

# Lecture Notes

# ELEC A6

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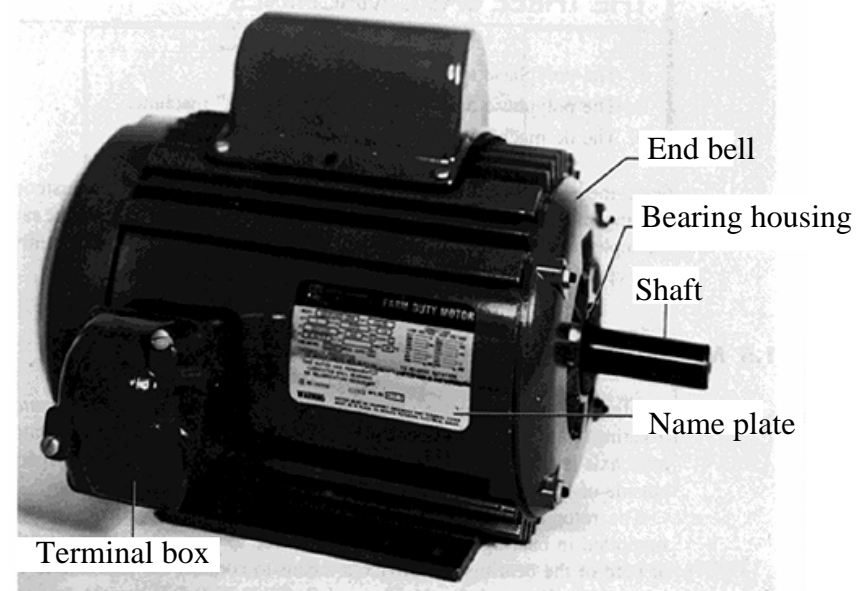
## **Induction Machines**

# INDUCTION MOTORS

## General

- The induction machine is used as a motor and as a generator. However, it is most frequently used as a motor. It is the Workhorse of industry.
- Majority of the motors used by industry are squirrel cage induction motors.
- Both three-phase and single-phase motors are widely used.
- The induction generators are seldom used. Their typical application is the wind power plant.

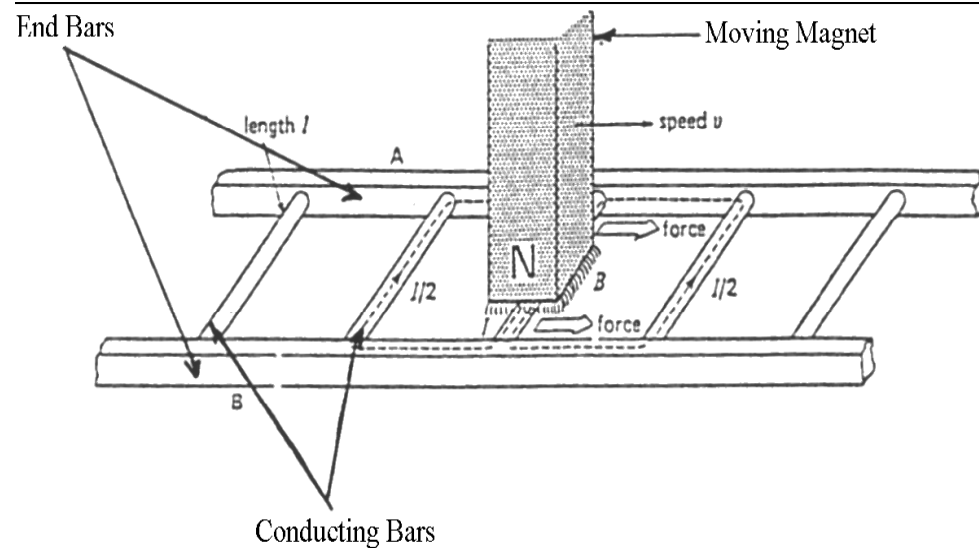
## Single phase induction motor



# INDUCTION MOTORS

## Basic principles:

- An emf is induced in the conducting bars as they are “cut” by the flux while the magnet is being moved.
- $E = BVL$  (Faraday’s Law)
- The emf induces or produces a current  $I$ , which in term produces a force,  $F$ .
- $F = BIL$  Lorentz Force



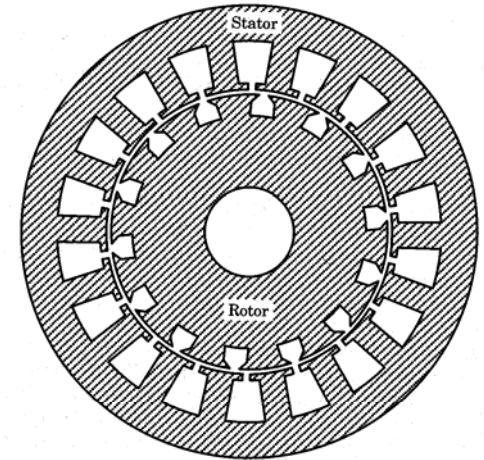
# INDUCTION MOTORS

- **Stator construction**

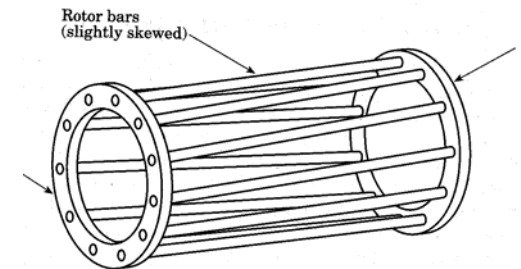
- The stator of an induction motor is similar to a stator of any synchronous motor.
- Laminated iron core with slots
- Coils are placed in the slots to form a three or single phase winding

- **Squirrel-cage rotor construction**

- Laminated Iron core with slots
- Metal bars are molded in the slots
- Two rings short circuits the bars
- The bars are slanted to reduce noise



(a)



