilde{g_1}|^2}\}}}{\|\mathbf{h}\|}|\tilde{\mathbf{g}}\right]\right] = \frac{\sqrt{2}\,\Gamma(N+\frac{1}{2})}{\sigma\Gamma(N)(2N-1)}$$
$$- O\left(\left(\frac{\epsilon}{P}\right)^{N-\frac{1}{2}}\right). \quad (21)$$

Using the same approach as in (20), since $\mathsf{E}\left[\frac{1}{\|\mathbf{h}\|^2}\right] = \frac{1}{2(N-1)\sigma^2}$ (see [21]), we obtain

$$\mathsf{E}_{\tilde{\mathbf{g}}}\left[\mathsf{E}\left[\frac{\mathbf{1}_{\{\|\mathbf{h}\|^2 > \frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\tilde{g_1}|^2}\}}}{\|\mathbf{h}\|^2}|\tilde{\mathbf{g}}\right]\right] = \frac{1}{2(N-1)\sigma^2} - O\left(\left(\frac{\epsilon}{P}\right)^{N-1}\right). \quad (22)$$

Therefore, by (19), (21), and (22), we can derive

$$\mathsf{E}_{\tilde{\mathbf{g}}}\left[\|\tilde{\mathbf{g}}\|^2 \cdot \mathsf{Pr}\left\{\|\mathbf{h}\|^2 \leq \frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\tilde{g_1}|^2}|\tilde{\mathbf{g}}\right\}\right] = \mathsf{E}\left[f_1(\tilde{\mathbf{g}})\right]O\left(\left(\frac{\epsilon}{P}\right)^N\right), \quad (23)$$

$$\mathsf{E}_{\tilde{\mathbf{g}}}\left[\|\tilde{\mathbf{g}}_{-\mathbf{1}}\|^2 \cdot \mathsf{Pr}\left\{\|\mathbf{h}\|^2 > \frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\tilde{g_1}|^2}|\tilde{\mathbf{g}}\right\}\right] = \mathsf{E}\left[\|\tilde{\mathbf{g}}_{-\mathbf{1}}\|^2\right]$$
$$- \mathsf{E}\left[f_2(\tilde{\mathbf{g}})\right]O\left(\left(\frac{\epsilon}{P}\right)^N\right), \quad (24)$$

$$\mathsf{E}_{\tilde{\mathbf{g}}}\left[|\tilde{g_1}|\|\tilde{\mathbf{g}}_{-\mathbf{1}}\|\mathsf{E}\left[\frac{\mathbf{1}_{\{\|\mathbf{h}\|^2 > \frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\tilde{g_1}|^2}\}}}{\|\mathbf{h}\|}|\tilde{\mathbf{g}}\right]\right]$$
$$= \frac{\sqrt{2}\,\Gamma(N+\frac{1}{2})}{\sigma\Gamma(N)(2N-1)}\mathsf{E}\left[|\tilde{g_1}|\|\tilde{\mathbf{g}}_{-\mathbf{1}}\|\right] - \mathsf{E}\left[f_3(\tilde{\mathbf{g}})\right]O\left(\left(\frac{\epsilon}{P}\right)^{N-\frac{1}{2}}\right), \quad (25)$$

$$\mathsf{E}_{\tilde{\mathbf{g}}}\left[\left(\|\tilde{\mathbf{g}}_{-\mathbf{1}}\|^2 - |\tilde{g_1}|^2\right)\mathsf{E}\left[\frac{\mathbf{1}_{\{\|\mathbf{h}\|^2 > \frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\tilde{g_1}|^2}\}}}{\|\mathbf{h}\|^2}|\tilde{\mathbf{g}}\right]\right]$$
$$= \frac{\mathsf{E}\left[\|\tilde{\mathbf{g}}_{-\mathbf{1}}\|^2 - |\tilde{g_1}|^2\right]}{2(N-1)\sigma^2} - \mathsf{E}\left[f_4(\tilde{\mathbf{g}})\right]O\left(\left(\frac{\epsilon}{P}\right)^{N-1}\right), \quad (26)$$

where f_1, f_2, f_3, f_4 are suitable functions of the entries of $\tilde{\mathbf{g}}$. Thus, following (17) and using (23)-(26), the expected value of G can be written as

$$\mathsf{E}[G] = P\mathsf{E}\left[\|\tilde{\mathbf{g}}_{-\mathbf{1}}\|^2\right] + \sqrt{2P\epsilon}\frac{\Gamma(N+\frac{1}{2})}{\sigma\Gamma(N)(N-\frac{1}{2})}\mathsf{E}\left[|\tilde{g_1}|\right]\mathsf{E}\left[\|\tilde{\mathbf{g}}_{-\mathbf{1}}\|\right]$$
$$+ \epsilon\frac{\mathsf{E}\left[|\tilde{g_1}|^2 - \|\tilde{\mathbf{g}}_{-\mathbf{1}}\|^2\right]}{2(N-1)\sigma^2} - O\left(\left(\frac{\epsilon}{P}\right)^{\frac{3}{2}}\right). \quad (27)$$

The random variable, $\frac{\|\tilde{\mathbf{g}}_{-\mathbf{1}}\|^2}{\sigma^2}$ (resp. $\frac{\|\tilde{\mathbf{g}}_{-\mathbf{1}}\|}{\sigma}$) is chi-square (resp. chi) distributed with $2(N-1)$ degrees of freedom. Therefore, we have $\mathsf{E}\left[\|\tilde{\mathbf{g}}_{-\mathbf{1}}\|^2\right] = 2(N-1)\sigma^2$, $\mathsf{E}\left[\|\tilde{\mathbf{g}}_{-\mathbf{1}}\|\right] = \sqrt{2}\sigma\frac{\Gamma(N-\frac{1}{2})}{\Gamma(N-1)}$, $\mathsf{E}\left[|\tilde{g_1}|^2\right] = 2\sigma^2$, and $\mathsf{E}\left[|\tilde{g_1}|\right] = \sqrt{2}\,\Gamma\left(\frac{3}{2}\right)\sigma$, (see [21]). Thus, for $\epsilon = \alpha P$, we obtain

$$\mathsf{E}[G] = 2P\sigma^2(N-1) + 2\sqrt{2P\epsilon}\,\sigma\left(\frac{\Gamma\left(\frac{3}{2}\right)\Gamma(N-\frac{1}{2})\Gamma\left(N+\frac{1}{2}\right)}{\Gamma(N)\Gamma(N-1)\left(N-\frac{1}{2}\right)}\right)$$
$$- \epsilon\left(\frac{N-2}{N-1}\right) - O\left(\left(\frac{\epsilon}{P}\right)^{\frac{3}{2}}\right) \quad (28)$$
$$= 2P\sigma^2(N-1) + 2\sqrt{2\alpha}P\sigma\left(\frac{\Gamma\left(\frac{3}{2}\right)\Gamma(N-\frac{1}{2})\Gamma\left(N+\frac{1}{2}\right)}{\Gamma(N)\Gamma(N-1)\left(N-\frac{1}{2}\right)}\right)$$
$$- \alpha P\left(\frac{N-2}{N-1}\right) - O\left(\alpha^{\frac{3}{2}}\right). \quad (29)$$

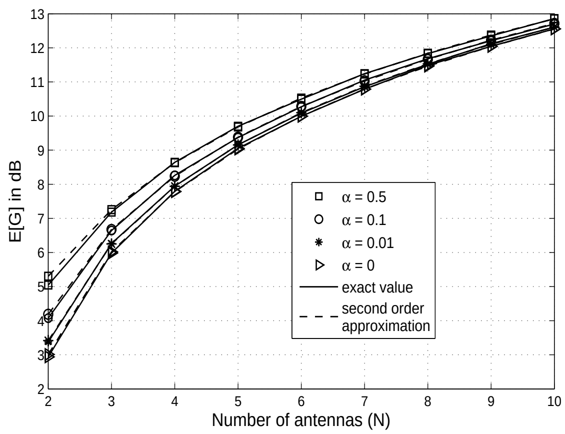

Fig. 2. The exact value of $\mathsf{E}[G]$ in the case of perfect CSI and its second order approximation versus N for different values of α when $P = 1$ and $\sigma = 1$. For different values of α from zero to 0.5, the approximation tracks well the exact value. For a fixed N, as α increases, $\mathsf{E}[G]$ increases, i.e., intentionally allowing additional interference to the primary provides additional received-power to the secondary. With the increase of N, the gap between the null-steering result (performance for $\alpha = 0$) and the performance corresponding to other values of α decreases.

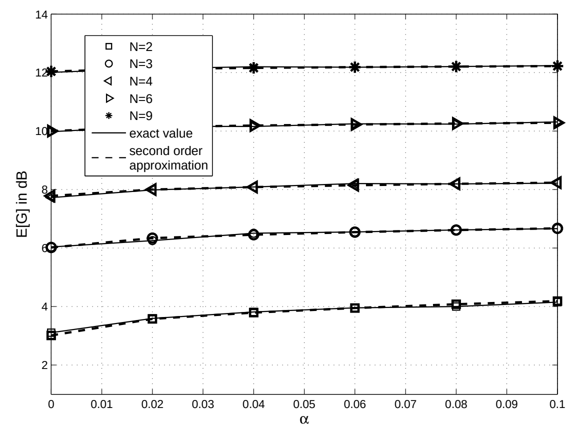

Fig. 3. The exact value of $\mathsf{E}[G]$ in the case of perfect CSI and its second order approximation versus α for different values of N when $P = 1$ and $\sigma = 1$. The approximation tracks well the exact value. $\mathsf{E}[G]$ increases as α increases. Smaller N leads to a higher increase in $\mathsf{E}[G]$. For $N = 2$, $N = 3$, $N = 4$, $N = 6$, and $N = 9$, the increase in $\mathsf{E}[G]$ is 1.2 dB (31%), 0.7 dB (16%), 0.5 dB (11%), 0.3 dB (6%), and 0.2dB (4%) respectively, when α increases from 0 to 0.1.

