Lecture Notes
ELE-A6

DC Machine

DIRECT CURRENT (DC) MACHINES Fundamentals

- **Generator action:** An emf (voltage) is induced in a conductor if it moves through a magnetic field.

- **Motor action:** A force is induced in a conductor that has a current going through it and placed in a magnetic field

- Any DC machine can act either as a generator or as a motor.

DC Generator Fundamentals

$$e = (\boldsymbol{v} \times \boldsymbol{B}) \cdot \ell$$

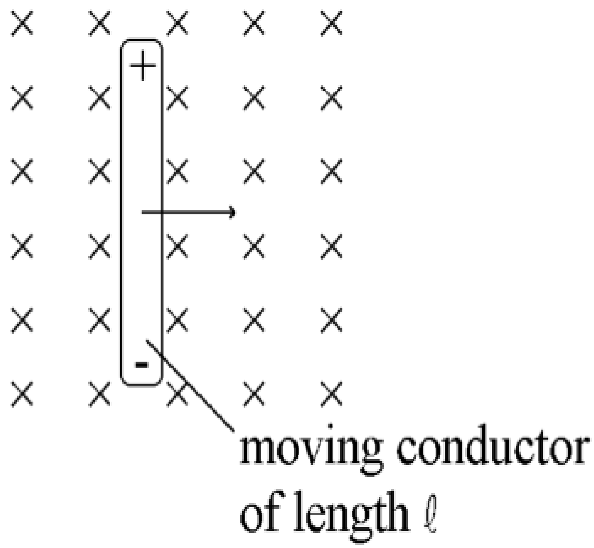
$$e = B\ell v \sin\alpha \cos\beta$$

e = induced voltage, v = velocity of the conductor, B = flux density and l is the length of the conductor

α - angle between the direction in which the conductor is moving and the flux is acting.

β - smallest possible angle the conductor makes with the direction of, the vector product, ($v \times \boldsymbol{B}$) and for maximum induction, $\beta = 0$. Hence, $e = Blv$ for most cases.

($v \times \boldsymbol{B}$) indicates the direction of the current flow in the conductor, or the polarity of the emf.



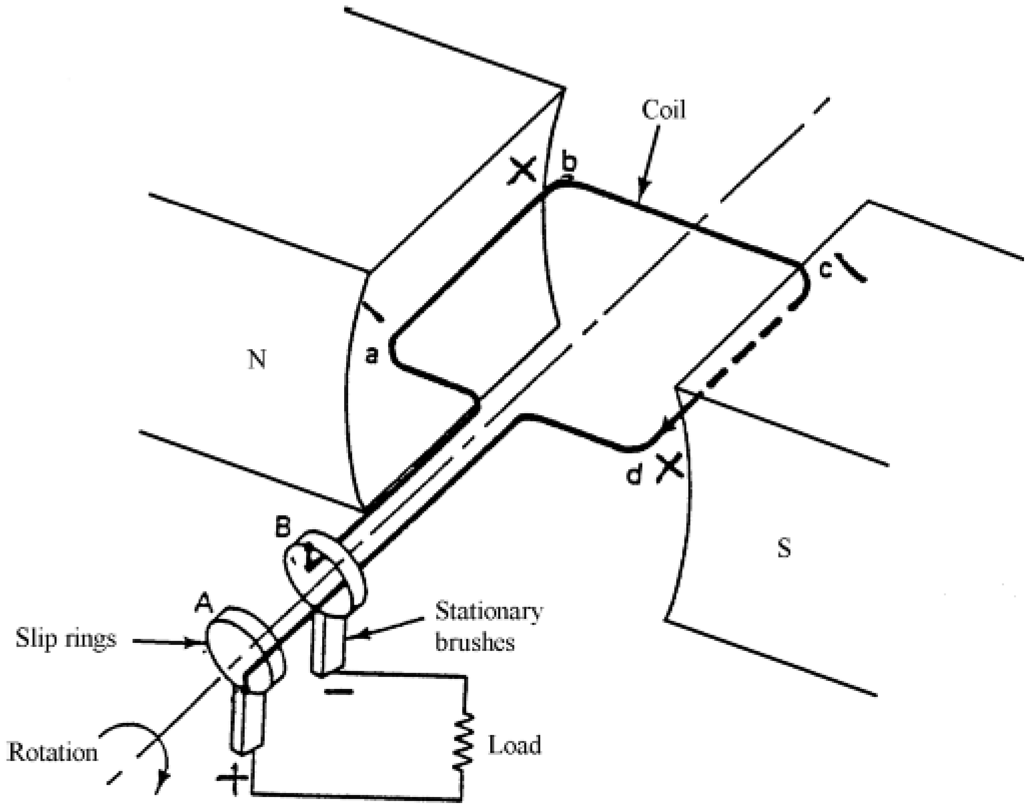
Generated Voltage in a Loop (a coil of one turn)

- For emf to be induced, the conductors must cut the flux lines as they move. Otherwise, ($v \times \boldsymbol{B}$) = 0.

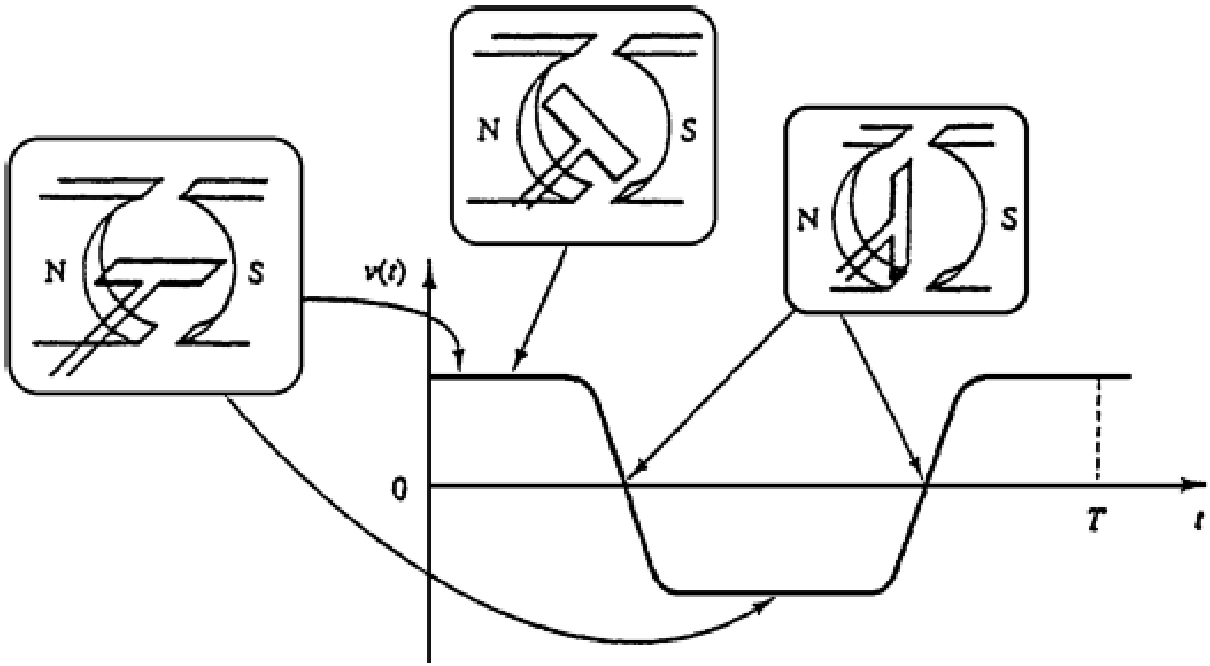
$$e_{loop} = e_{ab} + e_{bc} + e_{cd} + e_{da}$$

