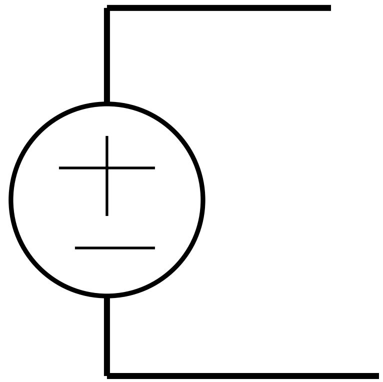
Lecture Notes
ELE 6A

Ramadan El-Shatshat

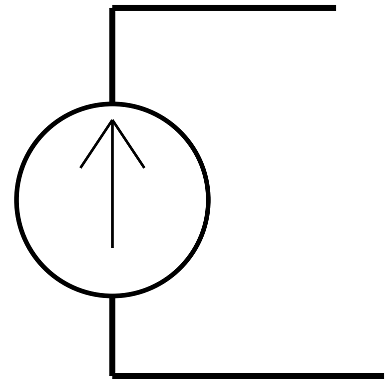
Single Phase circuit

Circuit Elements

- Sources

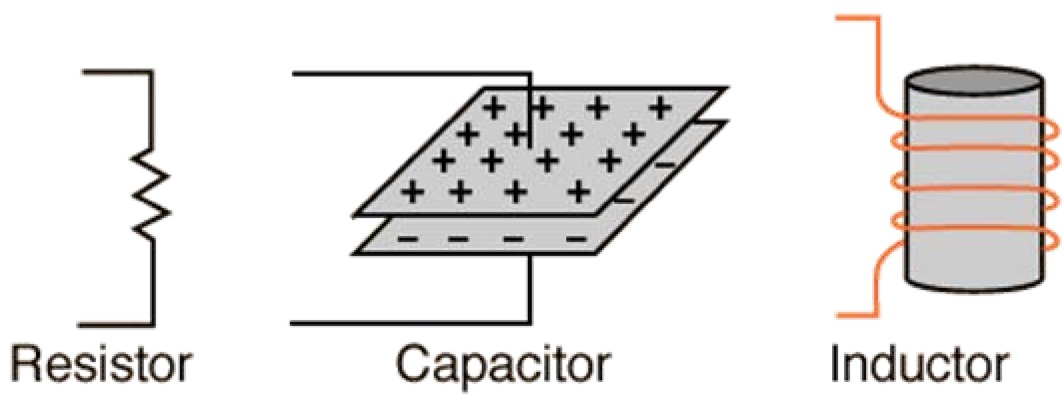


Voltage Source

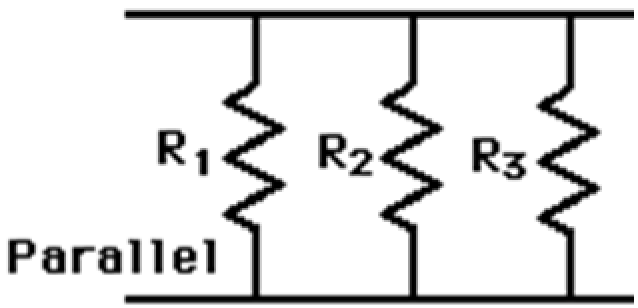


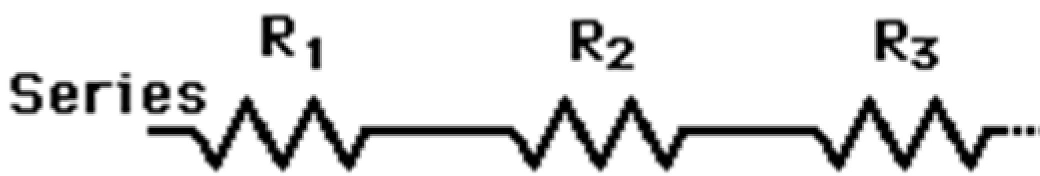
Current Source

- Sinks



Resistor Connections





$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$R_{total} = R_1 + R_2 + R_3$$