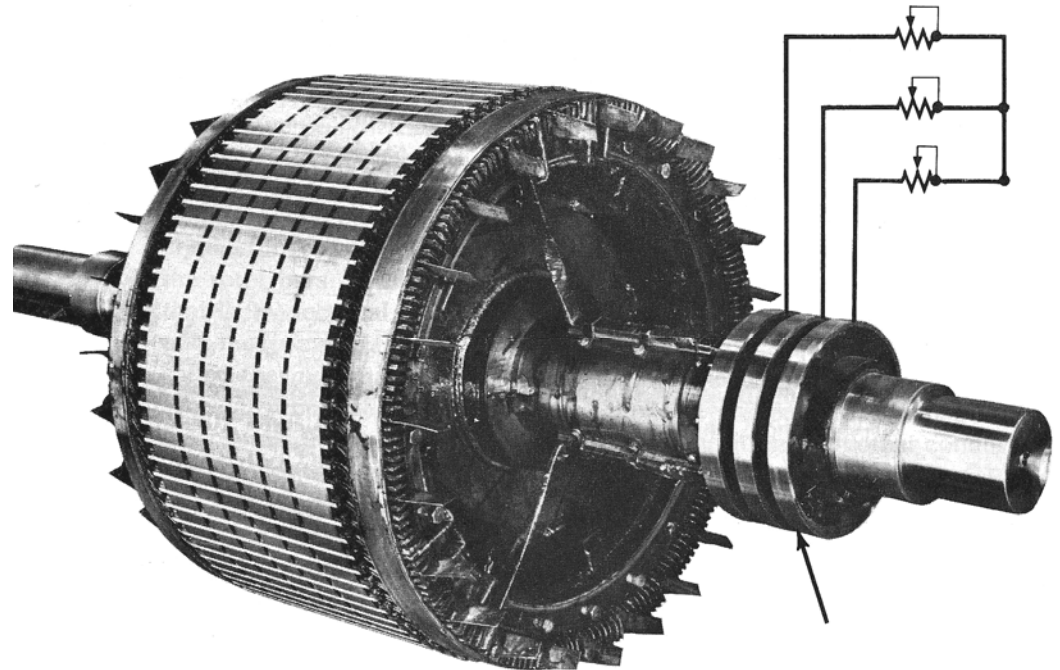
(b)

# INDUCTION MOTORS

## Wound-rotor

- The picture shows the rotor of a large wound-rotor motor
- The ends of each phase is connected to a slip ring.
- Three brushes contact the three slip-rings to three wye connected resistances.

## Rotor construction



# INDUCTION MOTORS

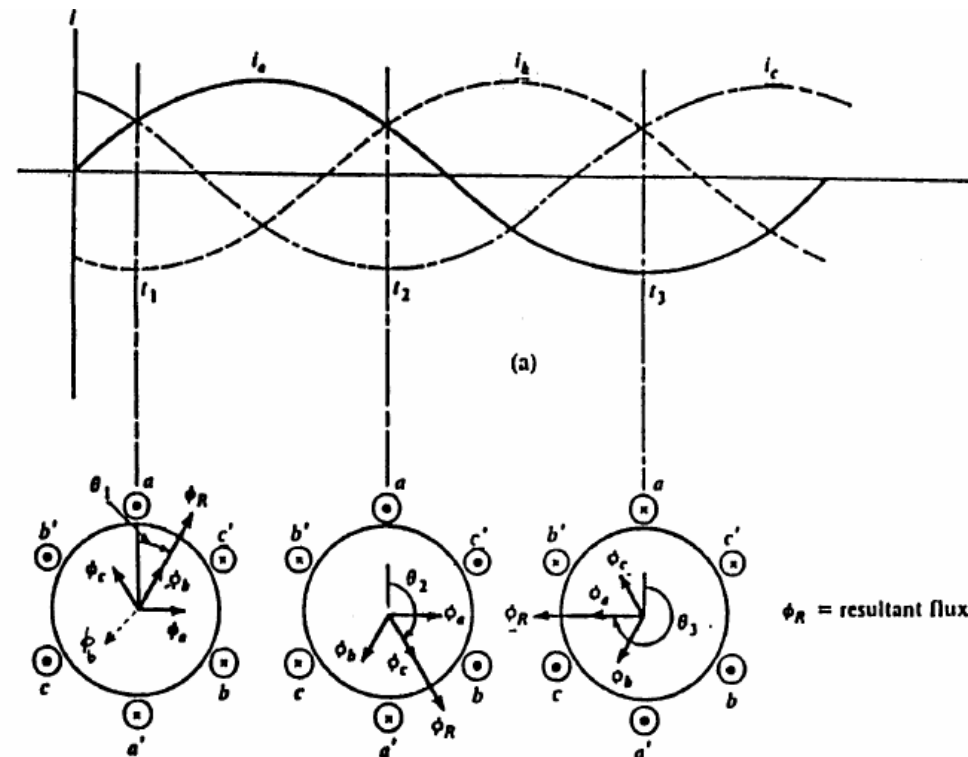
## Rotating Magnetic Field

- In ac machines, the three-phase currents  $i_a$ ,  $i_b$  and  $i_c$ , each of equal magnitude, but differing in phase by  $120^\circ$ , produce a magnetic field of constant magnitude that rotates in space. Such a magnetic field produced by balanced three phase currents flowing in three-phase windings is called a rotating magnetic field (RMF). Existence of a RMF is an essential condition for the operation of a ac rotating machine.

# INDUCTION MOTORS

## Production of RMF:

- The concept of RMF can be illustrated using the following graphical representation. Consider a set of balanced three-phase currents  $i_a$ ,  $i_b$  and  $i_c$ , flowing through the three-phase windings  $aa'$ ,  $bb'$  and  $cc'$  (for simplicity, only one coil per phase is considered).
- The coils  $aa'$ ,  $bb'$  and  $cc'$  are displaced in space, by  $120^\circ$ . The currents in each coil are responsible for producing their own magnetic flux,  $\phi_a$ ,  $\phi_b$  and  $\phi_c$  respectively.
- The following figure shows the resultant flux  $\phi_r$  that results from these three fluxes at any given instant in time.  $\phi_r$  is however, (i) constant in magnitude but (ii) rotates in space with time.



# INDUCTION MOTORS

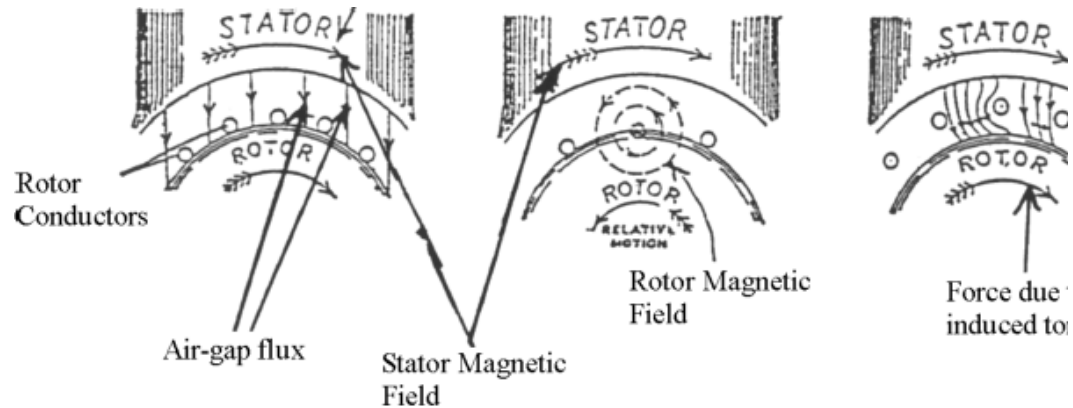
## Three-phase motors. Operation principles.