D. Performance Evaluation

The first term in (29) corresponds to null-steering beamforming ($\alpha = 0$) which has been studied in [18]. This term grows linearly with N due to spatial diversity and we refer to it as the *null-steering result*. If we exclude higher order terms in (29) (i.e. terms of order $\alpha^{\frac{3}{2}}$ and higher), we obtain a second order approximation of $\mathsf{E}[G]$ in $\sqrt{\alpha}$. Note that throughout this paper, second order approximation refers to second order approximation in $\sqrt{\alpha}$.

Figures 2 and 3 plot $\mathsf{E}[G]$ (the exact value) and its second order approximation versus N and α respectively, for $P = 1$ and $\sigma = 1$. As shown in the figures, the second order approximations track well the exact value. Therefore, the second order approximation of $\mathsf{E}[G]$ is accurate for α at least as large as 0.5.

As N increases, the terms $\frac{\Gamma(N-\frac{1}{2})\Gamma(N+\frac{1}{2})}{\Gamma(N)\Gamma(N-1)(N-\frac{1}{2})}$ and $\frac{N-2}{N-1}$ in (29), both converge to 1. Particularly, for $N = 10$, we have $\frac{\Gamma(N-\frac{1}{2})\Gamma(N+\frac{1}{2})}{\Gamma(N)\Gamma(N-1)(N-\frac{1}{2})} \simeq 0.97$ and $\frac{N-2}{N-1} \simeq 0.89$. Therefore, for large N, the second order approximation of $\mathsf{E}[G]$ can be written as

$$\mathsf{E}[G] \simeq 2P\sigma^2(N-1) + 2\sqrt{2\alpha}\,P\sigma\,\Gamma\left(\frac{3}{2}\right) - \alpha P. \quad (30)$$

We are interested in finding tradeoffs between the primary interference threshold, $\epsilon = \alpha P$, and the mean secondary received-power, $\mathsf{E}[G]$. In other words, we aim to study the improvement in $\mathsf{E}[G]$ as α grows slightly larger than zero. In (30), since N is sufficiently large, for small α the first term is clearly dominant. Thus, a small increase in α from zero results in a small relative increase in $\mathsf{E}[G]$. Therefore, when the number of secondary antennas is large, a nonzero interference threshold does not lead to a much greater $\mathsf{E}[G]$ than the null-steering result.

On the other hand, for smaller values of N, since the first term in the second order approximation of (29) is not dominant, an increase in α from zero does lead to a moderate increase in $\mathsf{E}[G]$. As shown in Fig. 3, for $N = 2$, $N = 4$, and $N = 6$, the change of α from 0 to 0.1 leads to an increase of 1.2dB (31%), 0.5dB (11%), and 0.3dB (6%) in $\mathsf{E}[G]$ respectively. In other words, intentionally allowing additional interference to the primary provides additional received-power to the secondary. For larger values of N, a small change of α has a relatively smaller impact on $\mathsf{E}[G]$. This fact can also be observed in Fig. 2 as the gap between the the performance for $\alpha = 0$ (null-steering result) and the performance corresponding to different values of α decreases as N increases.

IV. Imperfect CSI

A more general and realistic scenario is when the channel gains are not perfectly known at the secondary transmitter. This happens due to channel estimation errors which are nonzero. With estimation errors, the estimated channel gains can be expressed as

$$\hat{\mathbf{g}} = \mathbf{g} + \mathbf{w}, \quad \hat{\mathbf{h}} = \mathbf{h} + \mathbf{v}, \quad (31)$$

where $\mathbf{g}$ and $\mathbf{h}$ are the actual secondary channel gain and the actual primary channel gain, respectively. The vectors $\mathbf{w}$ and $\mathbf{v}$ in (31) are estimation error vectors. The random vectors $\mathbf{h}$, $\mathbf{g}$, $\mathbf{w}$, and $\mathbf{v}$ are assumed to be mutually independent. In addition, $\mathbf{w}$ and $\mathbf{v}$ are assumed to be distributed as $\mathcal{CN}(\mathbf{0}, 2\sigma_e^2\mathbf{I})$ (a Gaussian-distributed model for the estimated channel gains is often a reasonable model in estimation methods such as Maximum Likelihood estimation (MLE) [22]).

We assume that the secondary transmitter does not have a priori knowledge of the error vectors, σ_e^2, or any uncertainty regions containing the actual channel channels. Therefore, uncertainty analysis can not be performed. Instead, we find an optimal beamforming vector merely by using the estimates of channel gains instead of the actual channel gains. Consequently, we derive the secondary received-power (resp. primary interference-power) corresponding to this beamforming vector and refer to it as the "actual" secondary received-power (resp. primary interference-power). Then, we analyze the effect of channel estimation error on these results. Intuitively, estimation error leads to extra interference at the primary receiver and less power at the secondary receiver compared to the corresponding values in the case of perfect CSI. Therefore, the results in the case of perfect CSI are upperbounds for the results obtained in this section.

A. Problem Formulation

As stated earlier, in the imperfect CSI case, only the estimated channel gains are available at the secondary transmitter. Therefore, the optimization problem considered in this section has the same formulation as $\mathbf{P1}$ in Section III-A but with the estimated channel gains instead. Therefore, we obtain $\mathbf{P4}$ as

$$\text{maximize} \quad |\mathbf{x}^T \hat{\mathbf{g}}|^2 \tag{32}$$

$$\text{subject to:} \quad |\mathbf{x}^T \hat{\mathbf{h}}|^2 \le \epsilon, \tag{33}$$

$$\|\mathbf{x}\|^2 \le P. \tag{34}$$

To solve $\mathbf{P4}$, we follow the same approach as in Section III-B. We can similarly exploit $\mathbf{U}^{'}$ as a rotation matrix and finally obtain the optimal beamforming vector in two different cases.

B. Actual Primary Interference-power

Since $\mathbf{P4}$ has the same formulation as $\mathbf{P1}$ in Section III-A, the optimal solution to $\mathbf{P4}$ is similar to the optimal solution to $\mathbf{P1}$ with estimated channel gains instead (see Appendix A). Therefore, using (31) which relates the actual unknown channel gains to the estimated channel gains and having the optimal solution to $\mathbf{P4}$, the actual primary interference-power is

$$\begin{aligned} I &= |\mathbf{h}^T \mathbf{x}^{\mathsf{opt}}|^2 = |(\hat{\mathbf{h}}^T - \mathbf{v}^T)\mathbf{x}^{\mathsf{opt}}|^2 = |\hat{\mathbf{h}}^T \mathbf{x}^{\mathsf{opt}} - \mathbf{v}^T \mathbf{x}^{\mathsf{opt}}|^2 \\ &= |\|\hat{\mathbf{h}}\| y_1^{\mathsf{opt}} - \tilde{\mathbf{v}}^T \mathbf{y}^{\mathsf{opt}}|^2 = \|\hat{\mathbf{h}}\|^2 |y_1^{\mathsf{opt}}|^2 + |\tilde{\mathbf{v}}^T \mathbf{y}^{\mathsf{opt}}|^2 \\ &- \|\hat{\mathbf{h}}\| y_1^{\mathsf{opt}} \tilde{\mathbf{v}}^\dagger \mathbf{y}^{\mathsf{opt}*} - \|\hat{\mathbf{h}}\| y_1^{\mathsf{opt}*} \tilde{\mathbf{v}}^T \mathbf{y}^{\mathsf{opt}}, \end{aligned} \tag{35}$$

where $\tilde{\mathbf{v}}$ is the rotated version of $\mathbf{v}$ and thus it is also distributed as $\mathcal{CN}(\mathbf{0}, 2\sigma_e^2 \mathbf{I})$. It is worth noting that the term $\|\hat{\mathbf{h}}\|^2 |y_1^{\mathsf{opt}}|^2$ in (35) is the primary interference threshold $\epsilon = \alpha P$ (the maximum allowable interference) while its following terms are introduced by the estimation error. The mean $\mathsf{E}[I]$ is then derived as (see Appendix A)

$$\mathsf{E}[I] = 2P\sigma_e^2 + \alpha P - O\left(\alpha^N\right). \tag{36}$$

C. Actual Secondary Received-power

Having the optimal solution to $\mathbf{P4}$ and using (31), the actual power at the secondary receiver is

$$\begin{aligned} G &= |\mathbf{g}^T \mathbf{x}^{\mathsf{opt}}|^2 = |(\hat{\mathbf{g}}^T - \mathbf{w}^T)\mathbf{x}^{\mathsf{opt}}|^2 = |\hat{\mathbf{g}}^T \mathbf{x}^{\mathsf{opt}} - \mathbf{w}^T \mathbf{x}^{\mathsf{opt}}|^2 \\ &= |\tilde{\hat{\mathbf{g}}}^T \mathbf{y}^{\mathsf{opt}} - \tilde{\mathbf{w}}^T \mathbf{y}^{\mathsf{opt}}|^2 = |\tilde{\hat{\mathbf{g}}}^T \mathbf{y}^{\mathsf{opt}}|^2 + |\tilde{\mathbf{w}}^T \mathbf{y}^{\mathsf{opt}}|^2 \\ &- \tilde{\hat{\mathbf{g}}}^T \mathbf{y}^{\mathsf{opt}} \tilde{\mathbf{w}}^\dagger \mathbf{y}^{\mathsf{opt}*} - \tilde{\hat{\mathbf{g}}}^\dagger \mathbf{y}^{\mathsf{opt}*} \tilde{\mathbf{w}}^T \mathbf{y}^{\mathsf{opt}}, \end{aligned} \tag{37}$$