$$e_{loop} = Blv + 0 + Blv + 0$$

$$e_{loop} = 2\,B\,l\,v$$

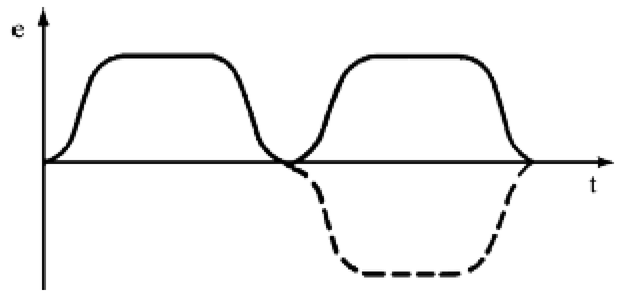


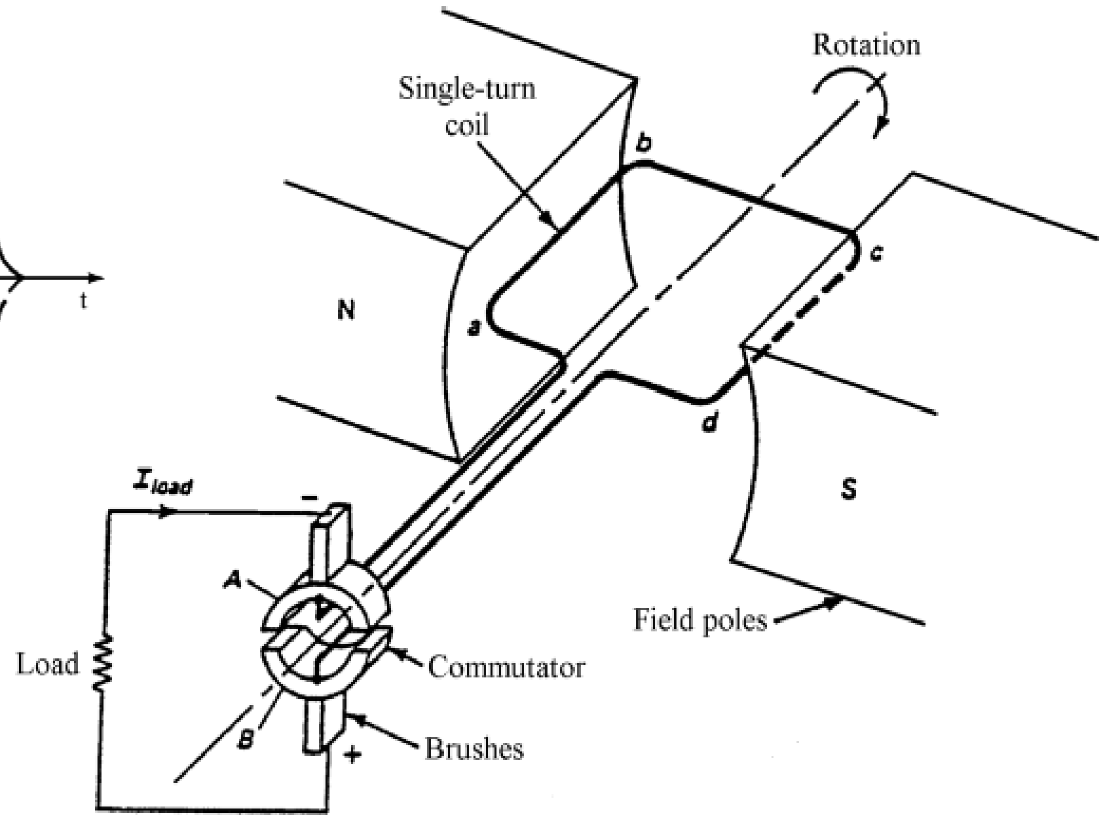
Generated Voltage in a Loop (a coil of one turn)



- Note: Induced voltages are always AC.

Generated Voltage in a Loop (a coil of one turn)



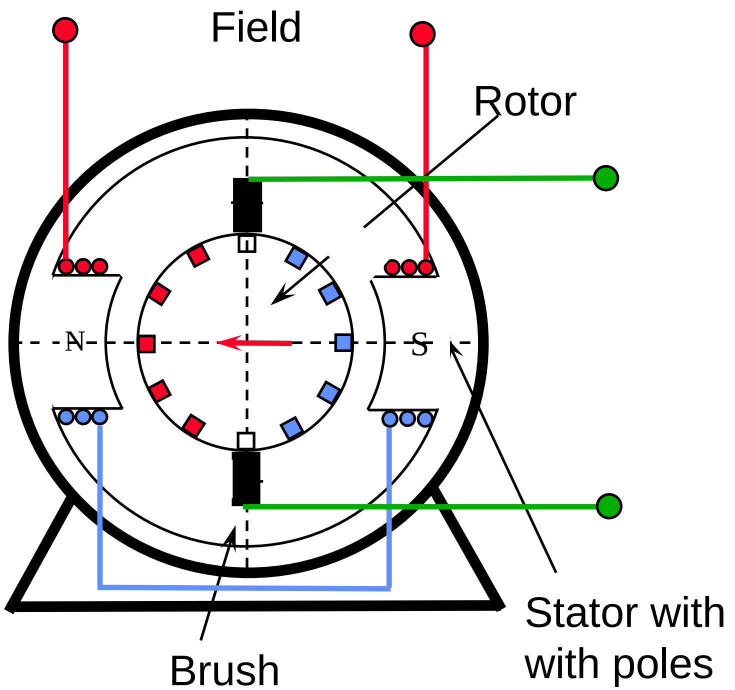


DC MACHINES

Real DC machine Construction

- **Stator:** Stationary part of the machine. The stator carries a field winding that is used to produce the required magnetic field by DC excitation. Often know as the field.

- **Rotor:** The rotor is the rotating part of the machine. The rotor carries a distributed winding, and is the winding where the emf is induced. Also known as the armature.