- 1) Energize the stator with three-phase voltage.
- 2) Currents in the stator winding produce a rotating magnetic field. This field revolves in the air gap.
- 3) The stator magnetic field links the rotor conductors through the air gap and voltage will be induced in the rotor conductors.
- 4) Currents in the rotor conductors will produce their own magnetic field which opposes the stator magnetic field.
- 5) The torque developed due to interaction of the stator and rotor magnetic fields pushes the rotor into rotation.
- 6) The direction of the rotation of the rotor is the same as the direction of the rotation of the revolving magnetic field in the air gap.



# INDUCTION MOTORS

Assume that the RMF produced by the stator currents rotates in the clockwise direction.



Hence the direction of the magnetic field (flux lines) produced by the rotor currents is counterclockwise.

The rotor conductors are therefore pushed from left (strong field region) to the right (weak field region). Hence, the rotor rotates in the same direction as that of the RMF.

# INDUCTION MOTORS

## Synchronous Speed and Slip

- The stator magnetic field (rotating magnetic field) rotates at a speed,  $n_s$ , the synchronous speed.
- If,  $n_m$  = speed of the rotor, the “slip”  $s$  for an induction motor is defined;

$$s = \frac{n_s - n_m}{n_s} \times 100\%$$

# INDUCTION MOTORS

## Synchronous Speed and Slip

- At stand still,  $s = 1$ , that is  $n_m = 0$ . At synchronous speed,  $n_m = n_s$ ,  $s = 0$ .
- The mechanical speed of the rotor, in terms of slip and synchronous speed:

$$n_m = (1 - s)n_s$$

$$\omega_m = (1 - s)\omega_s$$

# INDUCTION MOTORS

## Frequency of Rotor Currents and Voltages:

With the rotor at stand-still, the frequency of the induced voltages and currents is the same as that of the stator (supply) frequency,  $f_e$ .

If the rotor rotates at speed of  $n_m$ , then the relative speed is the slip speed

$$n_{slip} = n_s - n_m$$

$n_{slip}$  is the speed responsible for the induction.

But  $n_m = n_s(1 - s)$  by definition of slip.

Hence,  $n_{slip} = n_s - n_s(1 - s)$ , thus the frequency of the induced voltages

and currents is,  $f_r = sf_e$ .

# INDUCTION MOTORS

## Example no. 1:

A three-phase, 20 hp, 208 V, 60 Hz, six pole, wye connected induction motor delivers 15 kW at a slip of 5%.

Calculate:

- Synchronous speed
- Rotor speed
- Frequency of rotor current

## Solution

- Synchronous speed:  $n_s = 120 f / p = (120) / 6 = 1200 \text{ rpm}$
- Rotor speed:  $n_r = (1-s) n_s = (1- 0.05) (1200) = 1140 \text{ rpm}$
- Frequency of rotor current:  $f_r = s f = (0.05) (60) = 3 \text{ Hz}$

# INDUCTION MOTORS

## Three phase motors. Development of equivalent circuit

- The induction motor consists of a two magnetically connected systems: Stator and rotor.
- This is similar to a transformer that also has two magnetically connected systems: primary and secondary windings.
- The stator is supplied by a balanced three-phase voltage that drives a three-phase current through the winding. This current induces a voltage in the rotor.
- The applied voltage ( $V_1$ ) across phase A is equal to the sum of the
  - induced voltage ( $E_1$ ).
  - voltage drop across the stator resistance ( $I_1 R_1$ ).
  - voltage drop across the stator leakage reactance ( $I_1 j X_1$ ).

# INDUCTION MOTORS

$I_1$  = stator current/phase

$R_1$  = stator winding resistance/phase

$X_1$  = stator winding reactance/phase

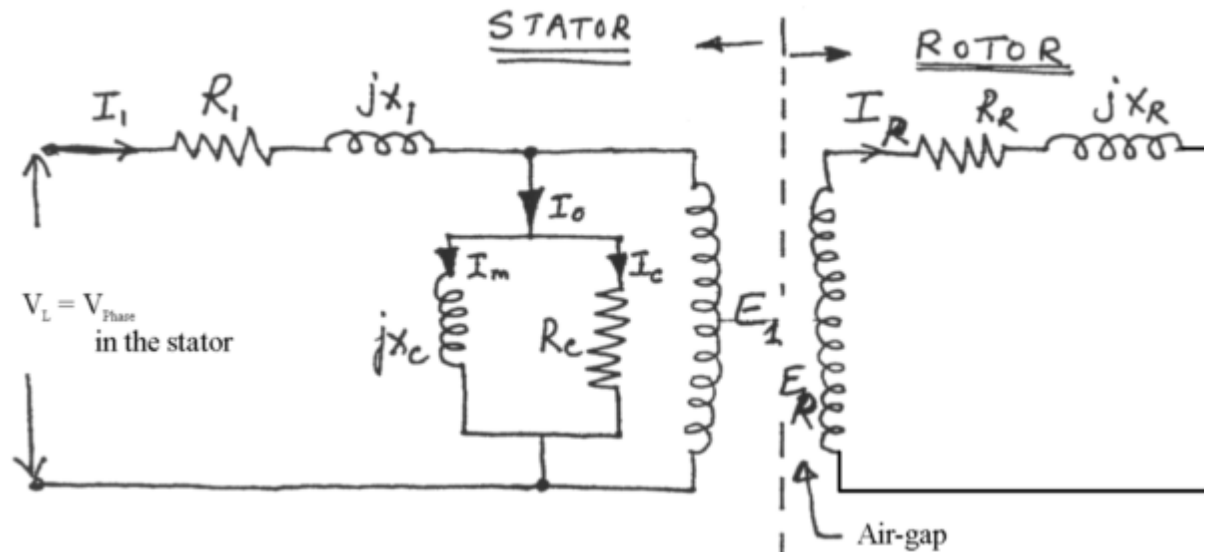
$R_R$  and  $X_R$  are the rotor winding resistance and reactance per phase, respectively