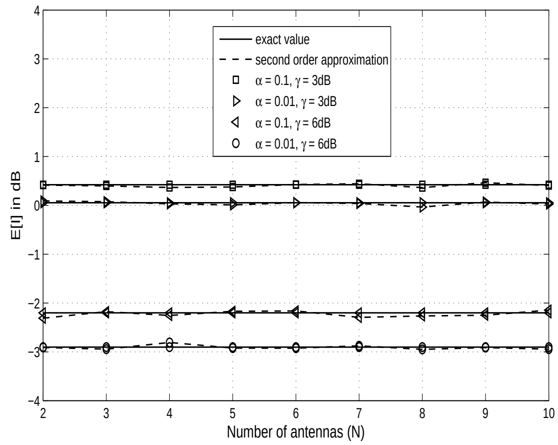

Fig. 4. The exact value of $\mathsf{E}[I]$ in the case of imperfect CSI and its second order approximation versus N for different values of α and γ when $P = 1$ and $\sigma = 1$. The second order approximation tracks well the exact value and it is insensitive to N.

where $\tilde{\mathbf{w}}$ is the rotated version of $\mathbf{w}$ and thus it is also distributed as $\mathcal{CN}(\mathbf{0}, 2\sigma_e^2 \mathbf{I})$. For $\epsilon = \alpha P$, $\mathsf{E}[G]$ is derived as (see Appendix B)

$$\mathsf{E}[G] = Q + \sqrt{\alpha} R + \alpha T + O\left(\alpha^{\frac{3}{2}}\right), \tag{38}$$

where

$$Q = 2P(N-1)\sigma^2 \left(1 - \frac{1}{\gamma} + \frac{1}{\gamma(1+\gamma)} + \frac{1}{(N-1)(1+\gamma)}\right), \tag{39}$$

$$R = 2\sqrt{2} P\sigma \frac{\Gamma\left(\frac{3}{2}\right)\Gamma\left(N - \frac{1}{2}\right)\Gamma\left(N + \frac{1}{2}\right)}{\Gamma(N)\Gamma(N-1)\left(N - \frac{1}{2}\right)} \left(\frac{\gamma}{1+\gamma}\right)^{\frac{3}{2}}, \tag{40}$$

$$\begin{aligned} T = P\Bigg(&-1 + \frac{1}{N-1} - \frac{1}{(1+\gamma)^2} - \frac{\gamma}{(N-1)(1+\gamma)^2} \\ &+ \frac{2N-3}{(N-1)(1+\gamma)}\Bigg), \end{aligned} \tag{41}$$

and $\gamma = \sigma^2/\sigma_e^2$ is the ratio of the scattering component's power to the power of the estimation error. Note that in the case of no channel estimation error ($\gamma = \infty$), (38) equals to (29).

D. Performance Evaluation

In (36), the first term is equal to the actual mean primary interference-power for null-steering beamforming ($\alpha = 0$) [18]. In the case of a relatively small α, for sufficiently large N, the first two terms in (36) (second order approximation in $\sqrt{\alpha}$) become dominant, and these terms are independent of N. Therefore, employing more antennas at the secondary transmitter to improve its performance generates *no significant* extra interference at the primary receiver.

The empirical average of the actual interference at the primary receiver (the exact $\mathsf{E}[I]$) and the second order approximation of $\mathsf{E}[I]$ in (36) are plotted versus N in Fig. 4. As shown in the figure, the approximation tracks well the exact value and is effectively insensitive to N. Even though the acceptable primary interference threshold is $\epsilon = \alpha P$,

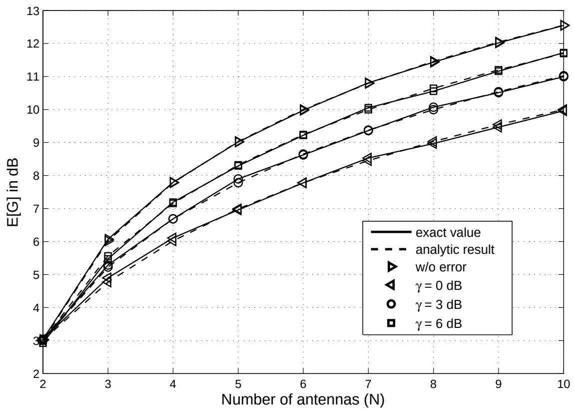

Fig. 5. The exact value of $\mathsf{E}[G]$ in the case of imperfect CSI and the corresponding analytic result versus N for different values of γ when $\alpha = 0$, $P = 1$, and $\sigma = 1$. The analytic result tracks well the exact value. For a fixed N, as γ increases, $\mathsf{E}[G]$ increases to approach the upperbound (the w/o error performance).

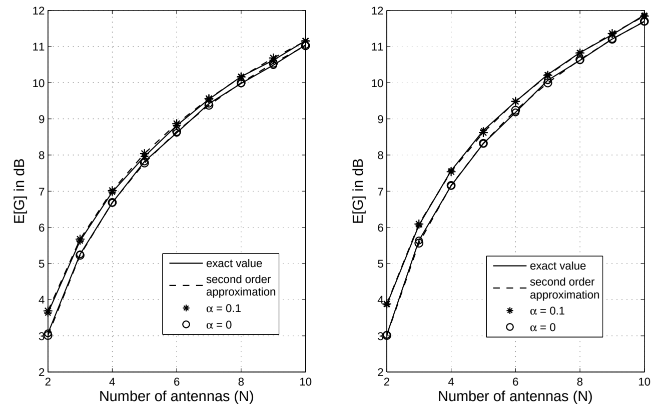

Fig. 6. The exact value of $\mathsf{E}[G]$ in the case of imperfect CSI and its second order approximation versus N for different values of α and γ when $P = 1$ and $\sigma = 1$. The second order approximation tracks well the exact value. Left: $\gamma = 3$dB, Right: $\gamma = 6$dB,

because of channel estimation inaccuracy, $\mathsf{E}[I]$ has an extra dominant term in (36), which is proportional to the power of the estimation error. Therefore, more accurate estimations of the channel gains result in values of interference-power that are closer to the desired threshold ϵ.

The mean secondary received-power found with perfect CSI in (29) is an upperbound for the actual secondary received-power derived in (38). In other words, as $\gamma \to \infty$, (38) converges to (29). Fig. 5 plots $\mathsf{E}[G]$ and the analytic result in (38) for $\alpha = 0$ versus N for different values of γ, when $P = 1$ and $\sigma = 1$. As shown in the figure, the analytic result tracks well the exact value. Furthermore, for a fixed N, as γ increases, $\mathsf{E}[G]$ increases to approach the upperbound (the w/o error performance).

As N increases, the second order approximation of (38) converges to

$$\begin{aligned}\mathsf{E}[G] \simeq\ & \frac{2P\sigma^2}{1+\gamma} + 2P\sigma^2\left(1 - \frac{1}{\gamma} + \frac{1}{\gamma(1+\gamma)}\right)(N-1) \\ & + 2\sqrt{2\alpha}P\,\sigma\,\Gamma\left(\frac{3}{2}\right)\left(\frac{\gamma}{1+\gamma}\right)^{\frac{3}{2}} \\ & - \alpha P\left(1 + \frac{1}{(1+\gamma)^2}\right). \qquad (42)\end{aligned}$$

Figures 6 and 7 plot the exact $\mathsf{E}[G]$ in (38) and its second order approximation versus N for different values of γ and α when $P = 1$ and $\sigma = 1$. As shown in the figures, the second order approximation tracks well the exact value in each case. Furthermore, according to these figures, for small N, $\mathsf{E}[G]$ increases moderately as α increases but for larger values of N, the increase of α leads to a small relative increase in $\mathsf{E}[G]$. In other words, with the increase of N, the gap between the performance for $\alpha = 0$ (the null-steering result) and the performance corresponding to different values of α decreases.

Fig. 8 plots the exact $\mathsf{E}[G]$ in (38) and its second order approximation versus α for $N = 5$ and $N = 9$, when $P = 1$ and $\sigma = 1$. According to the figure, the second order approximation tracks well the exact $\mathsf{E}[G]$ for different values of γ. Furthermore, $\mathsf{E}[G]$ does not significantly improve with the increase in α, the improvements in the case of imperfect CSI are less compared to the perfect CSI case, and the improvements decrease as the number of antennas increases.

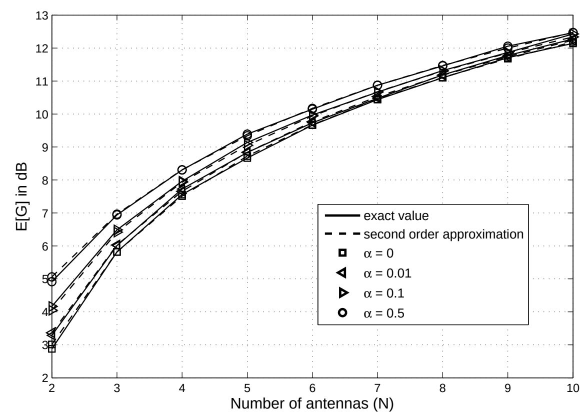

Fig. 7. The exact value of $\mathsf{E}[G]$ in the case of imperfect CSI and its second order approximation versus N for different values of α when $\gamma = 10$dB, $P = 1$, and $\sigma = 1$. The second order approximation tracks well the exact value. For a fixed N, as α increases, $\mathsf{E}[G]$ increases. With the increase of N, the gap between the performance for $\alpha = 0$ (the null-steering result) and the performance corresponding to different values of α decreases.

Thus, in the imperfect CSI case, the primary's allowance for a small nonzero interference is not a significant factor in improving the secondary's performance. Therefore, the secondary system can employ a large number of antennas to boost its performance without significantly impacting the primary, and perform null-steering beamforming instead. In this way, the mean actual interference at the primary receiver is only $2P\sigma_e^2$ according to (36) (recall that $\alpha = 0$ and the dominant terms in (36) are independent of N) and can be less than the interference threshold ϵ if the estimation error variance σ_e^2 is small enough.