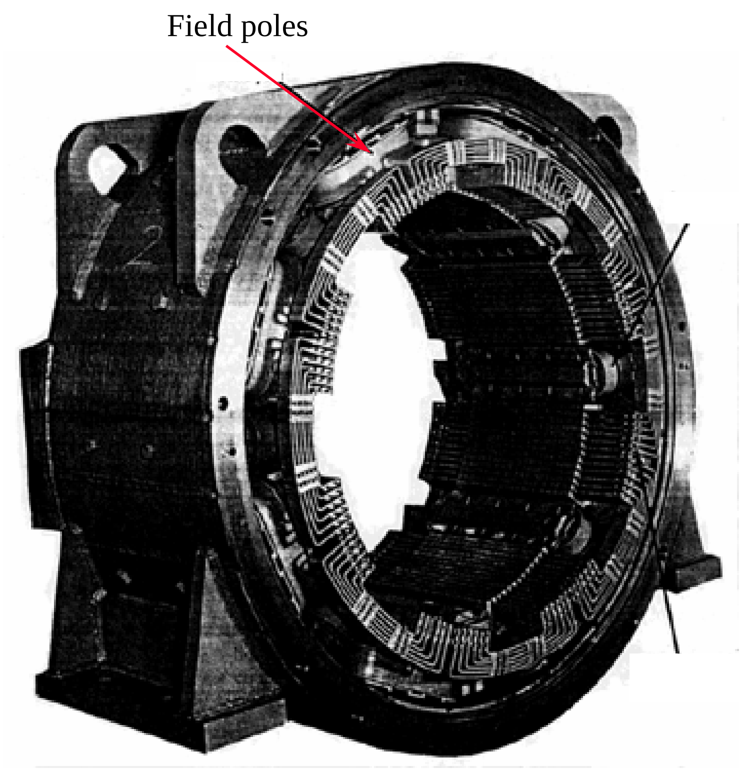


DIRECT CURRENT MACHINES

DC machine Construction

- The picture shows the stator of a large DC machine with several poles.
- The iron core is supported by a cast iron frame.

Dc motor stator construction

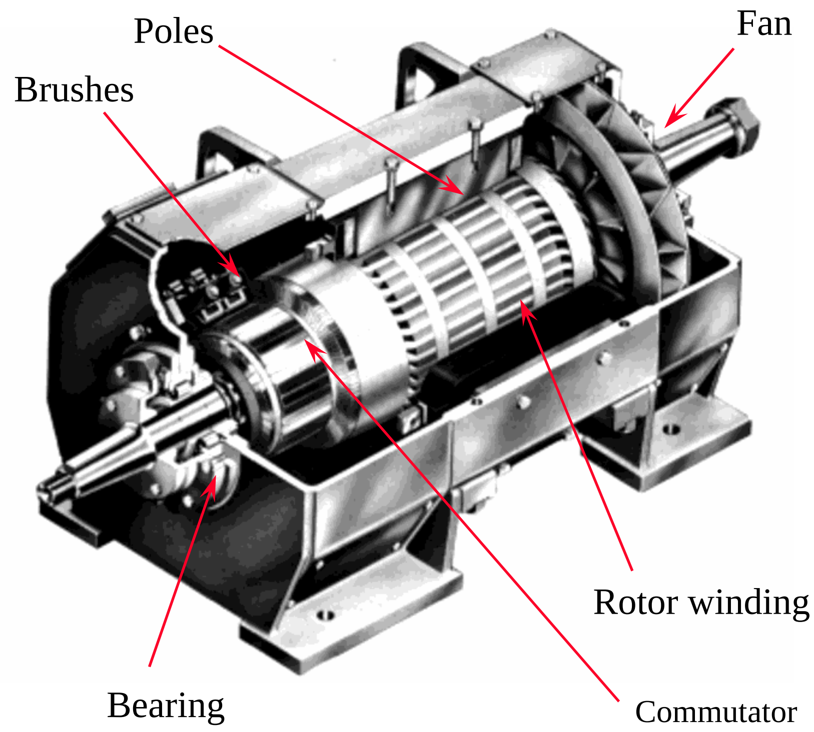


DIRECT CURRENT MACHINES

DC machine Construction

- The rotor iron core is mounted on the shaft.
- Coils are placed in the slots.
- The end of the coils are bent and tied together to assure mechanical strength.
- Note the commutator mounted on the shaft. It consists of several copper segments, separated by insulation.

DC motor rotor construction



DC MACHINES

Generated EMF in a Real DC Machine

$$E_G = \frac{ZP}{60a}\phi n = k_g \phi n = \frac{ZP}{2\pi a}\phi\omega = k_m \phi\omega$$

Where

Z = total number of conductors, P = total number of poles

a = *P* for lap winding, a = 2 for wave winding, ϕ = flux,

ω = speed in rad/s and n = speed in rpm.

DC Motor Fundamentals

$$F = (\ell \times B)i$$

F = induced force, B = flux density, I is the current passing in the conductor and l is the length of the conductor

L is a vector in the direction of the flow of the current.

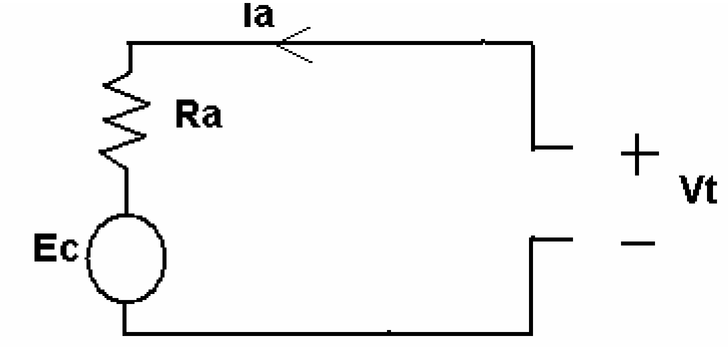
$(l \times B)$ direction indicates the direction of force.

DC Motor Fundamentals

Counter EMF:

When the motor is running, internally generated emf, ($E_G = E_C$) opposes the applied voltage, thus:

$$I_A = \frac{V_T - E_C}{R_A}$$



Where: V_T = terminal voltage, E_c = counter EMF, R_A is the armature resistance and I_A is the armature current

The relationship between the induced EMF and torque

$$E_{conductor} = B\ell v$$

$$T_{conductor} = B\ell ir$$

$$therefore,$$

$$\frac{E}{T} = \frac{B\ell v}{B\ell ir} = \frac{v}{ir}, v = \omega r$$

$$\frac{E}{T} = \frac{\omega}{i}$$

$$\underbrace{Ei}_{\substack{electric \\ power}} = \underbrace{T\omega}_{\substack{mechanical \\ power}}$$

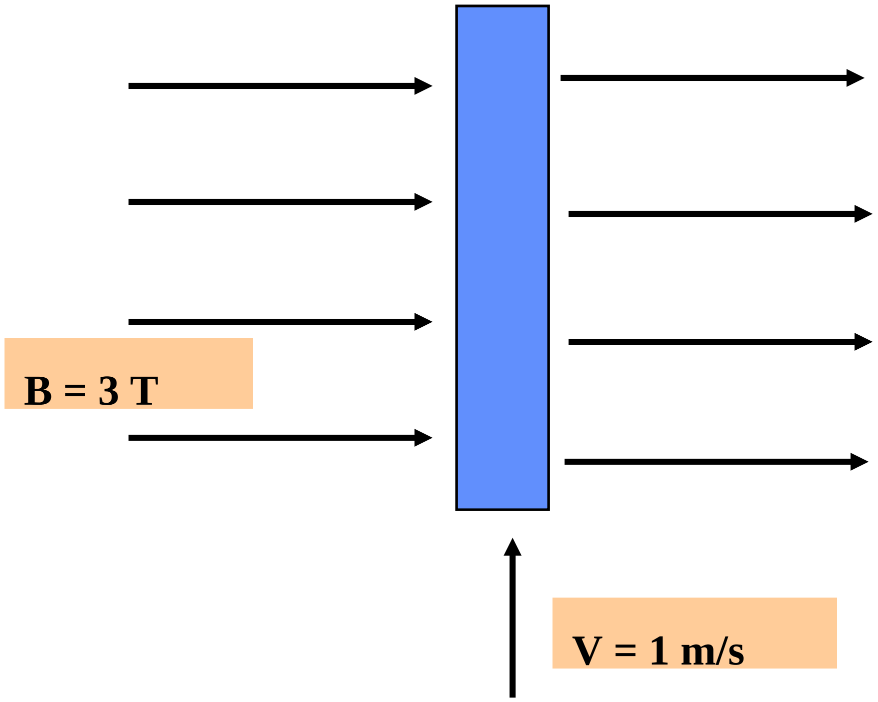
$$T = \frac{EI}{\omega} = \frac{ZP}{2\pi a}\phi I_A$$

$$T = k_m \phi I_A$$

Where: T is the torque,

Example 1

Find the induced voltage for a road with a length of 1 m.



