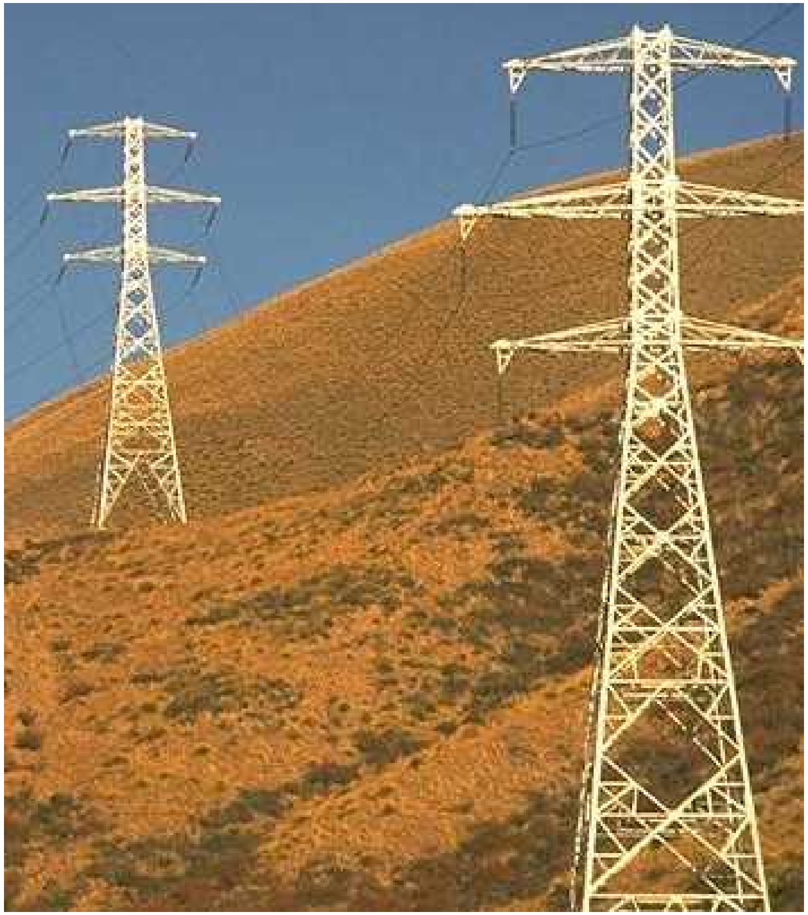
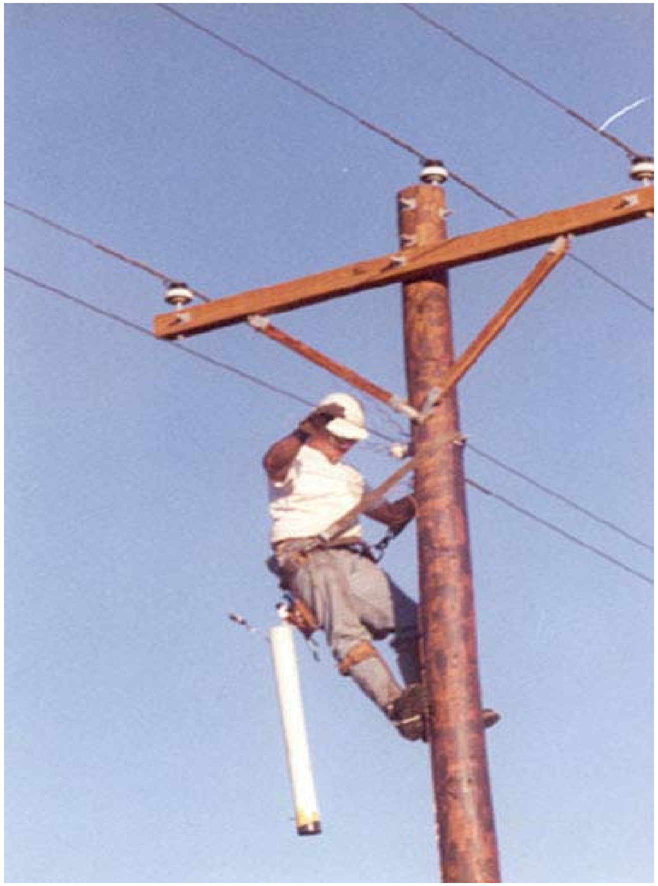
Lecture Notes
ELE A6

Ramadan El-Shatshat

Three Phase circuits

Three-phase Circuits





Advantages of Three-phase Circuits

- Smooth flow of power (instantaneous power is constant).
- Constant torque (reduced vibrations).
- The power delivery capacity tripled (increased by 200%) by increasing the number of conductors from 2 to 3 (increased by 50%).