Ohms Law

$$V = I * R$$

$$\text{Where:}$$

$$\text{V} = \text{voltage}$$

$$\text{I} = \text{current}$$

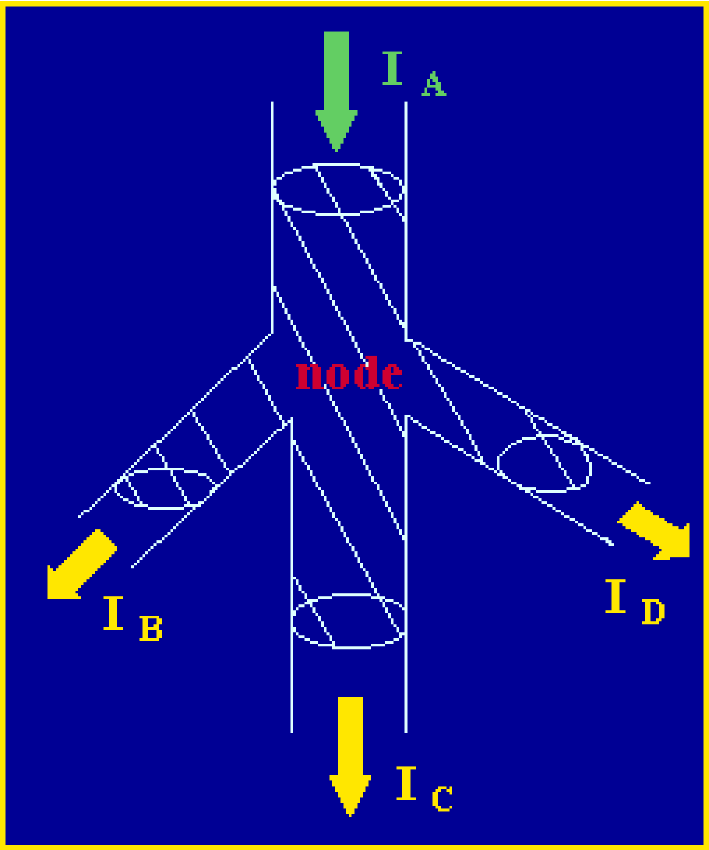
$$\text{R} = \text{resistance}$$

KCL

For any node in the circuit:

$$I_A = I_B + I_C + I_D$$

The summation of currents entering a node is equal to the currents leaving a node.

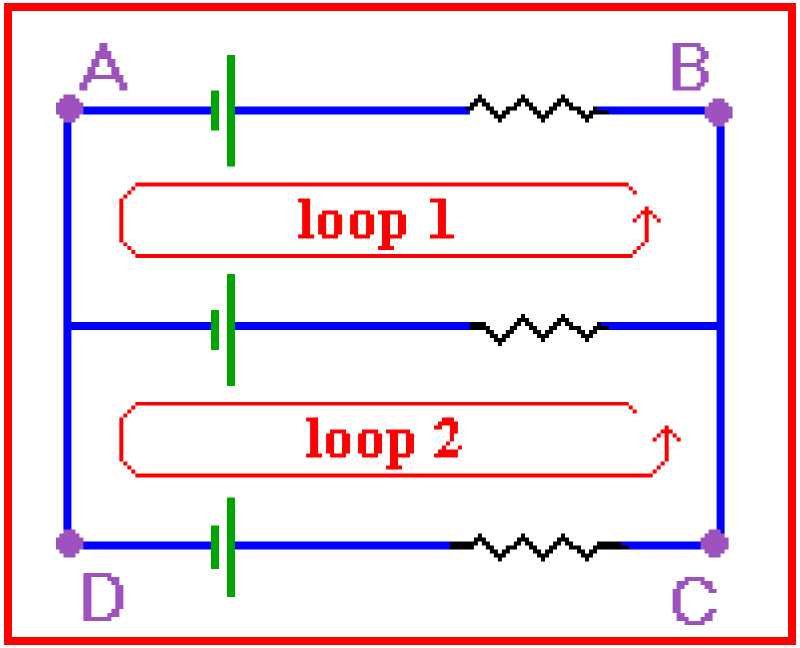


KVL

For any closed loop:

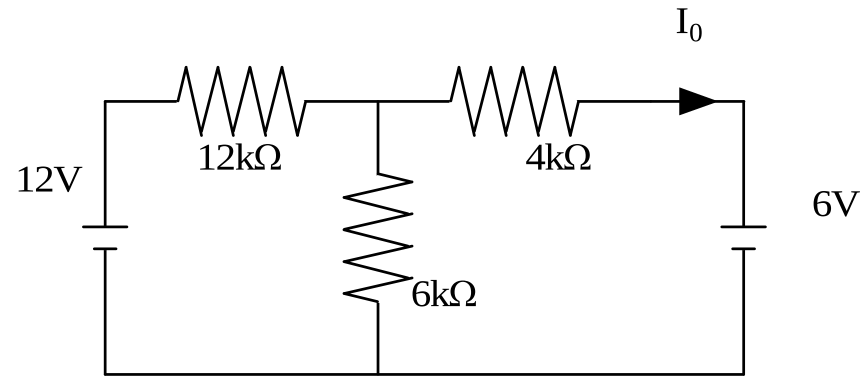
$$\sum V = 0$$

The summation of voltages in any closed loop is equal to zero.



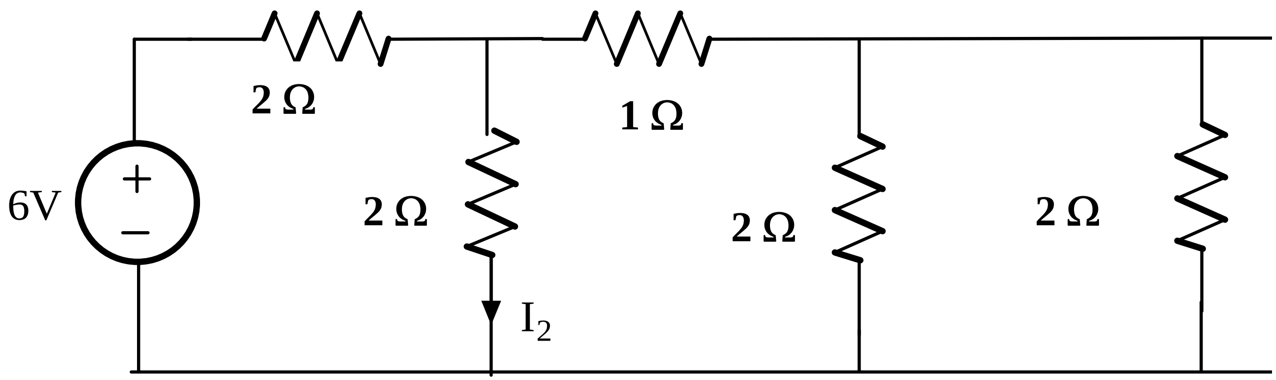
Example 1

- Find I_o (-0.25 mA)



Example 2

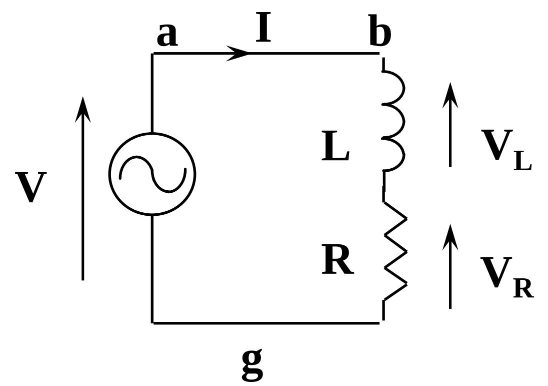
- Find I_2 (1 A)

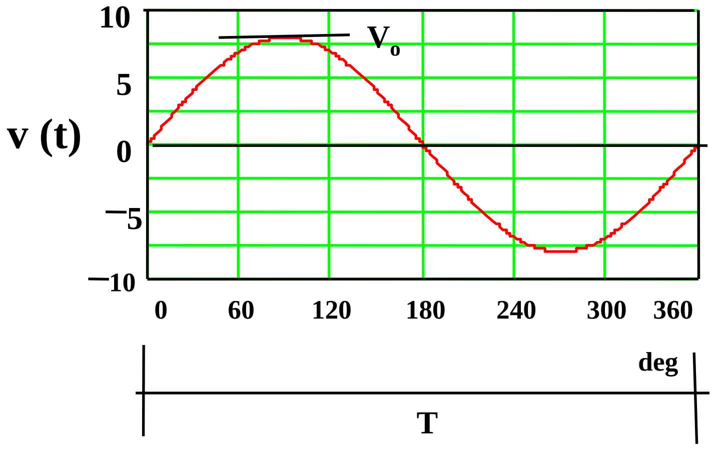


Single Phase Circuit (AC)