$$e_{ind} = (v \times B).l$$

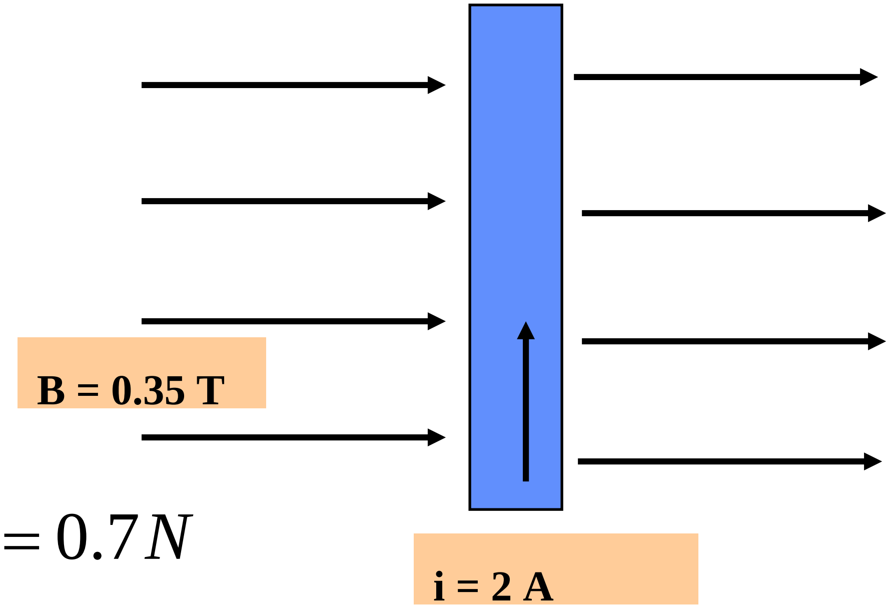
$$v \times B = vB = 3 \text{ (into the page)}$$

$$v \times B \text{ makes } 90^{\circ} \text{ with } l$$

$$(v \times B).l = 0$$

Example 2

Example: Find the force for a conductor with a length of 1 m.



$$F = (l \times B)i = (1)*(2)*(0.35) = 0.7N$$

The direction is into the page

Example 3

A 6 pole DC machine has an armature connected as a Wave winding. The aramature has 48 slots with 4 conductors/slot. The armature is rotating @ 600 rpm and the flux per pole is 30 mWb, calculate the induced voltage.

$$E_G = \frac{ZP}{60a}\phi n = \frac{48*4*6}{60*2}*600*30*10^{-3} = 172.8V$$

Example 4

A six-pole DC machine has a flux per pole of 30 mWb. The armature has 536 conductors connected as a lap winding. The DC machine runs at 1050 rpm and it delivers a rated armature current of 225 A to a load connected to its terminals, calculate:

A) Machine constant, K_m

B) Generated voltage, E_G

C) Conductor current

D) Electromagnetic torque.

E) Power delivered by the machine.

Example 4

$$a)\ k_m = \frac{ZP}{2\pi a} = \frac{6*536}{2\pi*6} = 85.31$$

$$b)\ \omega = \frac{2\pi n}{60} = \frac{2\pi*1050}{60} = 109.96 \text{ rad/s}$$

$$E_G = k_m \phi \omega = 85.31*0.03*109.96 = 281.4 \text{ V}$$

$c)$ Since it is lap winding :

$$I(conductor) = \frac{I_a}{a} = \frac{225}{6} = 37.5A$$

$$d)\ T = k_m \phi I_a = 85.31*0.03*225 = 575.84\ N.m$$

$$e)\ P = T\omega = E_G I_a = 63.32 \text{ kW}$$

DC Motors

Equivalent circuit.

The equivalent circuit of DC Motors (and Generators) has two components:

- **Armature circuit:** it can be represented by a voltage source and a resistance connected in series (the armature resistance). The armature winding has a resistance, R_a.

- **The field circuit:** It is represented by a winding that generates the magnetic field and a resistance connected in series. The field winding has resistance R_f.

Classification of DC Motors

- **Separately Excited and Shunt Motors**

- Field and armature windings are either connected separate or in parallel.

- **Series Motors**

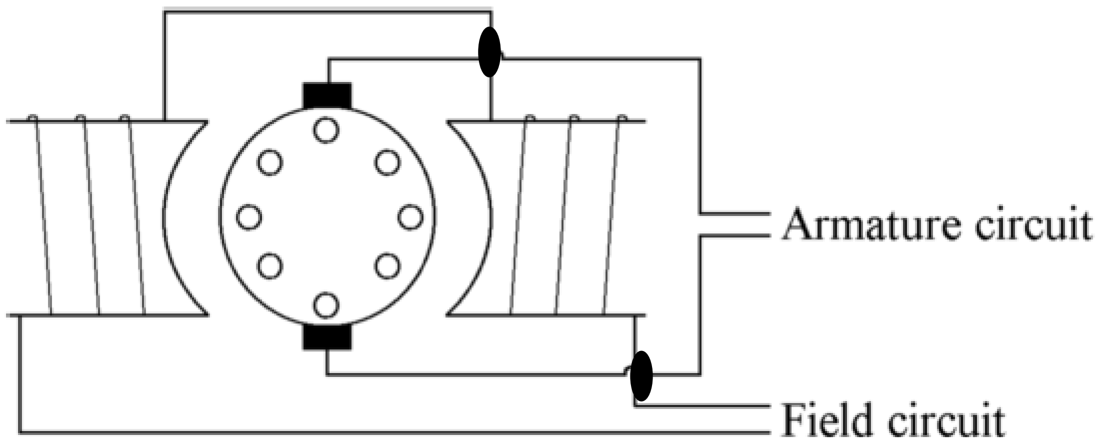
- Field and armature windings are connected in series.

- **Compound Motors**

- Has both shunt and series field so it combines features of series and shunt motors.

Shunt DC Motors

– The armature and field windings are connected in parallel.
– Constant speed operation.