Three-phase Circuits

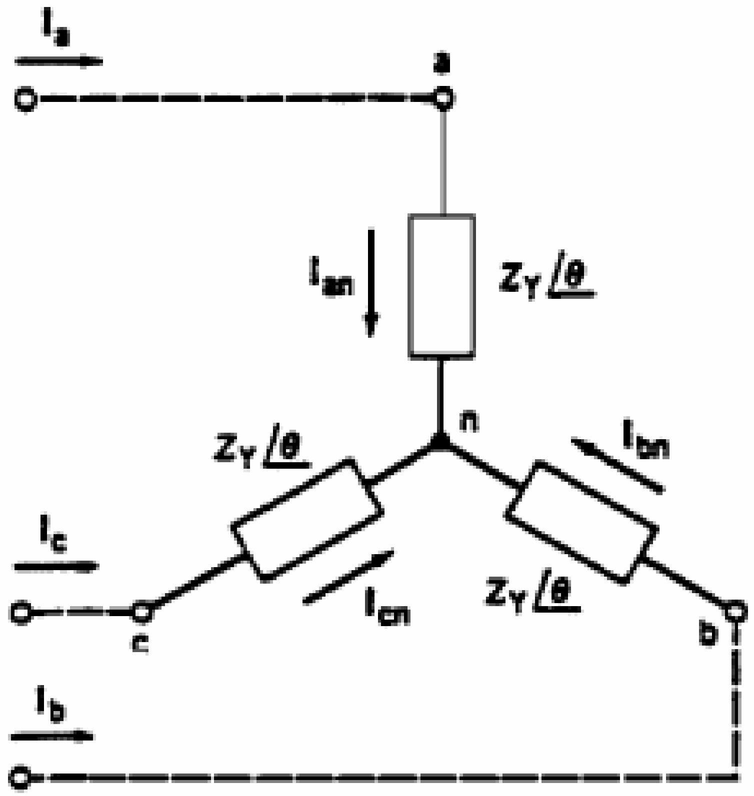
Wye-Connected System

- The neutral point is grounded
- The three-phase voltages have equal magnitude.
- The phase-shift between the voltages is 120 degrees.

$$\mathbf{V_{an}} = |V| \angle 0° = V$$

$$\mathbf{V_{bn}} = |V| \angle -120°$$

$$\mathbf{V_{cn}} = |V| \angle -240°$$



Three-phase Circuits

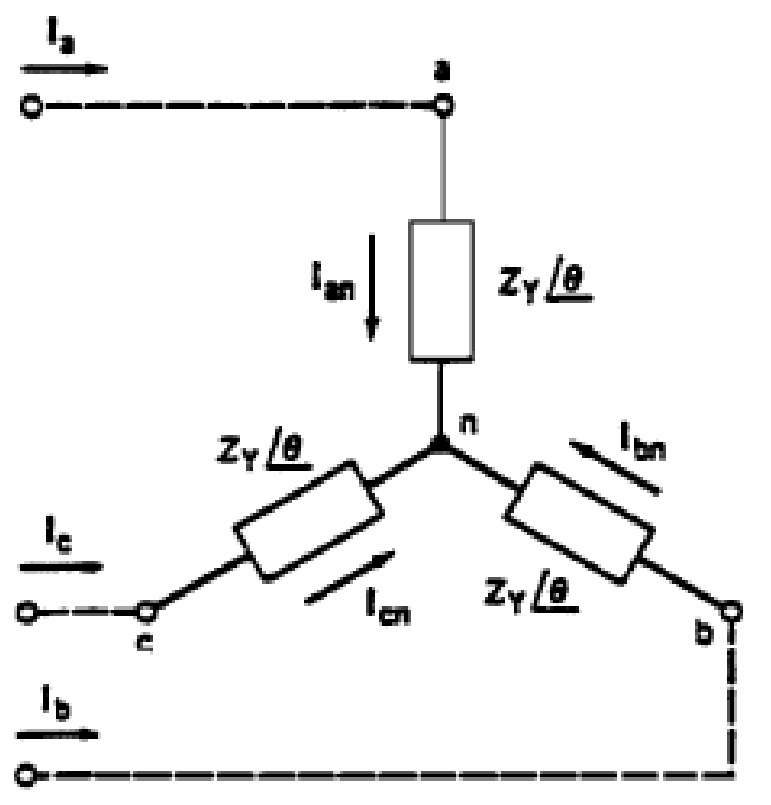
Wye-Connected System

- Line-to-line voltages are the difference of the phase voltages

$$\mathbf{V}_{\mathbf{ab}} = \mathbf{V}_{\mathbf{an}} - \mathbf{V}_{\mathbf{bn}} = \sqrt{3}\,\mathrm{V}\angle 30$$