Review

- **<u>Single phase circuit components</u>:**
- Voltage or current sources.
- Impedances (resistance, inductance, and capacitance).
- The components are connected in series or in parallel.
- The figure shows a simple circuit where a voltage source (generator) supplies a load (resistance and inductance in series).





Single Phase Circuit (AC)

Review

- **The voltage source produces a sinusoidal voltage wave**

$$v(t) = \sqrt{2}\, V_{rms} \sin(\omega t)$$

where: V_{rms} is the rms value of the voltage (volts)

w is the angular frequency of the sinusoidal function (rad/sec)

$$\omega = 2\pi f = \frac{2\pi}{T} \text{ rad/sec} \qquad f = \frac{1}{T} \text{ Hz}$$

f is the frequency (60 Hz in USA, 50 Hz in Europe).

T is the time period (seconds).

Single Phase Circuit (AC)

Review

The RMS value is calculated by $$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v(t)^2 \, dt}$$

For sinusoidal signal the RMS value = $$V_{rms} = \frac{V_{peak}}{\sqrt{2}}$$

Unless it is stated in the question, all the given values are in RMS value.

Single Phase Circuit (AC)

Review

- **The current is also sinusoidal**

$$i(t) = \sqrt{2}\, I_{rms} \sin(\omega t - \phi)$$

where: I_{rms} is the rms value of the current.

ϕ is the phase-shift between current and voltage.

- **The RMS current is calculated by the Ohm's Law:**

where: Z is the impedance

$$I_{rms} = \frac{V_{rms}}{Z}$$

Single Phase Circuit (AC)

Review

- **The impedances (in Ohms) are :**
  - **a) Resistance (R)**
  - **b) Inductive reactance** $X_L = \omega L$
  - **c) Capacitive reactance** $X_C = \dfrac{1}{\omega C}$

Single Phase Circuit (AC)

Review

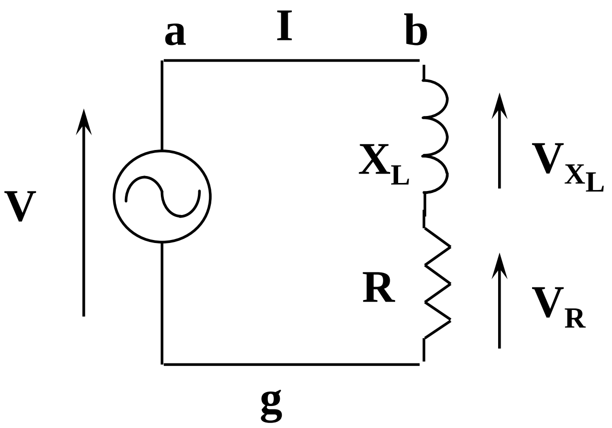
- **The impedance of a resistance and a reactance connected in series is :**