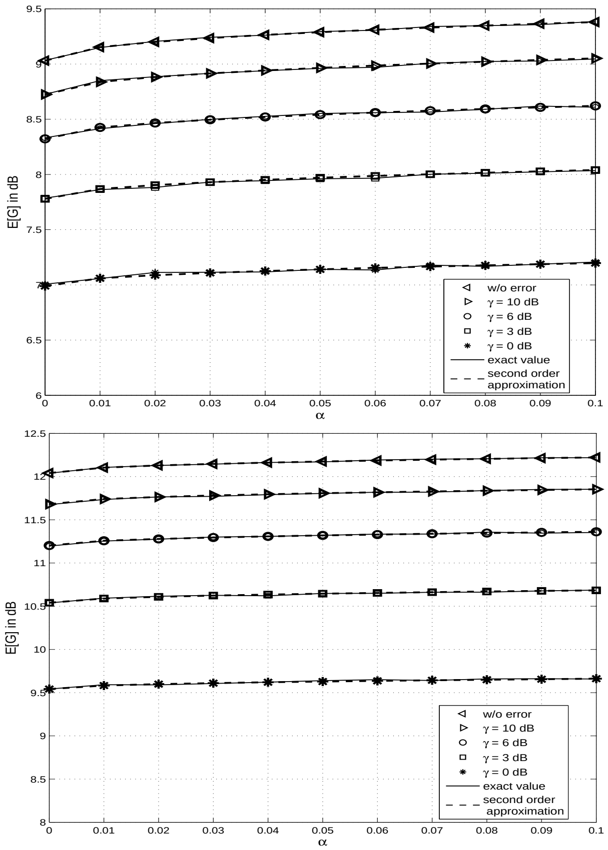

Fig. 8. The exact value of $\mathsf{E}[G]$ in the case of imperfect CSI and its second order approximation versus α for different values of N and γ, when $P=1$ and $\sigma=1$. Top; $N=5$, Bottom; $N=9$. The second order approximation tracks well the exact value of $\mathsf{E}[G]$ for different values of γ. The exact $\mathsf{E}[G]$ does not improve significantly with the increase in α. The improvements in the case of imperfect CSI are less compared to the perfect CSI case (w/o error performance).

V. Concluding Remarks

For a secondary transmitter, the optimal beamforming vector has been characterized to maximize the secondary received-power while limiting the primary interference-power to a threshold $\epsilon=\alpha P$. Both cases of perfect CSI and imperfect CSI have been studied. We have derived the mean secondary received-power $\mathsf{E}[G]$ in a closed-form expression for both cases and have shown that they both consist of a term which grows linearly with N, and additional terms dependent on α. Furthermore, under imperfect CSI, the dominant parts of $\mathsf{E}[I]$ are found to be independent of N. Therefore, each additional antenna provides a significant gain in the secondary received-power without generating significant extra interference at the primary receiver. Furthermore, since the primary's allowance for a small nonzero interference is not a significant factor in improving the secondary's performance, the secondary should perform null-steering beamforming instead. Thus, in the case of imperfect CSI, if the primary's interference threshold is greater than $2P\sigma_e^2$, then null-steering beamforming generates less mean interference at the primary receiver than the interference threshold. A direction for future work is the case when the primary receiver has multiple antennas or more than one single-antenna primary receiver exist as well as considering the case of bidirectional communication in the secondary link.

Appendix A

In the following, we derive the actual primary interference-power in the case of imperfect CSI.

When $\|\hat{\mathbf{h}}\|^2 \le \frac{\epsilon}{P}\frac{\|\tilde{\hat{\mathbf{g}}}\|^2}{|\tilde{\hat{g}}_1|^2}$: the optimal solution to $\mathbf{P4}$ is $\mathbf{y}^{\text{opt}} = \frac{\tilde{\hat{\mathbf{g}}}^*}{\|\tilde{\hat{\mathbf{g}}}\|}\sqrt{P}$. Therefore, using (35), the actual interference-power at the primary receiver is

$$I_1 = \frac{P\|\hat{\mathbf{h}}\|^2|\tilde{\hat{g}}_1|^2}{\|\tilde{\hat{\mathbf{g}}}\|^2} + \frac{P}{\|\tilde{\hat{\mathbf{g}}}\|^2}\left|\sum_{i=1}^{N}\tilde{v}_i\tilde{\hat{g}}_i^*\right|^2 - \frac{P\tilde{\hat{g}}_1^*\|\hat{\mathbf{h}}\|}{\|\tilde{\hat{\mathbf{g}}}\|^2}\sum_{i=1}^{N}\tilde{v}_i^*\tilde{\hat{g}}_i - \frac{P\tilde{\hat{g}}_1\|\hat{\mathbf{h}}\|}{\|\tilde{\hat{\mathbf{g}}}\|^2}\sum_{i=1}^{N}\tilde{v}_i\tilde{\hat{g}}_i^*. \tag{43}$$

Then, we derive

$$\begin{aligned}
\mathsf{E}\left[I_1\cdot\mathbf{1}_{\left\{\|\hat{\mathbf{h}}\|^2\le\frac{\epsilon}{P}\frac{\|\tilde{\hat{\mathbf{g}}}\|^2}{|\tilde{\hat{g}}_1|^2}\right\}}\right] &= P\mathsf{E}_{\tilde{\hat{\mathbf{g}}}}\left[\frac{|\tilde{\hat{g}}_1|^2}{\|\tilde{\hat{\mathbf{g}}}\|^2}\mathsf{E}\left[\{\|\hat{\mathbf{h}}\|^2\cdot\mathbf{1}_{\left\{\|\hat{\mathbf{h}}\|^2\le\frac{\epsilon}{P}\frac{\|\tilde{\hat{\mathbf{g}}}\|^2}{|\tilde{\hat{g}}_1|^2}\right\}}|\tilde{\hat{\mathbf{g}}}\right]\right] \\
&+ P\mathsf{E}_{\tilde{\hat{\mathbf{g}}}}\left[|\tilde{v}_1|^2\cdot\mathbf{1}_{\left\{\|\hat{\mathbf{h}}\|^2\le\frac{\epsilon}{P}\frac{\|\tilde{\hat{\mathbf{g}}}\|^2}{|\tilde{\hat{g}}_1|^2}\right\}}|\tilde{\hat{\mathbf{g}}}\right] \\
&+ PN(N-1)\mathsf{E}_{\tilde{\hat{\mathbf{g}}}}\left[\frac{\tilde{\hat{g}}_1^*\tilde{\hat{g}}_2}{\|\tilde{\hat{\mathbf{g}}}\|^2}\mathsf{E}\left[\tilde{v}_1\tilde{v}_2^*\cdot\mathbf{1}_{\left\{\|\hat{\mathbf{h}}\|^2\le\frac{\epsilon}{P}\frac{\|\tilde{\hat{\mathbf{g}}}\|^2}{|\tilde{\hat{g}}_1|^2}\right\}}|\tilde{\hat{\mathbf{g}}}\right]\right] \\
&- 2P\mathsf{E}_{\tilde{\hat{\mathbf{g}}}}\left[\frac{|\tilde{\hat{g}}_1|^2}{\|\tilde{\hat{\mathbf{g}}}\|^2}\mathsf{E}\left[\|\hat{\mathbf{h}}\|\tilde{v}_1^*\cdot\mathbf{1}_{\left\{\|\hat{\mathbf{h}}\|^2\le\frac{\epsilon}{P}\frac{\|\tilde{\hat{\mathbf{g}}}\|^2}{|\tilde{\hat{g}}_1|^2}\right\}}|\tilde{\hat{\mathbf{g}}}\right]\right] \\
&- 2P\sum_{i=2}^{N}\mathsf{E}_{\tilde{\hat{\mathbf{g}}}}\left[\frac{\tilde{\hat{g}}_1\tilde{\hat{g}}_i^*}{\|\tilde{\hat{\mathbf{g}}}\|^2}\mathsf{E}\left[\|\hat{\mathbf{h}}\|\tilde{v}_i\cdot\mathbf{1}_{\left\{\|\hat{\mathbf{h}}\|^2\le\frac{\epsilon}{P}\frac{\|\tilde{\hat{\mathbf{g}}}\|^2}{|\tilde{\hat{g}}_1|^2}\right\}}|\tilde{\hat{\mathbf{g}}}\right]\right].
\end{aligned} \tag{44}$$

Because of spherical symmetry, we have

$$\mathsf{E}\left[\tilde{v}_i\tilde{v}_j^*\cdot\mathbf{1}_{\left\{\|\hat{\mathbf{h}}\|^2\le\frac{\epsilon}{P}\frac{\|\tilde{\hat{\mathbf{g}}}\|^2}{|\tilde{\hat{g}}_1|^2}\right\}}|\tilde{\hat{\mathbf{g}}}\right]=0,\ \ \mathsf{E}\left[\|\hat{\mathbf{h}}\|\tilde{v}_i\cdot\mathbf{1}_{\left\{\|\hat{\mathbf{h}}\|^2\le\frac{\epsilon}{P}\frac{\|\tilde{\hat{\mathbf{g}}}\|^2}{|\tilde{\hat{g}}_1|^2}\right\}}|\tilde{\hat{\mathbf{g}}}\right]=0, \tag{45}$$

for all i and j where $i\neq j$. Furthermore, since the random variable $\frac{\|\hat{\mathbf{h}}\|^2}{\sigma^2}$ is chi-square distributed with $2N$ degrees of freedom, we can write

$$\begin{aligned}
\mathsf{E}\left[\|\hat{\mathbf{h}}\|^2\cdot\mathbf{1}_{\left\{\|\hat{\mathbf{h}}\|^2\le\frac{\epsilon}{P}\right\}}\right] &= \frac{\sigma^2+\sigma_e^2}{2^N\Gamma(N)}\int_0^{\frac{\epsilon}{P(\sigma^2+\sigma_e^2)}} x^{N+1}e^{-\frac{x}{2}}\,dx \\
&= \frac{\sigma^2+\sigma_e^2}{2^N\Gamma(N)}\int_0^{\frac{\epsilon}{P(\sigma^2+\sigma_e^2)}} x^{N+1}\left(1+\sum_{n=1}^{\infty}\frac{(-1)^n(x^n)}{2^n}\right)dx \\
&= \frac{4(\sigma^2+\sigma_e^2)}{(N+2)\Gamma(N)}\left(\frac{\epsilon}{2P(\sigma^2+\sigma_e^2)}\right)^{N+2}+O\left(\left(\frac{\epsilon}{P}\right)^{N+3}\right).
\end{aligned} \tag{46}$$