$$\mathbf{V}_{\mathbf{bc}} = \mathbf{V}_{\mathbf{bn}} - \mathbf{V}_{\mathbf{cn}} = \sqrt{3}\,\mathrm{V}\angle -90$$

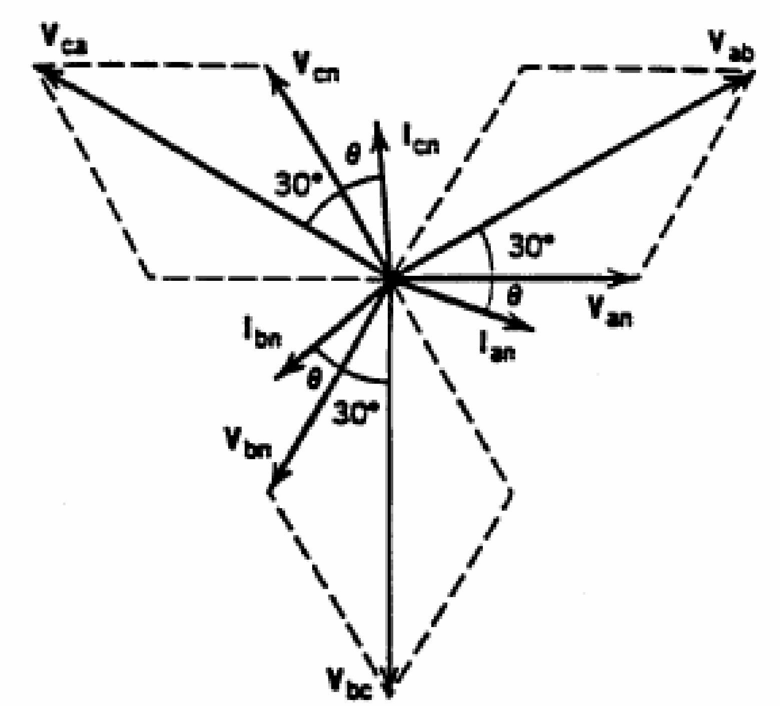
$$\mathbf{V}_{\mathbf{ca}} = \mathbf{V}_{\mathbf{cn}} - \mathbf{V}_{\mathbf{an}} = \sqrt{3}\,\mathrm{V}\angle 150$$



Three-phase Circuits

Wye-Connected System

- Phasor diagram is used to visualize the system voltages
- Wye system has two type of voltages: Line-to-neutral, and line-to-line.
- The line-to-neutral voltages are shifted with 120°
- The line-to-line voltage leads the line to neutral voltage with 30°
- The line-to-line voltage is $\sqrt{3}$ times the line-to-neutral voltage

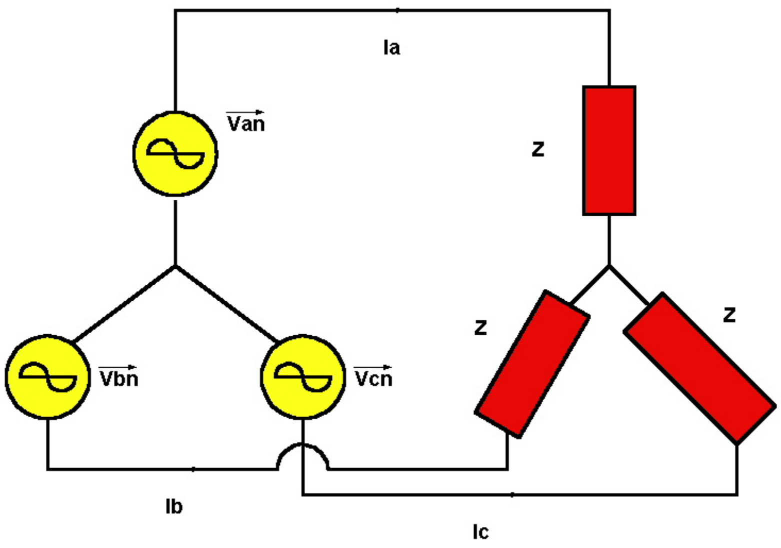


Three-phase Circuits

Wye-Connected Loaded System

- **The load is a balanced load and each one = Z**
- **Each phase voltage drives current through the load.**
- **The phase current expressions are:**