$I_R$  = rotor current

$V_1$  = applied voltage to the stator/phase

$$I_0 = I_c + I_m$$

( $I_m$  = magnetizing current,  $I_c$  = core-loss component of current)



# INDUCTION MOTORS

Induced voltages:

Let  $E_{R0}$  be the induced voltage in the rotor at stand-still

$$\square E_{R0} = 4.44 N_R \phi_m f_r$$

since,  $f_r = f_e$ , at stand-still,

$$E_{R0} = 4.44 N_R \phi_m f_e$$

If  $E_R$  is the induced voltage in the rotor winding with  $f_r = sf_e$ , ( $s \neq 1$ ) then,

$$E_R = 4.44 N_R \phi_m f_r$$

$$E_R = 4.44 N_R \phi_m s f_e$$

$$E_R = s E_{R0}$$

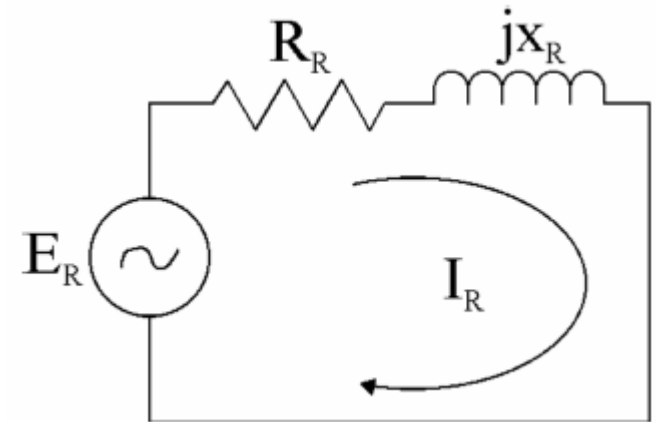


# INDUCTION MOTORS

## Rotor Circuit alone:

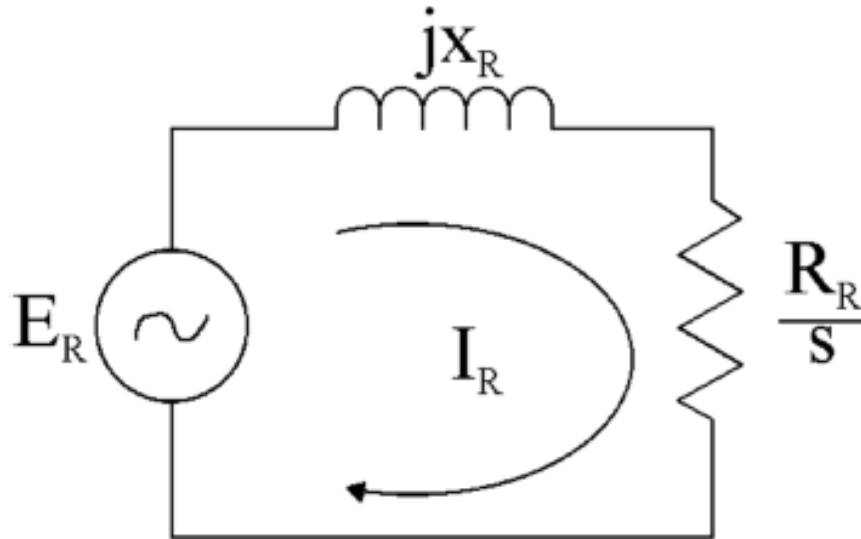
$$I_R = \frac{E_R}{R_R + jX_R} = \frac{s \cdot E_{R_0}}{R_R + s \cdot jX_{R_0}}$$

$$I_R = \frac{E_{R_0}}{\frac{R_R}{s} + jX_{R_0}}$$



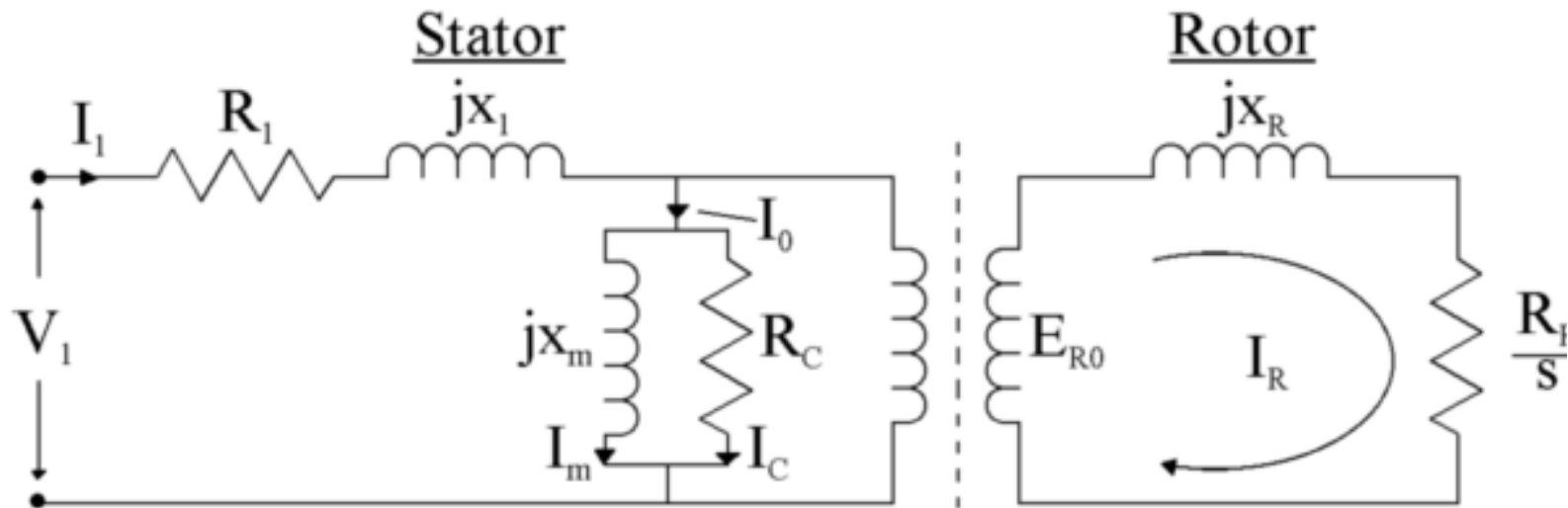
# INDUCTION MOTORS

The rotor circuit can be represented as:



# INDUCTION MOTORS

So, the Induction Motor circuit can be represented as:



# INDUCTION MOTORS

Transformation is done using the effective turns ratio,  $a_{eff}$  for currents.

$$I_2 = \frac{I_R}{a_{eff}}$$

Impedance transfer is made using the ration  $a_{eff}^2$ ; where  $R_2$  and  $X_2$  are transferred values.

$$R_2 = a_{eff}^2 R_R$$

$$X_2 = a_{eff}^2 X_R$$

# INDUCTION MOTORS

## Equivalent Circuit and Power Flow

$P_{in}$  = input power to the motor  
(3 phase)

$$P_{in} = \sqrt{3}V_L I_L \cos \theta = 3V_\phi I_\phi \cos \theta$$

$R_1$  = accounts for the stator copper losses ( $P_{SCL}$ )

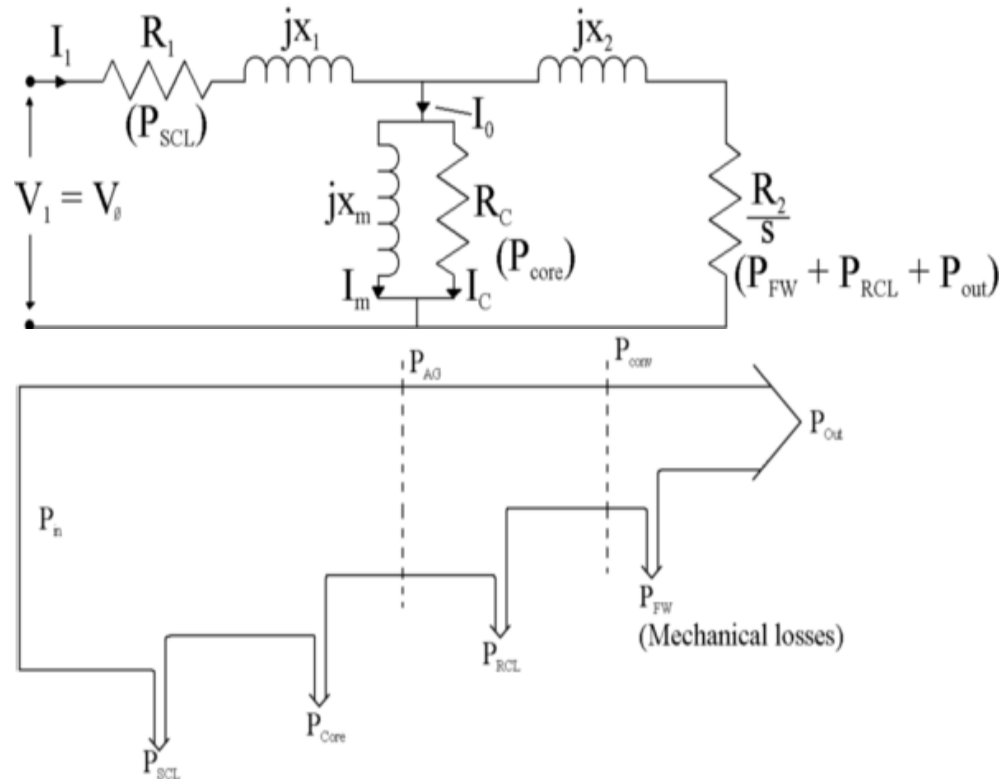
$R_C$  = accounts for the core losses

$R_2/s$  = accounts for the losses

$P_{FW}$ ,  $P_{RCL}$  and the output power,  $P_{out}$

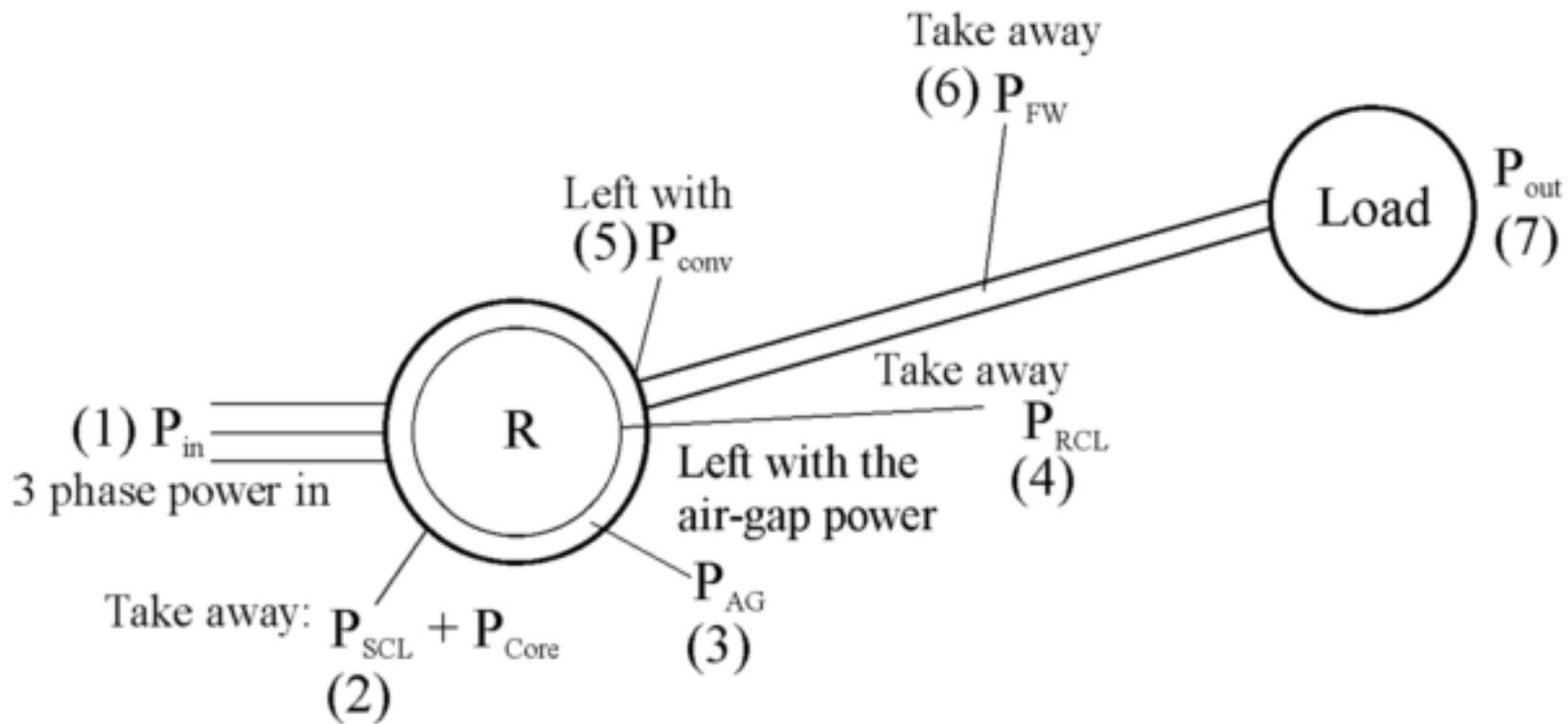