Consequently, we have

$$\begin{aligned}
&\mathsf{E}_{\tilde{\hat{\mathbf{g}}}}\left[\frac{|\tilde{\hat{g}}_1|^2}{\|\tilde{\hat{\mathbf{g}}}\|^2}\mathsf{E}\left[\{\|\hat{\mathbf{h}}\|^2\cdot\mathbf{1}_{\left\{\|\hat{\mathbf{h}}\|^2\le\frac{\epsilon}{P}\frac{\|\tilde{\hat{\mathbf{g}}}\|^2}{|\tilde{\hat{g}}_1|^2}\right\}}|\tilde{\hat{\mathbf{g}}}\right]\right] \\
&\qquad = \mathsf{E}\left[f_5(\tilde{\hat{\mathbf{g}}})\right]O\left(\left(\frac{\epsilon}{P}\right)^{N+2}\right),
\end{aligned} \tag{47}$$

where f_5 is a suitable function of the entries of $\tilde{\hat{\mathbf{g}}}$. Therefore, we obtain

$$\begin{aligned}
\mathsf{E}\left[I_1\cdot\mathbf{1}_{\left\{\|\hat{\mathbf{h}}\|^2\le\frac{\epsilon}{P}\frac{\|\tilde{\hat{\mathbf{g}}}\|^2}{|\tilde{\hat{g}}_1|^2}\right\}}\right] &= P\mathsf{E}_{\tilde{\hat{\mathbf{g}}}}\left[\mathsf{E}\left[|\tilde{v}_1|^2\cdot\mathbf{1}_{\left\{\|\hat{\mathbf{h}}\|^2\le\frac{\epsilon}{P}\frac{\|\tilde{\hat{\mathbf{g}}}\|^2}{|\tilde{\hat{g}}_1|^2}\right\}}|\tilde{\hat{\mathbf{g}}}\right]\right] \\
&+ O\left(\left(\frac{\epsilon}{P}\right)^{N+2}\right).
\end{aligned} \tag{48}$$

When $\|\hat{\mathbf{h}}\|^2 > \frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\hat{\tilde{g}}_1|^2}$: the optimal solution to $\mathbf{P4}$ is $y_1^{\mathsf{opt}} = \frac{\sqrt{\epsilon}e^{-j\arg(\tilde{\hat{g}}_1)}}{\|\hat{\mathbf{h}}\|}$ and $\mathbf{y_{-1}}^{\mathsf{opt}} = \frac{\tilde{\mathbf{g}}^*_{-1}}{\|\tilde{\mathbf{g}}_{-1}\|}\sqrt{P - \frac{\epsilon}{\|\hat{\mathbf{h}}\|^2}}$. Therefore, using (35), the actual interference-power is

$$
\begin{aligned}
I_2 = \epsilon &+ \left|\frac{\sqrt{\epsilon}}{\|\hat{\mathbf{h}}\|}e^{-j\arg(\tilde{\hat{g}}_1)}\tilde{v}_1 + \frac{\sqrt{P-\frac{\epsilon}{\|\hat{\mathbf{h}}\|^2}}}{\|\tilde{\mathbf{g}}_{-1}\|}\sum_{i=2}^{N}\tilde{v}_i\tilde{\hat{g}}_i^*\right|^2 \\
&- \sqrt{\epsilon}e^{-j\arg(\tilde{\hat{g}}_1)}\left(\frac{\sqrt{\epsilon}}{\|\hat{\mathbf{h}}\|}e^{j\arg(\tilde{\hat{g}}_1)}\tilde{v}_1^* + \frac{\sqrt{P-\frac{\epsilon}{\|\hat{\mathbf{h}}\|^2}}}{\|\tilde{\mathbf{g}}_{-1}\|}\sum_{i=2}^{N}\tilde{v}_i^*\tilde{\hat{g}}_i\right) \\
&- \sqrt{\epsilon}e^{j\arg(\tilde{\hat{g}}_1)}\left(\frac{\sqrt{\epsilon}}{\|\hat{\mathbf{h}}\|}e^{-j\arg(\tilde{\hat{g}}_1)}\tilde{v}_1 + \frac{\sqrt{P-\frac{\epsilon}{\|\hat{\mathbf{h}}\|^2}}}{\|\tilde{\mathbf{g}}_{-1}\|}\sum_{i=2}^{N}\tilde{v}_i\tilde{\hat{g}}_i^*\right).
\end{aligned} \tag{49}
$$

Then, we have

$$
\begin{aligned}
&\mathsf{E}\left[I_2\cdot\mathbf{1}_{\left\{\|\hat{\mathbf{h}}\|^2>\frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\hat{\tilde{g}}_1|^2}\right\}}\right] = \epsilon\,\mathsf{E}_{\tilde{\mathbf{g}}}\left[\mathsf{E}\left[\mathbf{1}_{\left\{\|\hat{\mathbf{h}}\|^2>\frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\hat{\tilde{g}}_1|^2}\right\}}|\tilde{\mathbf{g}}\right]\right] \\
&+\epsilon\,\mathsf{E}_{\tilde{\mathbf{g}}}\left[\mathsf{E}\left[\frac{|\tilde{v}_1|^2}{\|\hat{\mathbf{h}}\|^2}\cdot\mathbf{1}_{\left\{\|\hat{\mathbf{h}}\|^2>\frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\hat{\tilde{g}}_1|^2}\right\}}|\tilde{\mathbf{g}}\right]\right] \\
&+\mathsf{E}_{\tilde{\mathbf{g}}}\left[\mathsf{E}\left[|\tilde{v}_2|^2\left(P-\frac{\epsilon}{\|\hat{\mathbf{h}}\|^2}\right)\cdot\mathbf{1}_{\left\{\|\hat{\mathbf{h}}\|^2>\frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\hat{\tilde{g}}_1|^2}\right\}}|\tilde{\mathbf{g}}\right]\right] \\
&=\epsilon + P\mathsf{E}_{\tilde{\mathbf{g}}}\left[\mathsf{E}\left[|\tilde{v}_2|^2\cdot\mathbf{1}_{\left\{\|\hat{\mathbf{h}}\|^2>\frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\hat{\tilde{g}}_1|^2}\right\}}|\tilde{\mathbf{g}}\right]\right] - O\left(\left(\frac{\epsilon}{P}\right)^N\right).
\end{aligned} \tag{50}
$$

Note that in the derivation of (50), some terms are zero due to spherical symmetry. Since $\tilde{v}_1$ is equal in distribution with $\tilde{v}_2$, for $\epsilon = \alpha P$, we obtain

$$
\begin{aligned}
\mathsf{E}[I] &= \mathsf{E}\left[I_1\cdot\mathbf{1}_{\left\{\|\hat{\mathbf{h}}\|^2\le\frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\hat{\tilde{g}}_1|^2}\right\}}\right] + \mathsf{E}\left[I_2\cdot\mathbf{1}_{\left\{\|\hat{\mathbf{h}}\|^2>\frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\hat{\tilde{g}}_1|^2}\right\}}\right] \\
&= P\mathsf{E}\left[|\tilde{v}_1|^2\right] + \epsilon - O\left(\left(\frac{\epsilon}{P}\right)^N\right) \\
&= 2P\sigma_e^2 + \alpha P - O\left(\alpha^N\right).
\end{aligned} \tag{51}
$$

Appendix B

In the following, we derive the actual secondary received-power in the case of imperfect CSI.
When $\|\hat{\mathbf{h}}\|^2 \le \frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\hat{\tilde{g}}_1|^2}$: in this case, the optimal solution to $\mathbf{P4}$ is $\mathbf{y}^{\mathsf{opt}} = \frac{\tilde{\mathbf{g}}^*}{\|\tilde{\mathbf{g}}\|}\sqrt{P}$. Therefore, using (37), the actual secondary received-power is

$$
G_1 = P\|\tilde{\mathbf{g}}\|^2 + \frac{P}{\|\tilde{\mathbf{g}}\|^2}\left|\sum_{i=1}^{N}\tilde{w}_i\tilde{\hat{g}}_i^*\right|^2 - P\sum_{i=1}^{N}\tilde{w}_i^*\tilde{\hat{g}}_i - P\sum_{i=1}^{N}\tilde{w}_i\tilde{\hat{g}}_i^*. \tag{52}
$$

Thus, using (19) with $\hat{\mathbf{h}}$ in place of $\mathbf{h}$, we obtain

$$
\begin{aligned}
\mathsf{E}\left[G_1\cdot\mathbf{1}_{\left\{\|\hat{\mathbf{h}}\|^2\le\frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\hat{\tilde{g}}_1|^2}\right\}}\right] &= \mathsf{E}_{\tilde{\mathbf{g}}}\left[G_1\cdot\Pr\left\{\|\hat{\mathbf{h}}\|^2\le\frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\hat{\tilde{g}}_1|^2}|\tilde{\mathbf{g}}\right\}\right] \\
&= \mathsf{E}\left[f_6(\tilde{\mathbf{g}})\right]O\left(\left(\frac{\epsilon}{P}\right)^N\right),
\end{aligned} \tag{53}
$$

where f_6 is a suitable function of the entries of $\tilde{\mathbf{g}}$.