$$Z = \sqrt{R^2 + X^2}$$

- **The phase angle is :**

$$\phi = \tan^{-1}\frac{X}{R}$$

- Impedance calculation

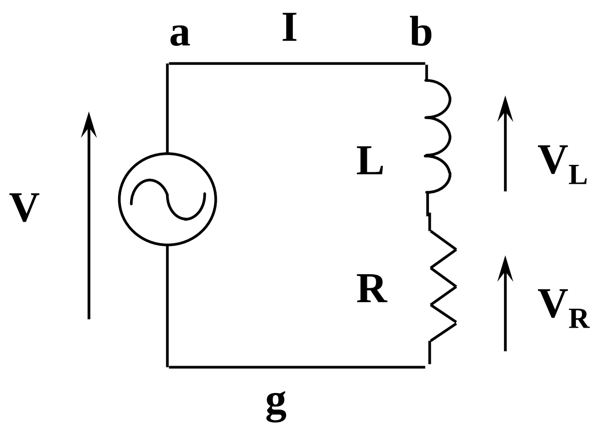


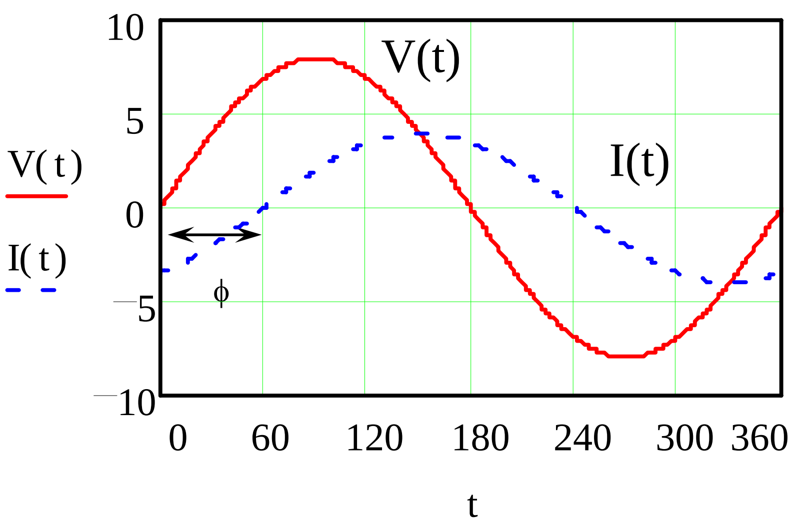
Single Phase Circuit (AC)

Review

Inductive circuit

- The ϕ phase-shift between the current and voltage is negative.
- The current is lagging with respect to the voltage.



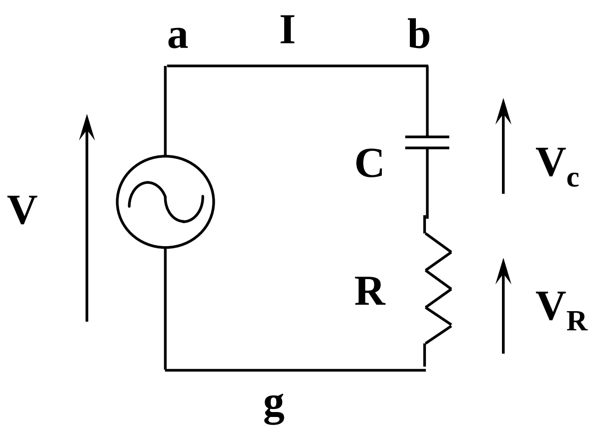


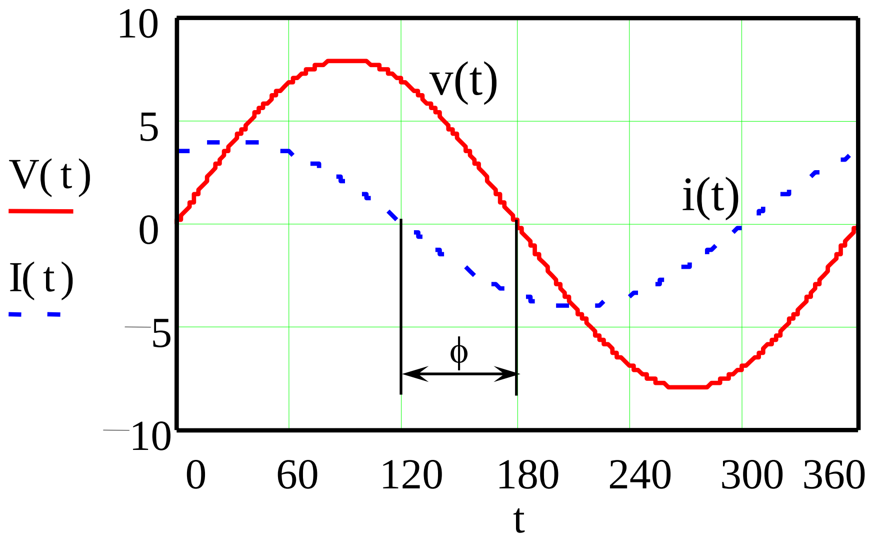
Single Phase Circuit (AC)

Review

Capacitive circuit

- The ϕ phase shift between the current and voltage is positive.
- The current is leading with respect to the voltage.