$P_{RCL}$  = rotor copper losses

$P_{FW}$  = friction and windage losses



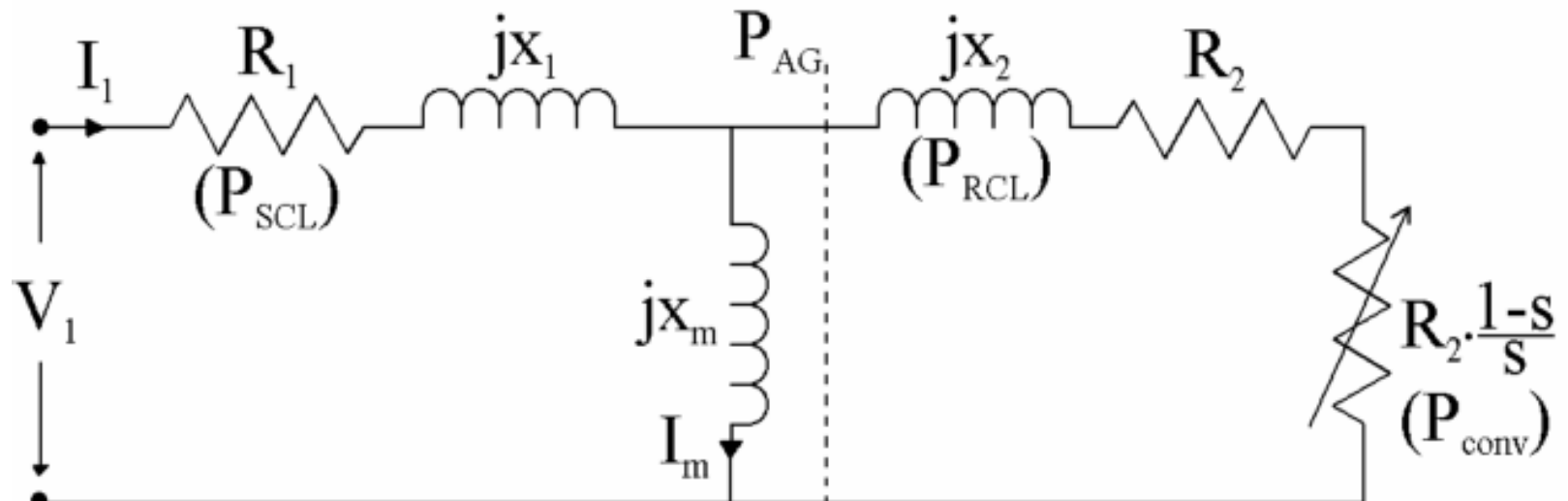
# INDUCTION MOTORS

## Equivalent Circuit and Power Flow



# INDUCTION MOTORS

- **Approximate Equivalent Circuit:**



# INDUCTION MOTORS

- **Approximate Equivalent Circuit:**

$$P_{AG} = P_{in} - P_{SCL} = \frac{3I_2^2 R_2}{s}$$

$$P_{conv} = 3I_2^2 \left( R_2 \left( \frac{1-s}{s} \right) \right) = P_{AG} - P_{RCL}$$

$$P_{out} = P_{conv} - (P_{core} + P_{FW})$$

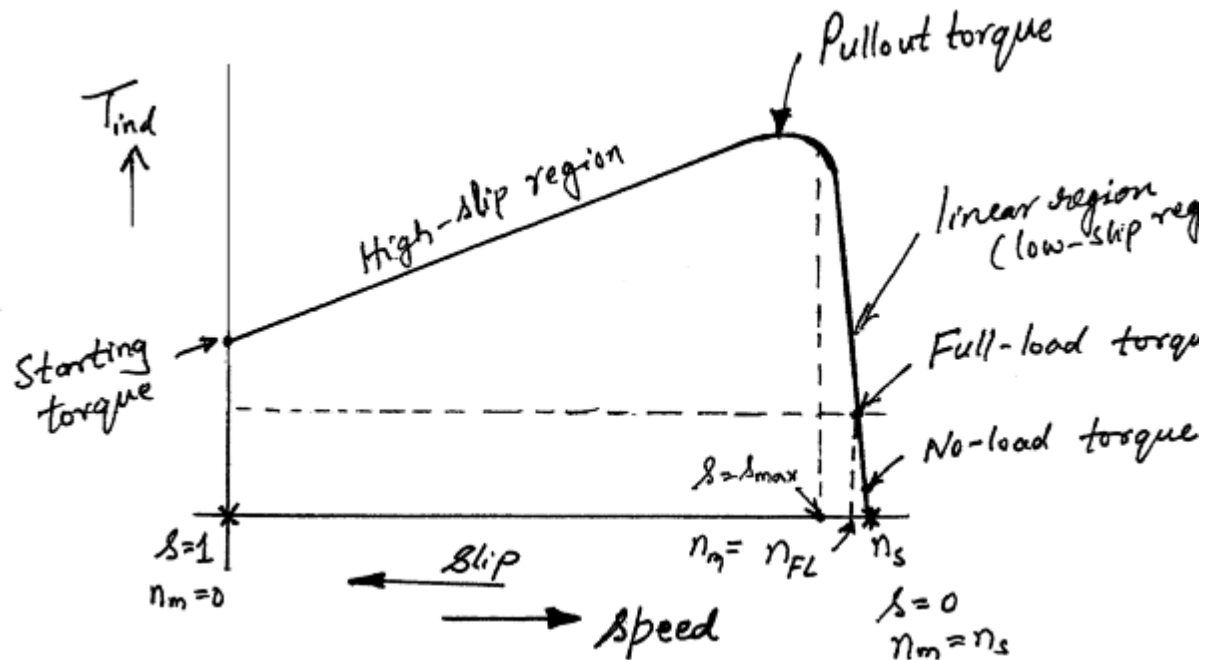
$$T_{dev} = \frac{P_{conv}}{\omega_m} = \frac{P_{AG} (1-s)}{\omega_s (1-s)} = \frac{P_{AG}}{\omega_s}$$



# INDUCTION MOTORS

## Torque-Speed Characteristic:

- For small values of  $s$ , the torque is directly proportional to  $s$ .
- For large values of  $s$ , the torque is inversely proportional to  $s$ .