When $\|\hat{\mathbf{h}}\|^2 > \frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\hat{\tilde{g}}_1|^2}$: the optimal solution has $y_1^{\mathsf{opt}} = \frac{\sqrt{\epsilon}e^{-j\arg(\tilde{\hat{g}}_1)}}{\|\tilde{\hat{\mathbf{h}}}\|}$ and $\mathbf{y}_{-1}^{\mathsf{opt}} = \frac{\tilde{\mathbf{g}}^*_{-1}}{\|\tilde{\mathbf{g}}_{-1}\|}\sqrt{P - \frac{\epsilon}{\|\hat{\mathbf{h}}\|^2}}$. Thus, using (37), the actual secondary received-power can be expressed as

$$
\begin{aligned}
G_2 = &\left(\frac{|\tilde{\hat{g}}_1|\sqrt{\epsilon}}{\|\hat{\mathbf{h}}\|} + \sqrt{P-\frac{\epsilon}{\|\hat{\mathbf{h}}\|^2}}\|\tilde{\mathbf{g}}_{-1}\|\right)^2 \\
&+\left|\frac{\tilde{w}_1\sqrt{\epsilon}e^{-j\arg(\tilde{\hat{g}}_1)}}{\|\tilde{\hat{\mathbf{h}}}\|} + \frac{\sqrt{P-\frac{\epsilon}{\|\hat{\mathbf{h}}\|^2}}}{\|\tilde{\mathbf{g}}_{-1}\|}\sum_{i=2}^{N}\tilde{w}_i\tilde{\hat{g}}_i{}^*\right|^2 \\
&-\left(\frac{\tilde{\hat{g}}_1\sqrt{\epsilon}e^{-j\arg(\tilde{\hat{g}}_1)}}{\|\tilde{\hat{\mathbf{h}}}\|} + \sqrt{P-\frac{\epsilon}{\|\hat{\mathbf{h}}\|^2}}\|\tilde{\mathbf{g}}_{-1}\|\right) \\
&\left(\frac{\tilde{w}_1^*\sqrt{\epsilon}e^{j\arg(\tilde{\hat{g}}_1)}}{\|\tilde{\hat{\mathbf{h}}}\|} + \frac{\sqrt{P-\frac{\epsilon}{\|\hat{\mathbf{h}}\|^2}}}{\|\tilde{\mathbf{g}}_{-1}\|}\sum_{i=2}^{N}\tilde{w}_i^*\tilde{\hat{g}}_i\right) \\
&-\left(\frac{\tilde{\hat{g}}_1^*\sqrt{\epsilon}e^{j\arg(\tilde{\hat{g}}_1)}}{\|\tilde{\hat{\mathbf{h}}}\|} + \sqrt{P-\frac{\epsilon}{\|\hat{\mathbf{h}}\|^2}}\|\tilde{\mathbf{g}}_{-1}\|\right) \\
&\left(\frac{\tilde{w}_1\sqrt{\epsilon}e^{-j\arg(\tilde{\hat{g}}_1)}}{\|\tilde{\hat{\mathbf{h}}}\|} + \frac{\sqrt{P-\frac{\epsilon}{\|\hat{\mathbf{h}}\|^2}}}{\|\tilde{\mathbf{g}}_{-1}\|}\sum_{i=2}^{N}\tilde{w}_i\tilde{\hat{g}}_i{}^*\right).
\end{aligned} \tag{54}
$$

Denote the first term in (54) as A. Thus we obtain

$$
\begin{aligned}
\mathsf{E}\left[A\cdot\mathbf{1}_{\left\{\|\hat{\mathbf{h}}\|^2>\frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\hat{\tilde{g}}_1|^2}\right\}}\right] &= 2P(N-1)\left(\sigma^2+\sigma_e^2\right) \\
&+2\sqrt{2P\epsilon(\sigma^2+\sigma_e^2)}\frac{\Gamma\left(\frac{3}{2}\right)\Gamma(N-\frac{1}{2})\Gamma\left(N+\frac{1}{2}\right)}{\Gamma(N)\Gamma(N-1)\left(N-\frac{1}{2}\right)} \\
&-\epsilon\left(\frac{N-2}{N-1}\right) - O\left(\left(\frac{\epsilon}{P}\right)^{\frac{3}{2}}\right),
\end{aligned} \tag{55}
$$

which is derived following the same approach as in Section III-C with $\hat{\mathbf{h}}$ in place of $\mathbf{h}$ and knowing

$$
\begin{aligned}
\mathsf{E}\left[\|\tilde{\mathbf{g}}_{-1}\|^2\right] &= \mathsf{E}\left[\sum_{i=2}^{N}|\tilde{g}_i+\tilde{w}_i|^2\right] = \mathsf{E}\left[\sum_{i=2}^{N}|\tilde{g}_i|^2\right] + \mathsf{E}\left[\sum_{i=2}^{N}|\tilde{w}_i|^2\right] \\
&= 2(N-1)\left(\sigma^2+\sigma_e^2\right).
\end{aligned} \tag{56}
$$

Denoting the second term in (54) as B, we obtain

$$
\begin{aligned}
&\mathsf{E}\left[B\cdot\mathbf{1}_{\left\{\|\hat{\mathbf{h}}\|^2>\frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\hat{\tilde{g}}_1|^2}\right\}}\right] = \epsilon\,\mathsf{E}_{\tilde{\mathbf{g}}}\left[|\tilde{w}_1|^2\mathsf{E}\left[\frac{\mathbf{1}_{\left\{\|\hat{\mathbf{h}}\|^2>\frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\hat{\tilde{g}}_1|^2}\right\}}}{\|\hat{\mathbf{h}}\|^2}|\tilde{\mathbf{g}}\right]\right] \\
&+\mathsf{E}_{\tilde{\mathbf{g}}}\left[\frac{\sum_{i=2}^{N}|\tilde{w}_i|^2|\tilde{\hat{g}}_i|^2}{\|\tilde{\mathbf{g}}_{-1}\|^2}\mathsf{E}\left[\left(P-\frac{\epsilon}{\|\hat{\mathbf{h}}\|^2}\right)\mathbf{1}_{\left\{\|\hat{\mathbf{h}}\|^2>\frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\hat{\tilde{g}}_1|^2}\right\}}|\tilde{\mathbf{g}}\right]\right] \\
&+\mathsf{E}_{\tilde{\mathbf{g}}}\left[\frac{\sum_{i=2,i\neq j}^{N}\sum_{j=2}^{N}\tilde{w}_i^*\tilde{w}_j\tilde{\hat{g}}_i\tilde{\hat{g}}_j^*}{\|\tilde{\mathbf{g}}_{-1}\|^2}\mathsf{E}\left[\left(P-\frac{\epsilon}{\|\hat{\mathbf{h}}\|^2}\right)\mathbf{1}_{\left\{\|\hat{\mathbf{h}}\|^2>\frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\hat{\tilde{g}}_1|^2}\right\}}|\tilde{\mathbf{g}}\right]\right] \\
&+\sqrt{\epsilon}\,\mathsf{E}_{\tilde{\mathbf{g}}}\left[\frac{\tilde{w}_1^*e^{j\arg(\tilde{\hat{g}}_1)}\sum_{i=2}^{N}\tilde{w}_i\tilde{\hat{g}}_i^*}{\|\tilde{\mathbf{g}}_{-1}\|}\mathsf{E}\left[\frac{\sqrt{P-\frac{\epsilon}{\|\hat{\mathbf{h}}\|^2}}}{\|\hat{\mathbf{h}}\|}\mathbf{1}_{\left\{\|\hat{\mathbf{h}}\|^2>\frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\hat{\tilde{g}}_1|^2}\right\}}|\tilde{\mathbf{g}}\right]\right]
\end{aligned} \tag{57}
$$

$$
+\sqrt{\epsilon}\,\mathsf{E}_{\tilde{\mathbf{g}}}\left[\frac{\tilde{w}_1e^{-j\arg(\tilde{\hat{g}}_1)}\sum_{i=2}^{N}\tilde{w}_i^*\tilde{\hat{g}}_i}{\|\tilde{\mathbf{g}}_{-1}\|}\mathsf{E}\left[\frac{\sqrt{P-\frac{\epsilon}{\|\hat{\mathbf{h}}\|^2}}}{\|\hat{\mathbf{h}}\|}\mathbf{1}_{\left\{\|\hat{\mathbf{h}}\|^2>\frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\hat{\tilde{g}}_1|^2}\right\}}|\tilde{\mathbf{g}}\right]\right]. \tag{58}
$$

Therefore, using the same argument as in (15), (21), and (22) with $\hat{\mathbf{h}}$ in place of $\mathbf{h}$, and knowing that (57) and (58) are equivalent due to spherical symmetry, we find

$$\mathsf{E}\left[B\cdot\mathbf{1}_{\left\{\|\hat{\mathbf{h}}\|^2>\frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\tilde{g}_1|^2}\right\}}\right]=\epsilon\mathsf{E}\left[|\tilde{w}_1|^2\right]\mathsf{E}\left[\frac{1}{\|\hat{\mathbf{h}}\|^2}\right]+\mathsf{E}\left[\frac{\sum_{i=2}^N|\tilde{w}_i|^2|\tilde{g}_i|^2}{\|\tilde{\mathbf{g}}_{-\mathbf{1}}\|^2}\right]\mathsf{E}\left[\left(P-\frac{\epsilon}{\|\hat{\mathbf{h}}\|^2}\right)\right]+\mathsf{E}\left[\frac{\sum_{i=2,i\neq j}^N\sum_{j=2}^N\tilde{w}_i^*\tilde{w}_j\tilde{g}_i\tilde{g}_j^*}{\|\tilde{\mathbf{g}}_{-\mathbf{1}}\|^2}\right]\mathsf{E}\left[\left(P-\frac{\epsilon}{\|\hat{\mathbf{h}}\|^2}\right)\right]+2\sqrt{\epsilon}\,\mathsf{E}\left[\tilde{w}_1^*e^{j\arg(\tilde{g}_1)}\right]\mathsf{E}\left[\frac{\sum_{i=2}^N\tilde{w}_i\tilde{g}_i^*}{\|\tilde{\mathbf{g}}_{-\mathbf{1}}\|}\right]\mathsf{E}\left[\frac{\sqrt{P}}{\|\hat{\mathbf{h}}\|}\right]+O\left(\left(\frac{\epsilon}{P}\right)^{\frac{3}{2}}\right). \tag{59}$$