Single Phase Circuit (AC)

Review

Complex Notation

- **Engineering calculations need the amplitude (rms value) and phase angle of voltage and current.**
- **The amplitude and phase angle can be calculated using complex notation.**
- **The voltage, current, and impedance are expressed by complex phasors.**

Single Phase Circuit (AC)

Review

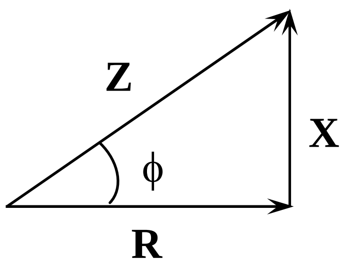
Complex Notation

Impedance phasor: (resistance, capacitor, and inductance connected in series)

Rectangular form:

$$\mathbf{Z} = R + j\omega L + \left(\frac{1}{j\omega C}\right) = R + j(X_L - X_C) = R + jX_T$$

Polar form: $\mathbf{Z} = |Z| \angle \varphi$



where: $\mathbf{Z} = \sqrt{R^2 + X_T^2}$ $\phi = \tan^{-1}\left(\frac{X}{R}\right)$

$$R = \mathbf{Z}\cos(\phi) \qquad X = \mathbf{Z}\sin(\phi)$$

Single Phase Circuit (AC)

Review

Complex Notation

- **Voltage phasor:**

$$\mathbf{V} = |V| \angle \delta = |V| \cos\delta + \mathbf{j}|V| \sin\delta$$

where : V is the rms value, and δ is the phase angle

Note: The supply voltage phase angle is often selected as the reference with δ = 0

Single Phase Circuit (AC)

Review

Complex Notation

- **Current phasor**

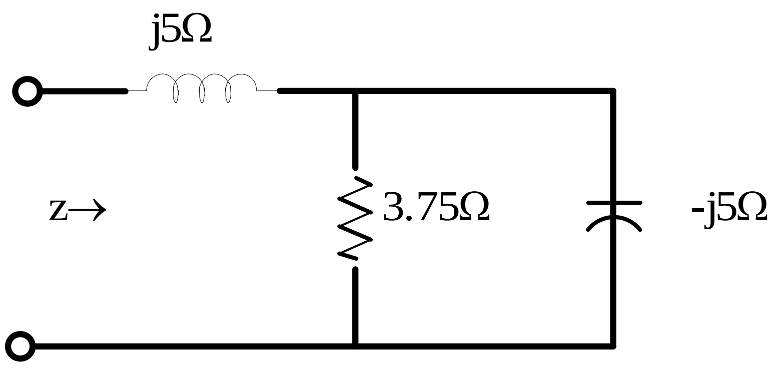
$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{|\mathrm{V}|\angle\delta}{|\mathrm{Z}|\angle\varphi} = \left|\frac{\mathrm{V}}{\mathrm{Z}}\right|\angle(\delta-\varphi) = \left|\frac{\mathrm{V}}{\mathrm{Z}}\right|\left[\cos(\delta-\phi) + j\sin(\delta-\phi)\right]$$

Single Phase Circuit (AC)

Review

Example 1:

$$Z = 5j + (-5j \,||\, 3.75) = 5j + \frac{-18.75j}{3.75 - 5j} = 5j + \frac{18.74\angle{-90^\circ}}{6.25\angle{-53.1^\circ}} = 5j + 3\angle{-36.9^\circ} = 2.4 + j3.2 = 4\angle{53^\circ}\Omega$$



Single Phase Circuit (AC)