# INDUCTION MOTORS

## Example no. 2:

A 480 V, 50 hp, three phase induction motor is drawing 60 A at 0.85 pf lagging. The stator copper losses are 2 kW and the rotor copper losses are 700 W. The friction and windage losses are 600 W, the core losses are 1800 W and the stray losses are negligible, find:

- The air gap power.
- The converted power.
- The output power.
- The efficiency of the motor.

# INDUCTION MOTORS

## Example no. 2 solution:

$$a) P_{in} = \sqrt{3}V_T I_L \cos(\theta)$$

$$P_{in} = \sqrt{3}(480)(60)(0.85) = 42.4 \text{ kW}$$

$$P_{AG} = P_{in} - P_{SCL} - P_{core} = 42.4 - 2 - 1.8 = 38.6 \text{ kW}$$

$$b) P_{conv} = P_{AG} - P_{RCL} = 38.6 - 0.7 = 37.9 \text{ kW}$$

$$c) P_{out} = P_{conv} - P_{F\&W} = 37.9 - 0.6 = 37.3 \text{ kW}$$

$$d) \eta = \frac{P_{out}}{P_{in}} = \frac{37.3}{42.4} = 88\%$$

# INDUCTION MOTORS

## Example no. 3:

A 460 V, 25 hp, 60 Hz, four pole, Y-connected induction motor has the following impedances:

$$R_1 = 0.641 \, \Omega$$

$$R_2 = 0.332 \, \Omega$$

$$X_1 = 1.106 \, \Omega$$

$$X_2 = 0.464 \, \Omega$$

$$X_m = 26.3 \, \Omega$$

The total rotational losses (including core losses) are 1100 W for a slip = 2.2%, find:

- (a) The speed.
- (b) The stator current.
- (c) Power factor
- (d) The converted and output power
- (e) The induced and load torque
- (f) Efficiency

# INDUCTION MOTORS

## Example no. 3 solution:

$$a) n_s \frac{120 f}{P} = \frac{(120)(60)}{4} = 1800 \text{ rpm}$$

$$n_m = (1 - s)n_s = (1 - .022)(1800) = 1760 \text{ rpm}$$

$$b) Z_{total} = \left\{ \left( \frac{R_2}{s} + jx_2 \right) \parallel (jx_m) \right\} + (R_1 + jx_1) = 14.07 \angle 33.6$$

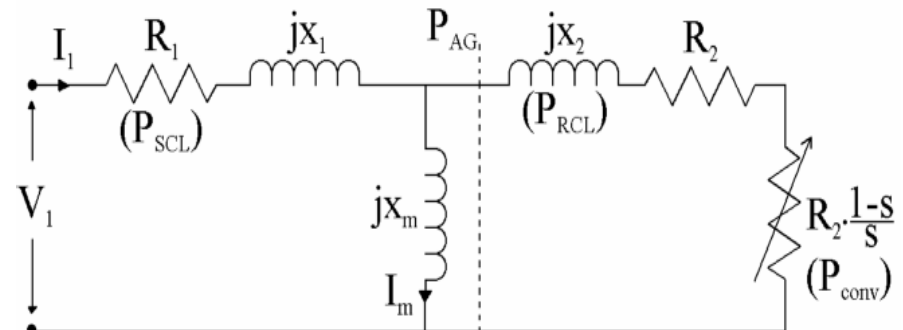
$$I_1 = \frac{V_{phase}}{Z_{total}} = 18.88 \angle -33.6$$

$$c) p.f. = \cos(33.6) = 0.833 \text{ lagging}$$

$$d) P_{in} = \sqrt{3}(480)(18.88)(0.833) = 12.53 \text{ kW}$$

$$P_{SCL} = 3I_1^2 R_1 = 3(18.88)^2 (0.641) = 685 \text{ W}$$

$$P_{AG} = P_{in} - P_{SCL} = 12,530 - 685 = 11.845 \text{ kW}$$



# INDUCTION MOTORS

## Example no. 3 solution:

$$P_{conv} = (1 - s)P_{AG} = (1 - 0.022)(11.845) = 11.858 \text{ kW}$$

$$P_{out} = P_{conv} - P_{rot} = 11.858 - 1.100 = 10.485 \text{ kW}$$

$$\text{e) } \tau_{ind} = \frac{P_{AG}}{\omega_s} = \frac{11,845}{188.5} = 62.8 \text{ N.m}$$

$$\tau_{out} = \frac{P_{out}}{\omega_m} = \frac{10,845}{184.4} = 56.9 \text{ N.m}$$

$$\text{f) } \eta = \frac{10,845}{12,530} = 83.7\%$$

# INDUCTION MOTORS

## Three-phase motors. Determination of parameters from test

- The motor parameters are determined from three tests:
  - No-load test. Provides the magnetizing reactance and core resistance ( $R_c$  and  $X_m$ ). In this course we will only find  $X_m$  and ignore  $R_c$
  - Blocked-Rotor Test (Short circuit test). Provides ( $R_1 + R_2$ ) and ( $X_1 + X_2$ ).
  - Stator DC resistance measurement. Determines the stator resistance value ( $R_1$ ).