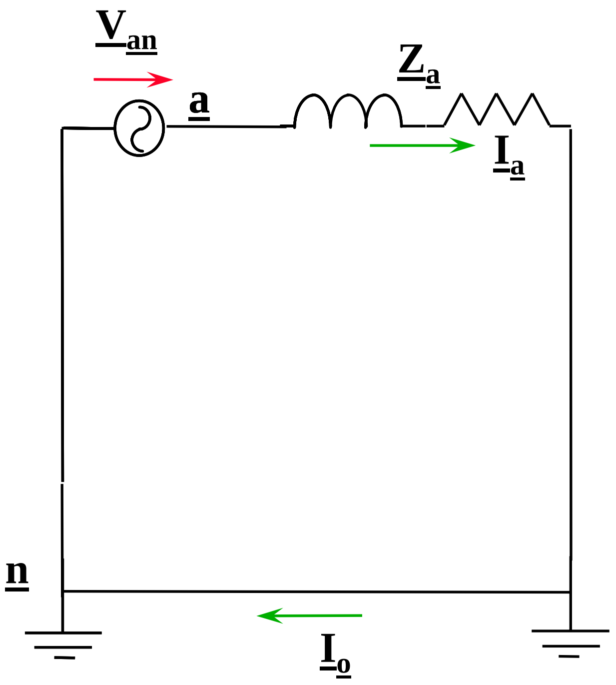
$$I_a = \frac{V_{an}}{Z}, \quad I_b = \frac{V_{bn}}{Z}, \quad I_c = \frac{V_{cn}}{Z}$$



Three-phase Circuits

Wye-Connected Loaded System

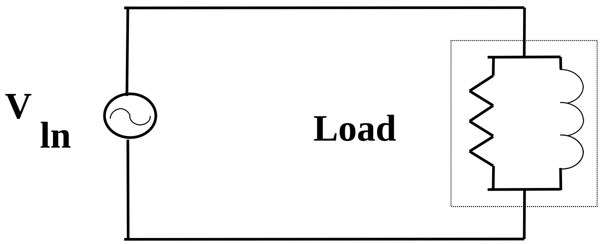
- **Since the load is balanced ($Z_a = Z_b = Z_c$) then: Neutral current = 0**
- **This case single phase equivalent circuit can be used (phase a, for instance, only)**
- **Phase b and c are eliminated**



Three-phase Circuits

<u>Wye-Connected System with balanced load</u>

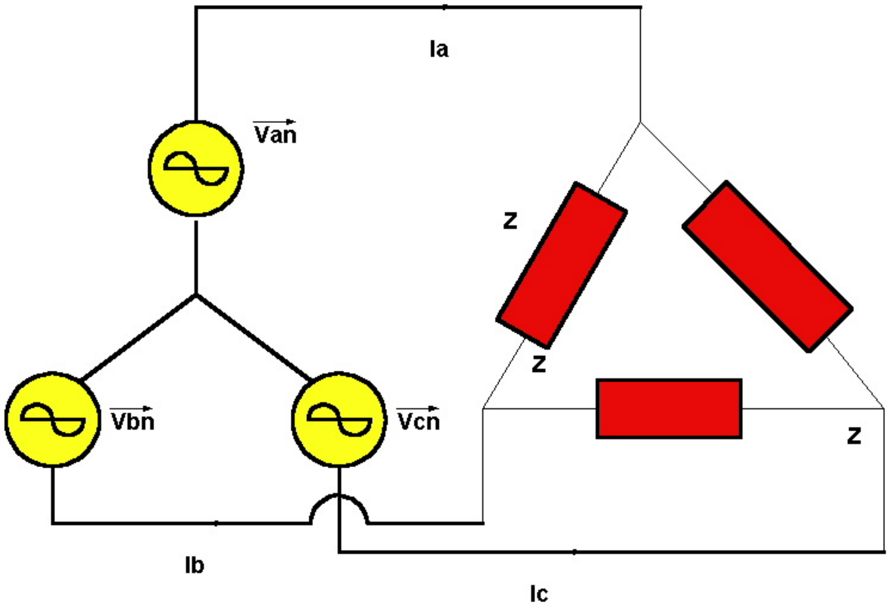
- A single-phase equivalent circuit is used
- Only phase **a** is drawn, because the magnitude of currents and voltages are the same in each phase. Only the phase angles are different (-120^{o} phase shift)
- The supply voltage is the **line to neutral voltage.**
- The single phase loads are connected to neutral or ground.