Shunt DC Motors

By KVL around the outer loop:

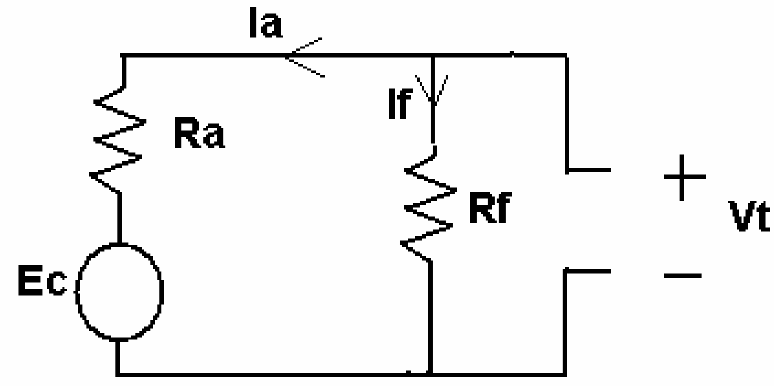
$V_T - I_A R_A - E_C = 0$

$$I_F = \frac{V_T}{R_F}$$

$$E_C = k_g \phi n \text{ but}$$

$$I_f \propto \phi, \text{so } I_f = m\phi$$

$$E_C = kI_f n$$



Starting of Shunt DC Motors

- At the starting of a DC motor, EC = 0, so:

$$I_A = \frac{V_T}{R_a}$$

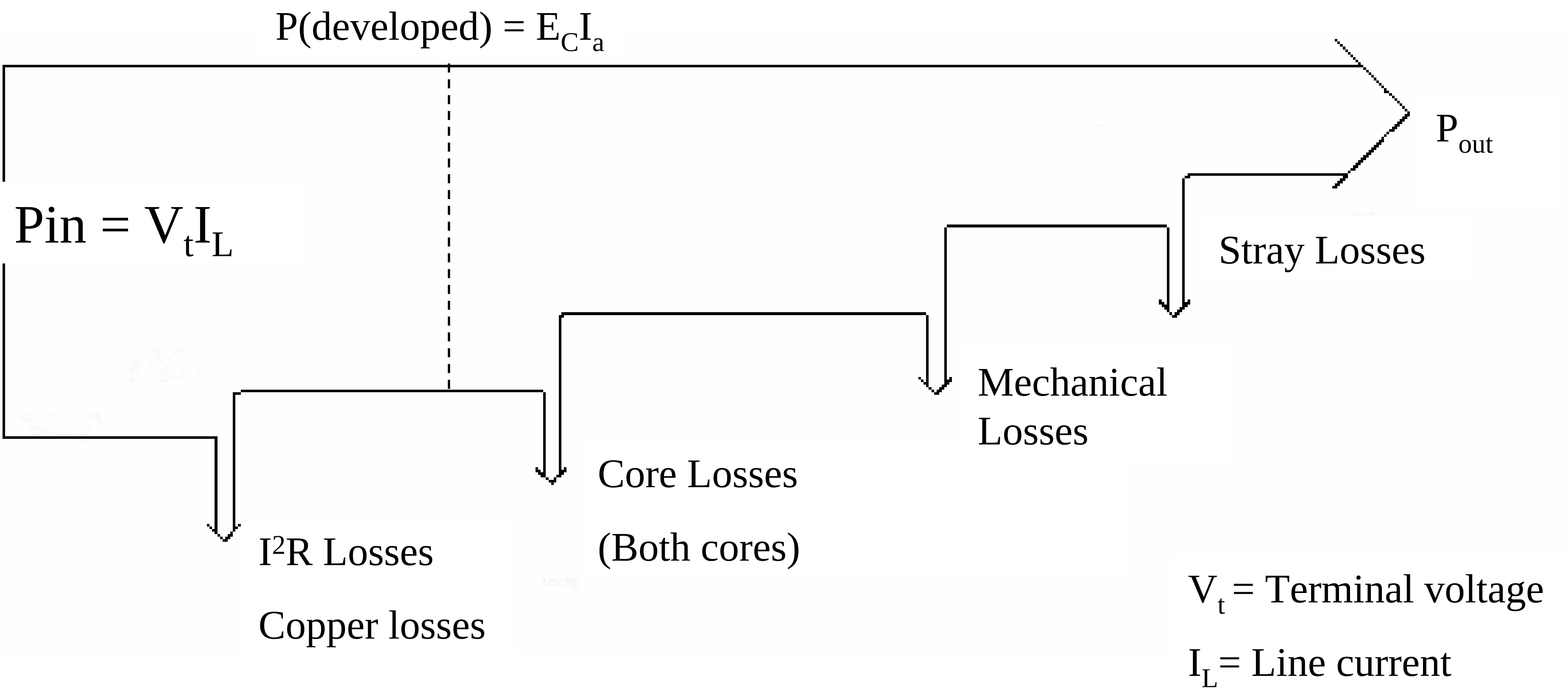
- To limit I_A, a resistance is inserted in series with R_A then removed after the development of E_C

Speed Regulation

Speed regulation is the percentage change in speed from no-load to full-load as a function of the full load speed.

$$SR = \frac{n_{NL} - n_{FL}}{n_{FL}} \times 100\%$$

Power Flow and Losses in DC Motors



V_t = Terminal voltage

I_L = Line current

Example 1

Q1) A 240 V, shunt DC motor takes an armature current of 20 A when running at 960 rpm. The armature resistance is 0.2 Ω. Determine the no load speed if the no load armature current is 1 A.

Example 1

$$V_t = I_a R_a + E_c$$

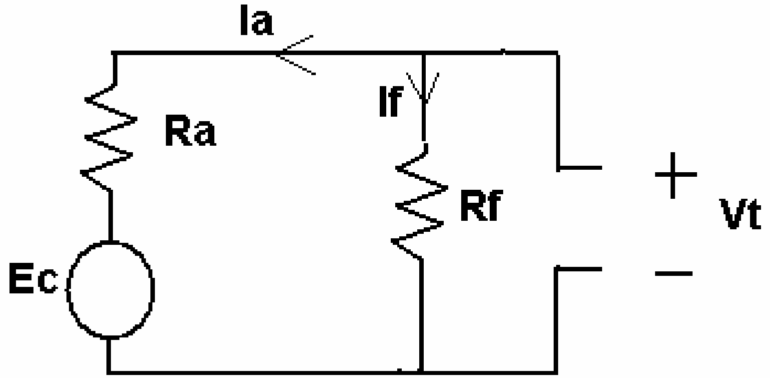
$$\Rightarrow E_c \text{ (full load)} = 240 - 20 * 0.2 = 236 \text{ V}$$

$$E_c \text{ (no load)} = 240 - 1 * 0.2 = 239.8 \text{ V}$$

assuming I_f is constant

$$\Rightarrow \frac{E_c(n.l.)}{E_c(F.l.)} = \frac{n(n.l.)}{n(F.l.)} \Rightarrow n(n.l.) = \frac{239.8}{236} * 960 = 975.45 \text{ rpm}$$

$n.l. =$ no load, $F.l. =$ full load



Example 2

Q2) A 120 V shunt motor has the following parameters: Ra = 0.4 Ω, RF = 120 Ω and rotational (core, mechanical and stray) losses are 240 W. On full load, the line current is 19.5 A and the motor runs at 1200 rpm, find:

- The developed power
- The output power, and
- The output torque.