Let

$$W=\frac{\sum_{i=2}^N|\tilde{w}_i|^2|\tilde{g}_i|^2}{\|\tilde{\mathbf{g}}_{-\mathbf{1}}\|^2},\quad V=\frac{\sum_{i=2,i\neq j}^N\sum_{j=2}^N\tilde{w}_i^*\tilde{w}_j\tilde{g}_i\tilde{g}_j^*}{\|\tilde{\mathbf{g}}_{-\mathbf{1}}\|^2}. \tag{60}$$

Conditioning on the entries of $\tilde{\mathbf{g}}$, we obtain

$$\mathsf{E}[W]=\sum_{i=2}^N\mathsf{E}_{\tilde{\mathbf{g}}}\left[\mathsf{E}\left[\frac{|\tilde{w}_i|^2|\tilde{g}_i|^2}{\|\tilde{\mathbf{g}}_{-\mathbf{1}}\|^2}|\tilde{\mathbf{g}}\right]\right]=\sum_{i=2}^N\mathsf{E}_{\tilde{\mathbf{g}}}\left[\frac{\mathsf{E}\left[|\tilde{w}_i|^2|\tilde{g}_i\right]|\tilde{g}_i|^2}{\|\tilde{\mathbf{g}}_{-\mathbf{1}}\|^2}\right], \tag{61}$$

$$\mathsf{E}[V]=\sum_{i=2,i\neq j}^N\sum_{j=2}^N\mathsf{E}_{\tilde{\mathbf{g}}}\left[\mathsf{E}\left[\frac{\tilde{w}_i\tilde{w}_j^*\tilde{g}_i^*\tilde{g}_j}{\|\tilde{\mathbf{g}}_{-\mathbf{1}}\|^2}|\tilde{\mathbf{g}}\right]\right]=\sum_{i=2,i\neq j}^N\sum_{j=2}^N\mathsf{E}_{\tilde{\mathbf{g}}}\left[\frac{\mathsf{E}\left[\tilde{w}_i|\tilde{g}_i\right]\mathsf{E}\left[\tilde{w}_j^*|\tilde{g}_j\right]\tilde{g}_i^*\tilde{g}_j}{\|\tilde{\mathbf{g}}_{-\mathbf{1}}\|^2}\right]. \tag{62}$$

Knowing that $\tilde{g}_i=\tilde{g}_i+\tilde{w}_i$, define $Y_i=-\frac{\sigma_e^2}{\sigma^2}\tilde{g}_i+\tilde{w}_i$. Random variables $\tilde{g}_i$ and Y_i are independent for all $i=1,2,\ldots,N$ since they are zero-mean Gaussian distributed and satisfy $\mathsf{E}[\tilde{g}_iY_i]=0$. Furthermore, we have $\mathsf{E}\left[|Y_i|^2\right]=2\sigma_e^2(1+\frac{\sigma_e^2}{\sigma^2})$. Since $\tilde{w}_i=\frac{\sigma_e^2}{\sigma^2+\sigma_e^2}\tilde{g}_i+\frac{\sigma^2}{\sigma^2+\sigma_e^2}Y_i$, we obtain

$$\mathsf{E}\left[|\tilde{w}_i|^2|\tilde{g}_i\right]=\mathsf{E}\left[\left|\frac{\sigma_e^2}{\sigma^2+\sigma_e^2}\tilde{g}_i+\frac{\sigma^2}{\sigma^2+\sigma_e^2}Y_i\right|^2\right]=\left(\frac{\sigma_e^2}{\sigma^2+\sigma_e^2}\right)^2|\tilde{g}_i|^2+\left(\frac{\sigma^2}{\sigma^2+\sigma_e^2}\right)^2\mathsf{E}\left[|Y_i|^2\right]=\left(\frac{\sigma_e^2}{\sigma^2+\sigma_e^2}\right)^2|\tilde{g}_i|^2+\frac{2\sigma^2\sigma_e^2}{\sigma^2+\sigma_e^2}. \tag{63}$$

Similarly, we can show that $\mathsf{E}\left[\tilde{w}_i|\tilde{g}_i\right]=\frac{\sigma_e^2}{\sigma^2+\sigma_e^2}\tilde{g}_i$ and $\mathsf{E}\left[\tilde{w}_j^*|\tilde{g}_j\right]=\frac{\sigma_e^2}{\sigma^2+\sigma_e^2}\tilde{g}_j^*$. Therefore, we obtain

$$\mathsf{E}[W]=\left(\frac{\sigma_e^2}{\sigma^2+\sigma_e^2}\right)^2\sum_{i=2}^N\mathsf{E}\left[\frac{|\tilde{g}_i|^4}{\|\tilde{\mathbf{g}}_{-\mathbf{1}}\|^2}\right]+\frac{2\sigma^2\sigma_e^2}{\sigma^2+\sigma_e^2}\sum_{i=2}^N\mathsf{E}\left[\frac{|\tilde{g}_i|^2}{\|\tilde{\mathbf{g}}_{-\mathbf{1}}\|^2}\right]=\left(\frac{\sigma_e^2}{\sigma^2+\sigma_e^2}\right)^2\sum_{l=2}^N\mathsf{E}\left[\frac{|\tilde{g}_i|^4}{\|\tilde{\mathbf{g}}_{-\mathbf{1}}\|^2}\right]+\frac{2\sigma^2\sigma_e^2}{\sigma^2+\sigma_e^2}, \tag{64}$$

and

$$\mathsf{E}[V]=\left(\frac{\sigma_e^2}{\sigma^2+\sigma_e^2}\right)^2\sum_{i=2,i\neq j}^N\sum_{j=2}^N\mathsf{E}\left[\frac{|\tilde{g}_i|^2|\tilde{g}_j|^2}{\|\tilde{\mathbf{g}}_{-\mathbf{1}}\|^2}\right]. \tag{65}$$

Therefore, we get

$$\mathsf{E}[W+V]=\left(\frac{\sigma_e^2}{\sigma^2+\sigma_e^2}\right)^2\mathsf{E}\left[\sum_{i=2}^N|\tilde{g}_i|^2\right]+\frac{2\sigma^2\sigma_e^2}{\sigma^2+\sigma_e^2}=\frac{2\sigma_e^4}{\sigma^2+\sigma_e^2}(N-1)+\frac{2\sigma^2\sigma_e^2}{\sigma^2+\sigma_e^2}. \tag{66}$$

Furthermore, we can write

$$\mathsf{E}\left[\tilde{w}_1^*e^{j\arg(\tilde{g}_1)}\right]=\mathsf{E}\left[\frac{\tilde{w}_1^*\tilde{g}_1}{|\tilde{g}_1|}\right]=\mathsf{E}_{\tilde{g}_1}\left[\mathsf{E}\left[\frac{\tilde{w}_1^*\tilde{g}_1}{|\tilde{g}_1|}|\tilde{g}_1\right]\right]=\mathsf{E}\left[\left(\frac{\sigma_e^2}{\sigma^2+\sigma_e^2}\tilde{g}_1^*+\frac{\sigma^2}{\sigma^2+\sigma_e^2}Y_1^*\right)\frac{\tilde{g}_1}{|\tilde{g}_1|}\right]=\frac{\sigma_e^2}{\sigma^2+\sigma_e^2}\mathsf{E}\left[|\tilde{g}_1|\right]=\sqrt{2}\,\Gamma\left(\frac{3}{2}\right)\frac{\sigma_e^2}{\sqrt{\sigma^2+\sigma_e^2}}. \tag{67}$$

Also, we can write

$$\mathsf{E}\left[\frac{\sum_{i=2}^N\tilde{w}_i\tilde{g}_i^*}{\|\tilde{\mathbf{g}}_{-\mathbf{1}}\|}\right]=\mathsf{E}\left[\frac{\sum_{i=2}^N\tilde{g}_i^*\mathsf{E}\left[\tilde{w}_i|\tilde{g}_i\right]}{\|\tilde{\mathbf{g}}_{-\mathbf{1}}\|}\right]=\mathsf{E}\left[\frac{\sum_{i=2}^N\mathsf{E}\left[\left(\frac{\sigma_e^2}{\sigma^2+\sigma_e^2}\tilde{g}_i+\frac{\sigma^2}{\sigma^2+\sigma_e^2}Y_i\right)\tilde{g}_i^*\right]}{\|\tilde{\mathbf{g}}_{-\mathbf{1}}\|}\right]=\frac{\sigma_e^2}{\sigma^2+\sigma_e^2}\mathsf{E}\left[\frac{\sum_{i=2}^N|\tilde{g}_i|^2}{\|\tilde{\mathbf{g}}_{-\mathbf{1}}\|}\right]=\frac{\sigma_e^2}{\sigma^2+\sigma_e^2}\mathsf{E}\left[\|\tilde{\mathbf{g}}_{-\mathbf{1}}\|\right]=\frac{\sqrt{2}\,\sigma_e^2\,\Gamma(N-\frac{1}{2})}{\sqrt{\sigma^2+\sigma_e^2}\,\Gamma(N-1)}. \tag{68}$$