Three-phase Circuits

<u>Balanced Delta-Connected System</u>

- **The system has only one voltage : the line-to-line voltage (V_{LL})**
- **The system has two currents**
  - **line current**
  - **phase current**
- **The phase currents are:**



$$I_a = \frac{V_{ab}}{z}$$

Three-phase Circuits

<u>Delta-Connected System</u>

The line currents are:

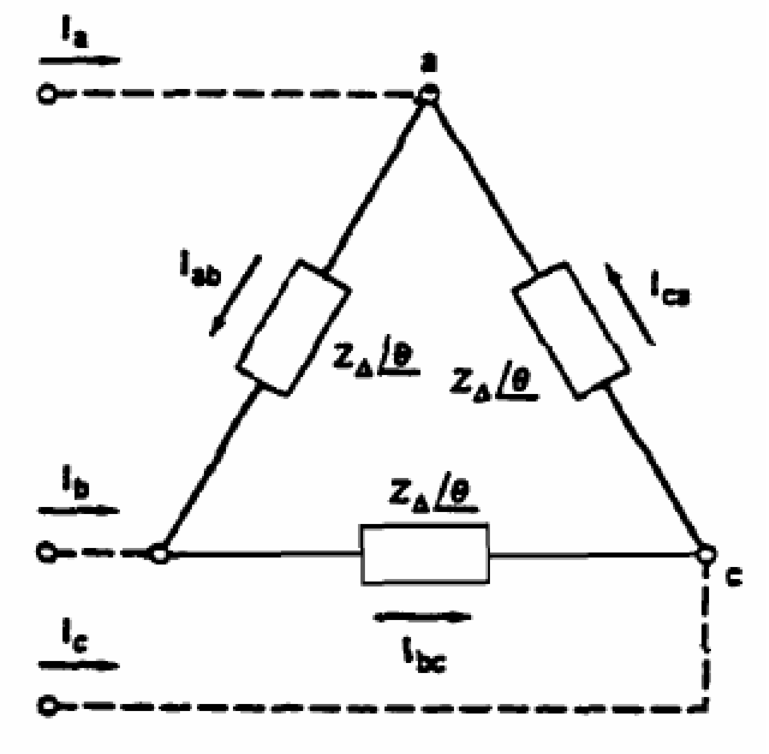
$$\mathbf{I_a = I_{ab} - I_{ca}}$$

$$\mathbf{I_b = I_{bc} - I_{ab}}$$

$$\mathbf{I_c = I_{ca} - I_{bc}}$$

- **In a balanced case the line currents are:**

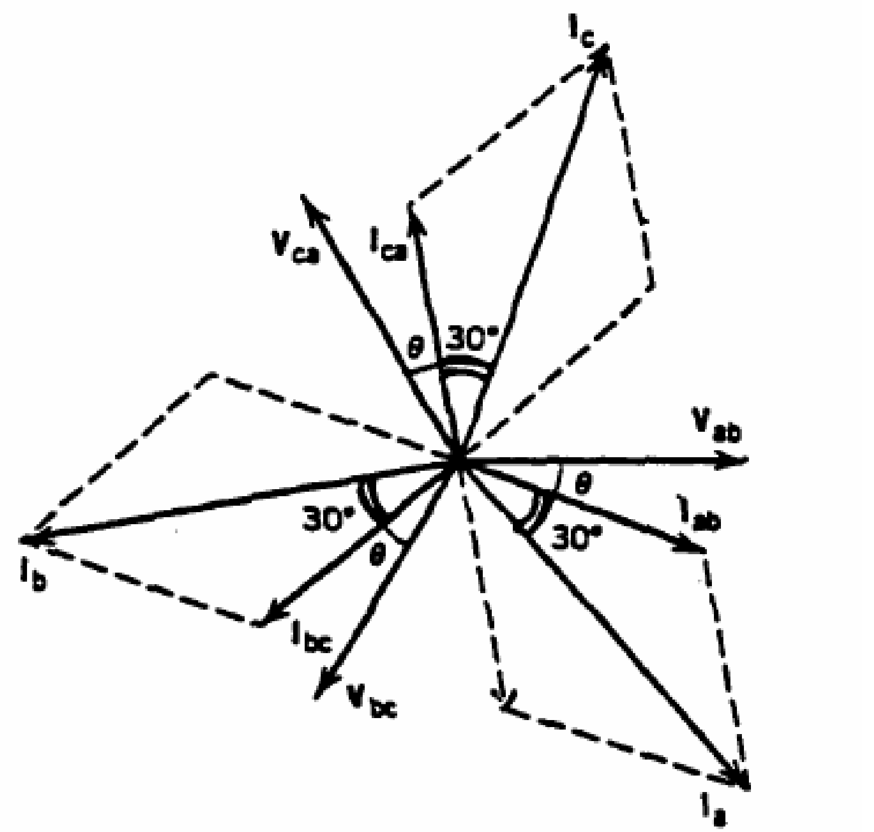
$$I_{line} = \sqrt{3} I_{phase} \angle -30$$