Example 2

$$(a)\ I_a = I_L - I_f$$

$$I_f = \frac{120}{120} = 1A \Rightarrow I_a = 19.5 - 1 = 18.5 \text{ A}$$

$$E_c = V_T - I_a R_a = 120 - (18.5)*(0.4) = 112.6 \text{ V}$$

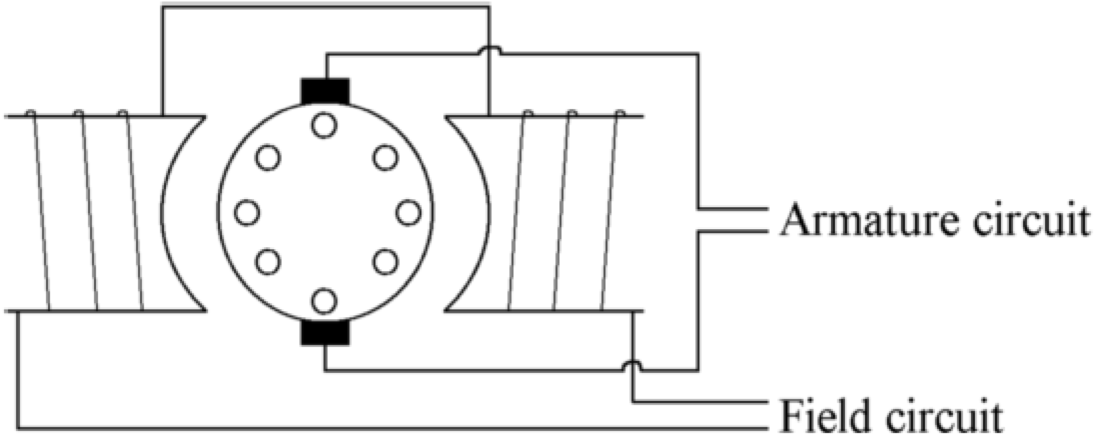
$$P_{developed} = E_c * I_a = 112.6*18.5 = 2083.1 \text{ watt}$$

$$\text{(b) } P_{out} = P_{developed} - rotational\ losses = 2083 - 240 = 1843 \text{ watt}$$

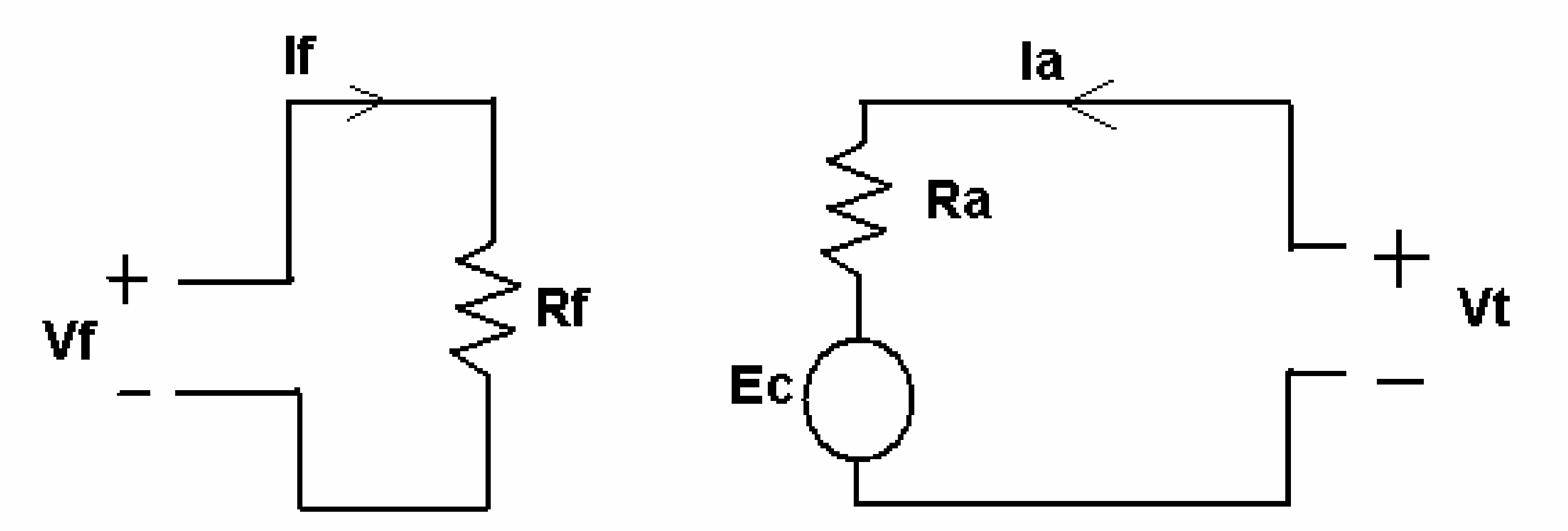
$$(c)\ \tau_{out} = \frac{P_{out}}{\omega} = \frac{1843}{\frac{2\pi}{60}1200} = 14.67 \text{ N.m}$$

Separately Excited DC Motors

- The armature winding supplies the load.
- The field winding is supplied by a separate DC source whose voltage is variable.
- Good speed control.

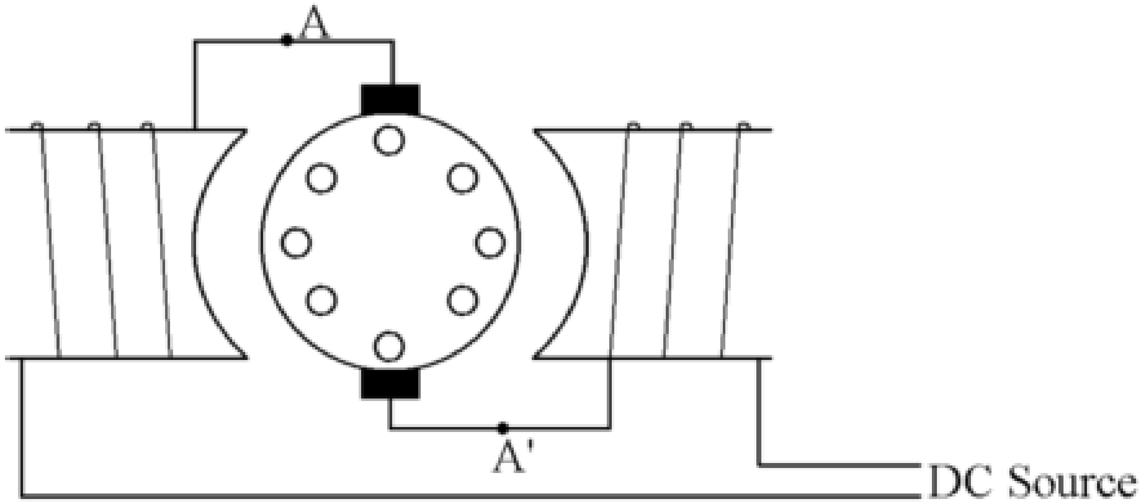


Separately Excited DC Motors



Series DC Motors

– The armature and field winding are connected in series.
– High starting torque.