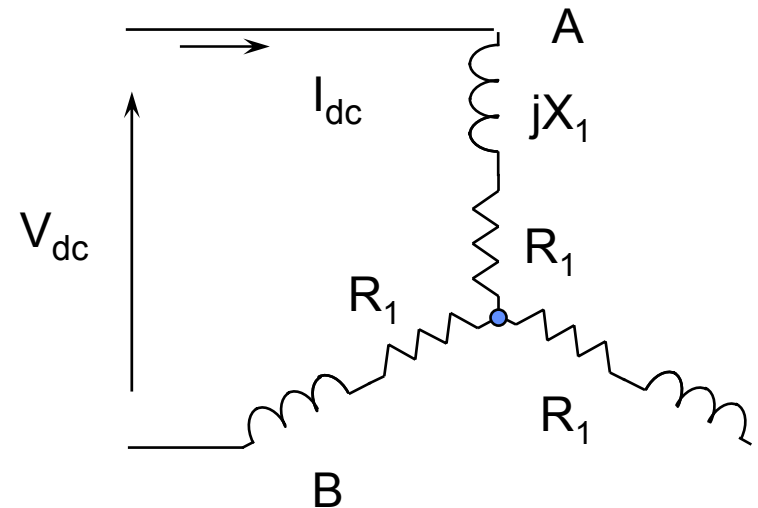
# INDUCTION MOTORS

## Three-phase motors. Determination of parameters from test

### Stator DC resistance measurement

- The motor is supplied by DC voltage between two terminals ( A and B at the figure).
- The dc voltage and current are measured.
- The resistance is:

$$R_1 = \frac{V_{dc}}{2 \cdot I_{dc}}$$





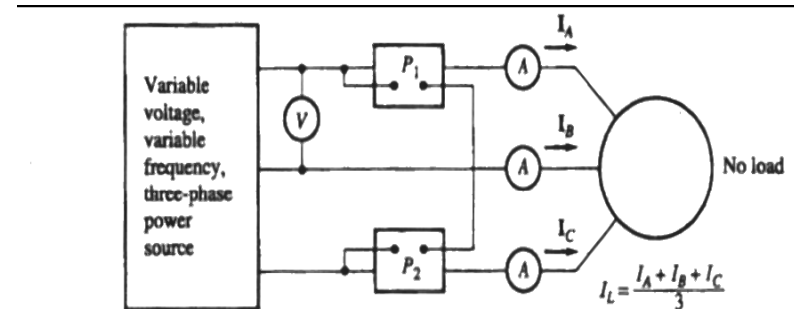
# INDUCTION MOTORS

## Three-phase motors. Determination of parameters from test

### • No-load test

- The motor is supplied by rated line -to-line voltage ( $V_{ml}$ ) and the no-load current  $I_{nl}$  and the no load input power  $P_{nl}$  are measured.
- The no-load input power includes magnetizing and rotational losses.
- Using the measured values,  $X_m$  can be calculated as follows

$$X_m = \frac{V_{n.l.}}{\sqrt{3}I_{n.l.}}$$



# INDUCTION MOTORS

## Three-phase motors. Determination of parameters from test.

- Blocked-Rotor Test

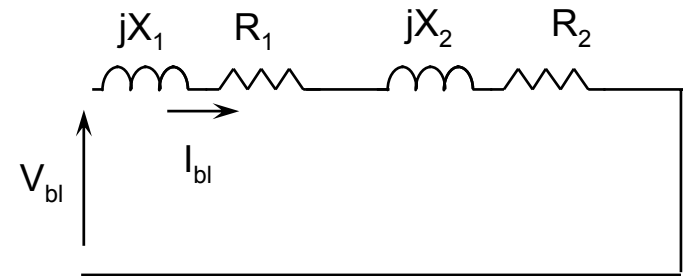
- The motor is supplied by reduced voltage  $V_{br}$  (line-to-line) and lower frequency voltage. Approximate frequency value is:  $f_{test} = (0.258)(60) = 15$  Hz. Reduced frequency simulates that rotor current frequency is small in normal operation.
- The voltage  $V_{br}$ , current  $I_{br}$ , the input power  $P_{br}$  are measured.
- The rotor is blocked slip is  $s = 1$ . Magnetizing reactance and resistance are neglected because of reduced supply voltage.

# INDUCTION MOTORS

## Three-phase motors. Determination of parameters from test.

- Blocked-Rotor Test

The approximate equivalent circuit is:



- Blocked rotor resistance is:

$$R_{br} = \frac{P_{br}}{3I_{br}^2}$$

- - Blocked rotor impedance is:  $Z_{br} = \frac{V_{br}}{\sqrt{3} I_{br}}$

# INDUCTION MOTORS

## Three-phase motors. Determination of parameters from test.

- Blocked-Rotor Test

- Blocked rotor reactance at the test frequency  $f_{\text{test}}$  is:

$$X_{\text{brtest}} = \sqrt{Z_{\text{br}}^2 - R_{\text{br}}^2}$$

- Blocked rotor reactance at the rated frequencies:

$$X_{\text{br}} = X_{\text{br, test}} (f_{\text{rated}} / f_{\text{test}})$$

- The equivalent circuit parameters are calculated from:

$$R_{\text{br}} = R_1 + R_2 \quad \text{and} \quad X_{\text{br}} = X_1 + X_2$$

- $R_1$  is determined by stator resistance measurement.

# INDUCTION MOTORS

Q1:

A 4-pole, 208 V, 3 $\phi$ , 60 Hz, 10 hp, Y-connected SCIM has the following characteristics:

$$R_1 = 0.4 \Omega$$

$$X_1 = 0.35 \Omega$$

$$R_2' = 0.14 \Omega$$

$$X_1' = 0.35 \Omega$$

$$X_m = 16 \Omega$$

Rotor losses are 360 W. Draw an equivalent circuit showing the given parameters, and, at a motor speed of 1746 rpm, calculate, *on a per-phase basis*, the following:

- a. the stator armature current;
- b. the rotor current;
- c. stator input power;
- d. stator copper loss;
- e. rotor power input;
- f. rotor power developed;
- g. total output power in watts and horsepower;
- h. motor efficiency; and,
- j. output torque.

# INDUCTION MOTORS

**Q2:**

The full-load slip of a 4-pole, 60 Hz, 440 V (line-to-line) three-phase induction motor is 0.05.

- i. What is the speed of the rotating stator field? What is the frequency (in Hz) of the rotor current?
- ii. When the output power of the motor is 90 hp at a speed of 1732 rpm, rotational losses (which include frictional and core losses) are 10,100 W, while copper losses for both the rotor and stator total 3700 W. Determine the motor efficiency and line current, assuming the motor has a 0.8 pf lagging under these conditions.

# INDUCTION MOTORS

## Q3:

A three-phase, 230 V, 30 hp, 60 Hz, six-pole induction motor is operating with a shaft load that requires 21.3 kW to cross the air gap to the rotor. Rotor copper loss is 1.05 kW, and mechanical and stray load losses are 300 W. Determine:

- a. shaft speed;
- b. mechanical power developed;
- c. developed torque;
- d. shaft torque; and,
- e. percent of rated hp the machine is required to deliver to the load.