Review

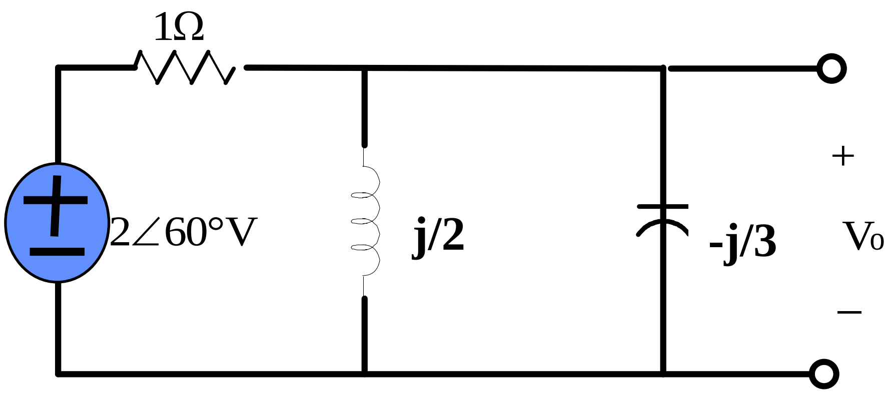
Example 2:

– Find V0

$$Z = \frac{j}{2} \parallel \frac{-j}{3} = \frac{\frac{1}{6}}{\frac{j}{2} - \frac{j}{3}} = -j$$

$$I = \frac{2\angle 60}{1-j} = \frac{2\angle 60}{\sqrt{2}\angle -45} = \sqrt{2}\angle 105$$

$$V_o = I * Z = \sqrt{2}\angle 15$$



Single Phase Circuit (AC)

Review

Power calculation.

1) **Instantaneous power**

$$p(t) = v(t)i(t) = \sqrt{2}\ V\ \mathbf{sin}(\omega t)\sqrt{2}\ I\ \mathbf{sin}(\omega t - \phi)$$

2) **Real Power (Average Power)**

$$P = VI\cos(\phi)$$

Power factor

3) **Reactive Power**

$$Q = VI\sin(\phi)$$

Single Phase Circuit (AC)

Review

4) Complex Power

- **The complex notation can be used for power calculation.**
- **The complex power is defined as : Voltage times the conjugate of the current.**

$$\mathbf{S} = \mathbf{V}\,\bar{\mathbf{I}} = \mathrm{V}\,\mathrm{I}\left[\cos(\phi) \pm \mathbf{j}\sin(\phi)\right] = \mathrm{P} \pm \mathbf{j}\,\mathrm{Q}$$

Positive for Inductive load

Negative for Capacitive load

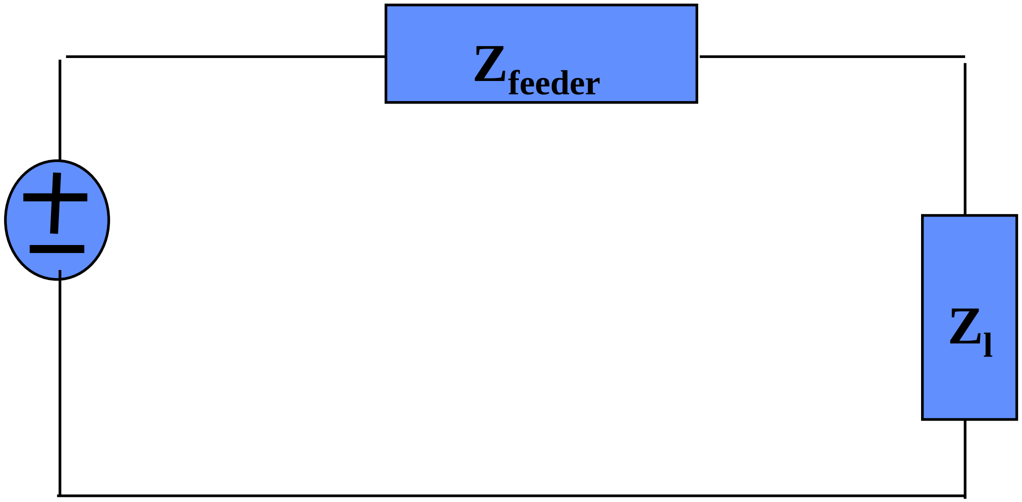
Single Phase Circuit (AC)

Review

Example 3

A generator supplies a load through a feeder whose impedance is $Z_{feeder} = 1 + j2$. The load impedance is $Z_l = 8 + j6$. The voltage across the load is 120 V.

Find the real power and reactive power supplied by the generator.



(P=1295W, Q=1154 VAR)