Series DC Motors

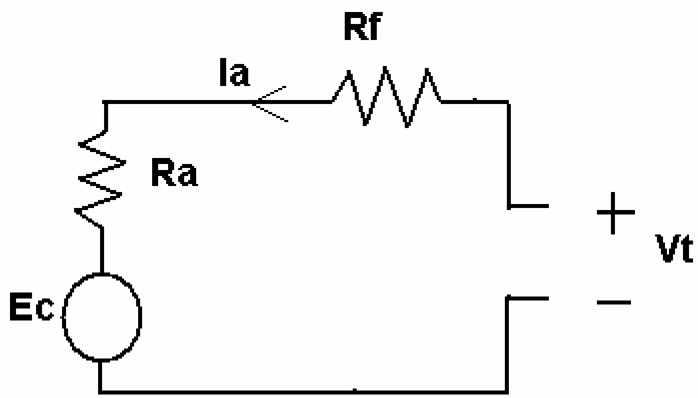
By KVL around the loop:

V_T - I_A (R_A + $R_{f)}$ - E_C = 0

$$E_C = k_g \phi n \text{ but}$$

$$I_A \propto \phi, \text{so } I_A = m\phi$$

$$E_C = kI_A n$$



Example 3

Q1) A DC series motor is operated at full load from a 240 V supply at a speed of 600 rpm. The EC is found to be 217.2 V at a line current of 38 A, find:

a) The armature resistance assuming the series field resistance is 0.2 Ω.

b) Find the no-load speed given that the no-load current is 1 A.

Example 3

$$(a)\, V_t = E_c + I_a * (R_a + R_f)$$

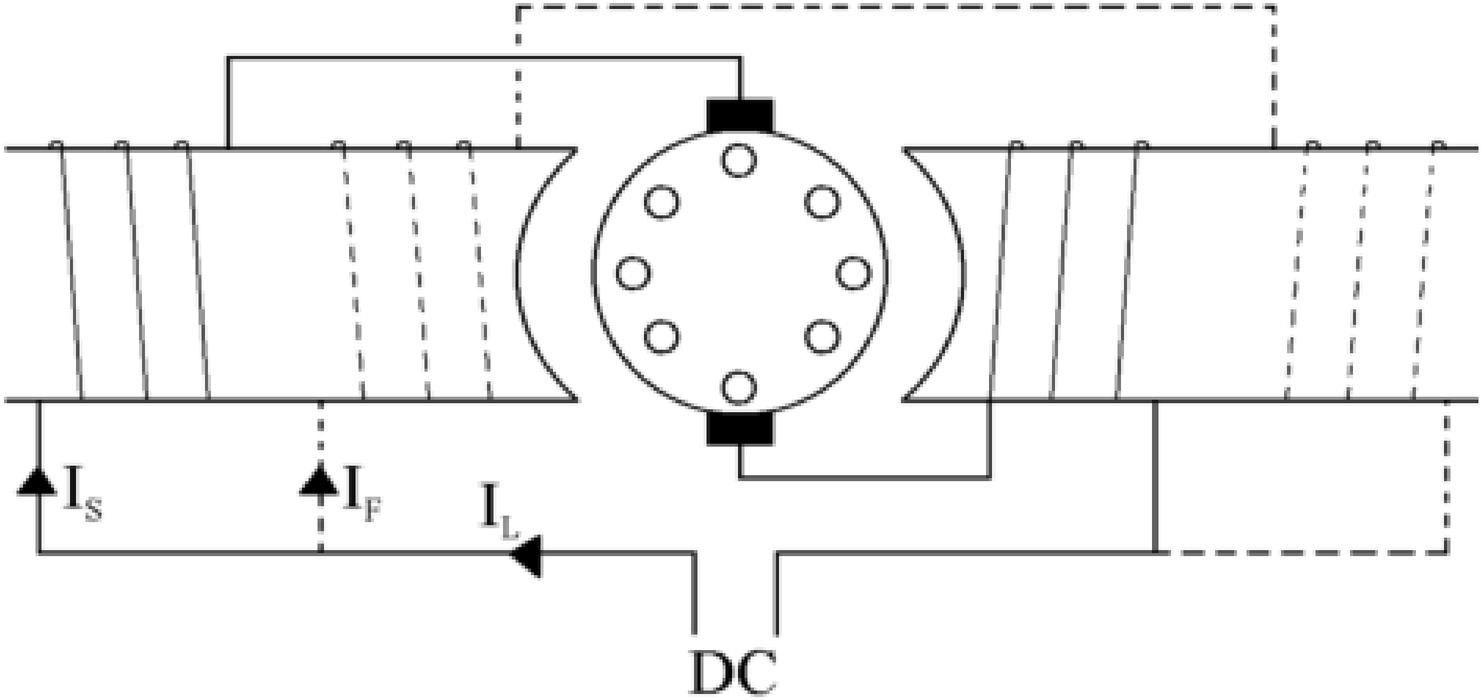
$$\Rightarrow R_a + R_f = \frac{240 - 217.2}{38} = 0.6\,\Omega$$

(b) At no load:

$$E_c = 240 - 1 * 0.6 = 239.6\text{ V}$$

$$\frac{E_{c2}}{E_{c1}} = \frac{n_2 I_{a2}}{n_1 I_{a1}} \Rightarrow n_2 = 25{,}151\text{ rpm}$$

Compound DC Motors



Speed Control of DC Motors

Speed can be controlled by varying:

1) Armature circuit resistance using an external resistance $R_{A\ Ext}$.
2) I_F can be varied by using an external resistance R_{adj} in series with R_F to control the flux, hence the speed.
3) The applied voltage to the armature circuit resistance, if the motor is separately excited

Torque developed by shunt motor

$$V_T = E_C + I_A R_A \quad \text{so}$$

$$V_T = k_m \phi \omega + I_A R_A \quad \text{but}$$

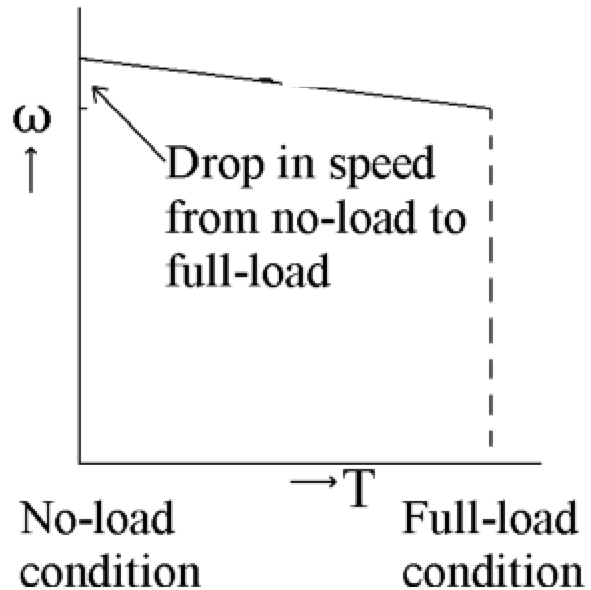
$$I_A = \frac{T}{k_m \phi} \quad \text{so}$$

$$V_T = k_m \phi \omega + \frac{T}{k_m \phi} R_A \quad \text{solving for } \omega$$