Therefore, knowing that $\mathsf{E}\left[\frac{1}{\|\hat{\mathbf{h}}\|}\right]=\frac{\sqrt{2}\,\Gamma\left(N+\frac{1}{2}\right)}{\sqrt{\sigma^2+\sigma_e^2}\,\Gamma(N)\,(2N-1)}$ and $\mathsf{E}\left[\frac{1}{\|\hat{\mathbf{h}}\|^2}\right]=\frac{1}{2(N-1)(\sigma^2+\sigma_e^2)}$, from (21) and (22) we obtain

$$\mathsf{E}\left[B\cdot\mathbf{1}_{\left\{\|\hat{\mathbf{h}}\|^2>\frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\tilde{g}_1|^2}\right\}}\right]=\frac{2P\sigma_e^2}{\sigma^2+\sigma_e^2}\left(\sigma_e^2(N-1)+\sigma^2\right)+2\sqrt{2P\epsilon}\,\Gamma\left(\frac{3}{2}\right)\frac{\sigma_e^4\,\Gamma\left(N-\frac{1}{2}\right)\,\Gamma\left(N+\frac{1}{2}\right)}{(\sigma^2+\sigma_e^2)^{\frac{3}{2}}\,\Gamma(N-1)\Gamma(N)\left(N-\frac{1}{2}\right)}+\frac{\epsilon\sigma_e^2}{\sigma^2+\sigma_e^2}\left(\frac{1}{N-1}-\frac{\sigma_e^2}{\sigma^2+\sigma_e^2}-\frac{\sigma^2}{(N-1)\left(\sigma^2+\sigma_e^2\right)}\right)+O\left(\left(\frac{\epsilon}{P}\right)^{\frac{3}{2}}\right). \tag{69}$$

Denoting the third term in (54) as C, we obtain

$$-\mathsf{E}\left[C\cdot\mathbf{1}_{\left\{\|\hat{\mathbf{h}}\|^2>\frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\tilde{g}_1|^2}\right\}}\right]=\epsilon\,\mathsf{E}_{\tilde{\mathbf{g}}}\left[|\tilde{w}_1|^2\mathsf{E}\left[\frac{\mathbf{1}_{\left\{\|\hat{\mathbf{h}}\|^2>\frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\tilde{g}_1|^2}\right\}}}{\|\hat{\mathbf{h}}\|^2}|\tilde{\mathbf{g}}\right]\right]+\sqrt{\epsilon}\,\mathsf{E}_{\tilde{\mathbf{g}}}\left[\frac{|\tilde{g}_1|\sum_{i=2}^N\tilde{w}_i^*\tilde{g}_i}{\|\tilde{\mathbf{g}}_{-1}\|}\mathsf{E}\left[\frac{\sqrt{P-\frac{\epsilon}{\|\hat{\mathbf{h}}\|}}}{\|\hat{\mathbf{h}}\|}\cdot\mathbf{1}_{\left\{\|\hat{\mathbf{h}}\|^2>\frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\tilde{g}_1|^2}\right\}}|\tilde{\mathbf{g}}\right]\right]+\sqrt{\epsilon}\,\mathsf{E}_{\tilde{\mathbf{g}}}\left[\tilde{w}_1^*e^{j\arg(\tilde{g}_1)}\|\tilde{\mathbf{g}}_{-1}\|\mathsf{E}\left[\frac{\sqrt{P-\frac{\epsilon}{\|\hat{\mathbf{h}}\|^2}}}{\|\hat{\mathbf{h}}\|}\cdot\mathbf{1}_{\left\{\|\hat{\mathbf{h}}\|^2>\frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\tilde{g}_1|^2}\right\}}|\tilde{\mathbf{g}}\right]\right]+\mathsf{E}_{\tilde{\mathbf{g}}}\left[\sum_{i=2}^N\tilde{g}_i\tilde{w}_i^*\mathsf{E}\left[\left(P-\frac{\epsilon}{\|\hat{\mathbf{h}}\|^2}\right)\cdot\mathbf{1}_{\left\{\|\hat{\mathbf{h}}\|^2>\frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\tilde{g}_1|^2}\right\}}|\tilde{\mathbf{g}}\right]\right], \tag{70}$$

which can be written as

$$-\mathsf{E}\left[C\cdot\mathbf{1}_{\left\{\|\hat{\mathbf{h}}\|^2>\frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\tilde{g}_1|^2}\right\}}\right]=\epsilon\,\mathsf{E}\left[|\tilde{w}_1|^2\right]\mathsf{E}\left[\frac{1}{\|\hat{\mathbf{h}}\|^2}\right]$$
$$+\sqrt{\epsilon}\,\mathsf{E}\left[|\tilde{g}_1|\right]\mathsf{E}\left[\frac{\sum_{i=2}^N\tilde{w}_i^*\tilde{g}_i}{\|\tilde{\mathbf{g}}_{-1}\|}\right]\mathsf{E}\left[\frac{\sqrt{P}}{\|\hat{\mathbf{h}}\|}\right]$$
$$+\sqrt{\epsilon}\,\mathsf{E}\left[\tilde{w}_1^*e^{j\arg(\tilde{g}_1)}\right]\mathsf{E}\left[\|\tilde{\mathbf{g}}_{-1}\|\right]\mathsf{E}\left[\frac{\sqrt{P}}{\|\hat{\mathbf{h}}\|}\right]$$
$$+\mathsf{E}\left[\sum_{i=2}^N\tilde{g}_i\tilde{w}_i^*\right]\mathsf{E}\left[P-\frac{\epsilon}{\|\hat{\mathbf{h}}\|^2}\right]+O\left(\left(\frac{\epsilon}{P}\right)^{\frac{3}{2}}\right).\qquad(71)$$

Therefore, we derive

$$-\mathsf{E}\left[C\cdot\mathbf{1}_{\left\{\|\hat{\mathbf{h}}\|^2>\frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\tilde{g}_1|^2}\right\}}\right]=2P\sigma_e^2(N-1)-\frac{\epsilon\,(N-2)\sigma_e^2}{(N-1)\,(\sigma^2+\sigma_e^2)}$$
$$+\frac{2\sqrt{2P\epsilon}\,\sigma\Gamma\left(\frac{3}{2}\right)\Gamma\left(N+\frac{1}{2}\right)\Gamma\left(N-\frac{1}{2}\right)}{\sqrt{\gamma(1+\gamma)}\Gamma(N)\Gamma\left(N-1\right)\left(N-\frac{1}{2}\right)}$$
$$+O\left(\left(\frac{\epsilon}{P}\right)^{\frac{3}{2}}\right).\qquad(72)$$

The fourth term in (54) is equal in distribution with C. Therefore, we obtain

$$\mathsf{E}\left[G_2\cdot\mathbf{1}_{\left\{\|\hat{\mathbf{h}}\|^2>\frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\tilde{g}_1|^2}\right\}}\right]=\mathsf{E}\left[A\cdot\mathbf{1}_{\left\{\|\hat{\mathbf{h}}\|^2>\frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\tilde{g}_1|^2}\right\}}\right]$$
$$+\mathsf{E}\left[B\cdot\mathbf{1}_{\left\{\|\hat{\mathbf{h}}\|^2>\frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\tilde{g}_1|^2}\right\}}\right]$$
$$+2\mathsf{E}\left[C\cdot\mathbf{1}_{\left\{\|\hat{\mathbf{h}}\|^2>\frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\tilde{g}_1|^2}\right\}}\right]$$
$$=Q+\sqrt{\frac{\epsilon}{P}}R+\frac{\epsilon}{P}T+O\left(\left(\frac{\epsilon}{P}\right)^{\frac{3}{2}}\right),\qquad(73)$$

where

$$Q=2P(N-1)\sigma^2\left(1-\frac{1}{\gamma}+\frac{1}{\gamma(1+\gamma)}+\frac{1}{(N-1)(1+\gamma)}\right),\qquad(74)$$

$$R=2\sqrt{2}P\sigma\frac{\Gamma\left(\frac{3}{2}\right)\Gamma\left(N-\frac{1}{2}\right)\Gamma\left(N+\frac{1}{2}\right)}{\Gamma(N)\Gamma(N-1)\left(N-\frac{1}{2}\right)}\left(\frac{\gamma}{1+\gamma}\right)^{\frac{3}{2}},\qquad(75)$$

$$T=P\left(-1+\frac{1}{N-1}-\frac{1}{(1+\gamma)^2}-\frac{\gamma}{(N-1)(1+\gamma)^2}\right.$$
$$\left.+\frac{2N-3}{(N-1)(1+\gamma)}\right),\qquad(76)$$

and $\gamma=\sigma^2/\sigma_e^2$ is the ratio of the scattering component's power to the power of the estimation error.

Therefore, for $\epsilon=\alpha P$, the actual secondary received-power has the expected value

$$\mathsf{E}[G]=\mathsf{E}\left[G_1\cdot\mathbf{1}_{\left\{\|\hat{\mathbf{h}}\|^2\le\frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\tilde{g}_1|^2}\right\}}\right]+\mathsf{E}\left[G_2\cdot\mathbf{1}_{\left\{\|\hat{\mathbf{h}}\|^2>\frac{\epsilon}{P}\frac{\|\tilde{\mathbf{g}}\|^2}{|\tilde{g}_1|^2}\right\}}\right]$$
$$=Q+\sqrt{\frac{\epsilon}{P}}R+\frac{\epsilon}{P}T+O\left(\left(\frac{\epsilon}{P}\right)^{\frac{3}{2}}\right)$$
$$=Q+\sqrt{\alpha}R+\alpha T+O\left(\alpha^{\frac{3}{2}}\right).\qquad(77)$$

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