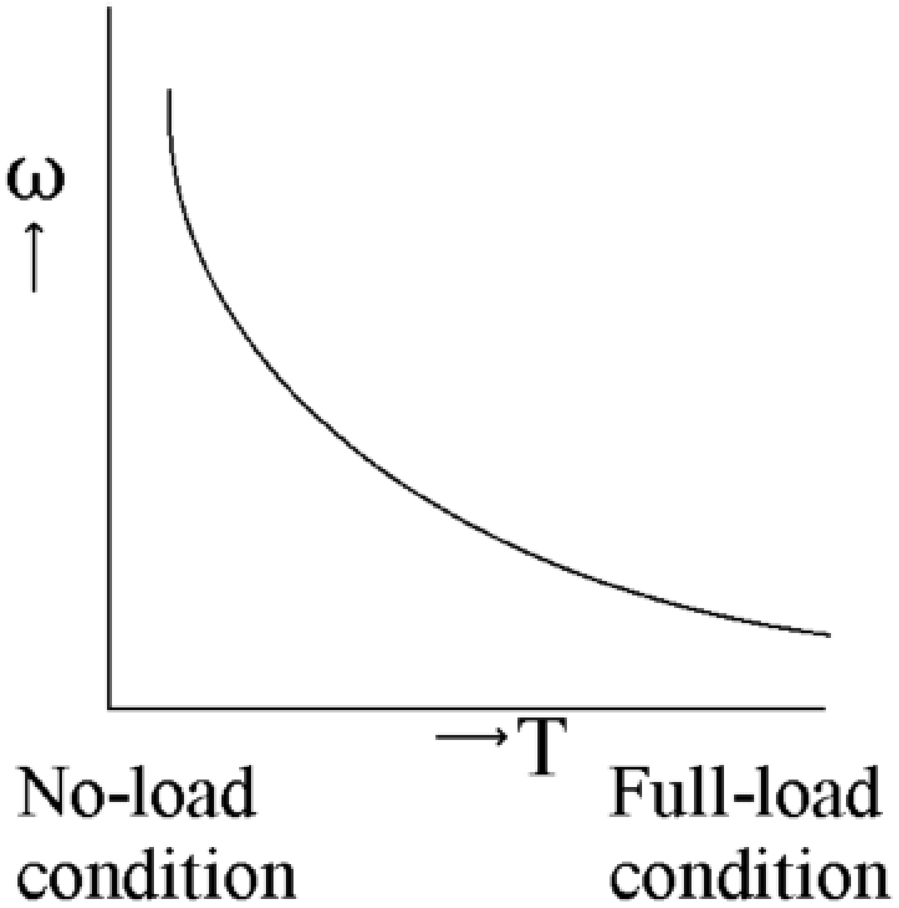
$$\omega = \frac{V_T}{k_m \phi} - \frac{R_A}{\left(K_m \phi\right)^2} T$$

Torque developed by shunt motor

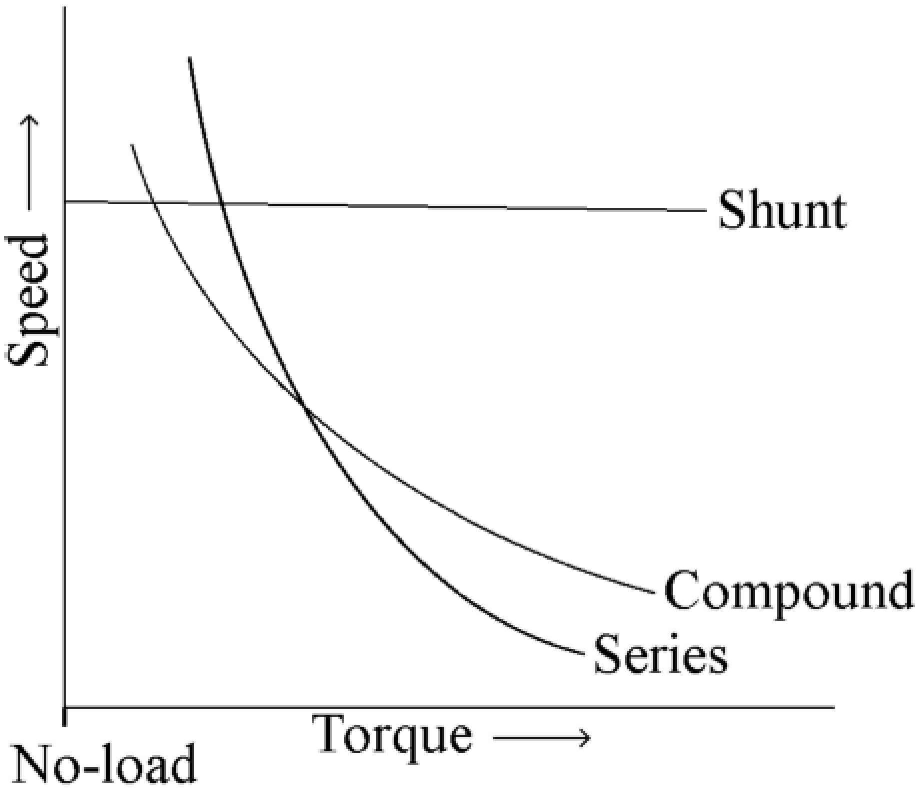
If V_T and I_F (hence ☠) are constant, speed is directly proportional to the torque



Torque developed by series motor



Comparison between different DC Motors



Comparison of DC Motors

Shunt Motors: **"Constant speed" motor (speed regulation is very good). Adjustable speed, medium starting torque. ($_{Start}$ = 1.4 × T_{FL})**

Applications: **centrifugal pump, machine tools, blowers fans, reciprocating pumps, etc.**

Series Motors: **Variable speed motor which changes speed drastically from one load condition to another. It has a high starting torque.**

Applications: **hoists, electric trains, conveyors, elevators, electric cars, etc.**

Compound motors: **Variable speed motors. It has a high starting torque and the no-load speed is controllable unlike in series motors.**

Applications: **Rolling mills, sudden temporary loads, heavy machine tools, punches, etc**

Home work problems

Q no.1

A 30 hp, 240 V, 1150 rpm DC shunt motor, operating at rated conditions, has an efficiency of 88.5 percent. The armature resistance is 0.064 Ω and the field resistance is 93.6 Ω. Draw an equivalent circuit for the motor, and determine:

a. what percentage of the total losses are due to rotation losses;
b. the external resistance needed in series with the armature circuit to limit the starting current to 175 percent of rated armature current on start-up; and,
c. the new speed if the flux is reduced by 10 percent and the shaft load is adjusted to maintain rated armature current.

Home work problems

Q no.2

A 230 V DC shunt motor has a full-load efficiency of 89.9%. Rated (full) load is 40 hp. Motor losses, expressed as a percentage of output power are as follows: rotational losses - 3.7%; armature copper loss - 3.9%; and, field copper loss - 2.5%. Stray load losses can be neglected. At full load, the motor speed is 1150 rpm. Neatly sketch an equivalent circuit for the DC shunt motor, and determine:

a. the field current;
b. the armature current at full load;
c. the armature current at half load;
d. the armature current at no load; and,
e. the shaft torque at full load.