Three-phase Circuit

Delta-Connected System

- **The phasor diagram is used to visualize the system currents**
- **The system has two type of currents: line and phase currents.**
- **The delta system has only line-to-line voltages, that are shifted by**
- **The phase currents lead the line currents by 30°**
- **The line current is $\sqrt{3}$ times the phase current and shifted by 30 degree.**



Three-phase Circuit

- **Circuit conversions**
  - A delta circuit can be converted to an equivalent wye circuit. The equation for phase a is (will not be used for this course):

$$\mathbf{Z_a} = \frac{\mathbf{Z}_{ab}\,\mathbf{Z_{ca}}}{\mathbf{Z}_{ab} + \mathbf{Z_{bc}} + \mathbf{Z_{ca}}}$$

  - Conversion equation for a balanced system is:

$$\mathbf{Z_a} = \frac{\mathbf{Z}_{ab}}{3}$$

Three-phase Circuit

Power Calculation

- **The three phase power is equal the sum of the phase powers**

$$\mathbf{P = P_a + P_b + P_c}$$

- **If the load is balanced:**

$$\mathbf{P = 3\,P_{phase} = 3\,V_{phase} I_{phase}\,\cos(\phi)}$$

- **Wye system:** $\mathbf{V_{phase} = V_{LN}}$ $\mathbf{I_{phase} = I_L}$ $\mathbf{V_{LL} = \sqrt{3}\,V_{LN}}$

$$\mathbf{P = 3\,V_{phase} I_{phase}\,\cos(\phi) = \sqrt{3}\,V_{LL} I_L\,\cos(\phi)}$$

- **Delta system:** $\mathbf{I_{Line} = \sqrt{3}\,I_{phase}}$ $\mathbf{V_{LL} = V_{phase}}$

$$\mathbf{P = 3\,V_{phase} I_{phase}\,\cos(\phi) = \sqrt{3}\,V_{LL} I_L\,\cos(\phi)}$$

Three-phase Circuit

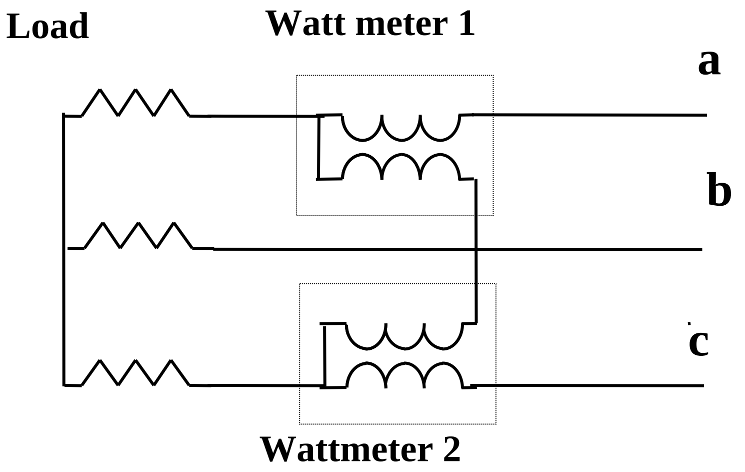
Power measurement

- **In a four-wire system (3 phases and a neutral) the real power is measured using three single-phase watt-meters.**
- **In a three-wire system (three phases without neutral) the power is measured using only two single-phase watt-meters.**

  - The watt-meters are supplied by the line current and the line-to-line voltage.

Three-phase Circuit

Power measurement



- **The total power is the algebraic sum of the two watt-meters reading.**

$$W_1 = V_{ab} I_a \cos(\theta_{vab} - \theta_{Ia})$$

$$W_2 = V_{cb} I_c \cos(\theta_{vcb} - \theta_{Ic})$$

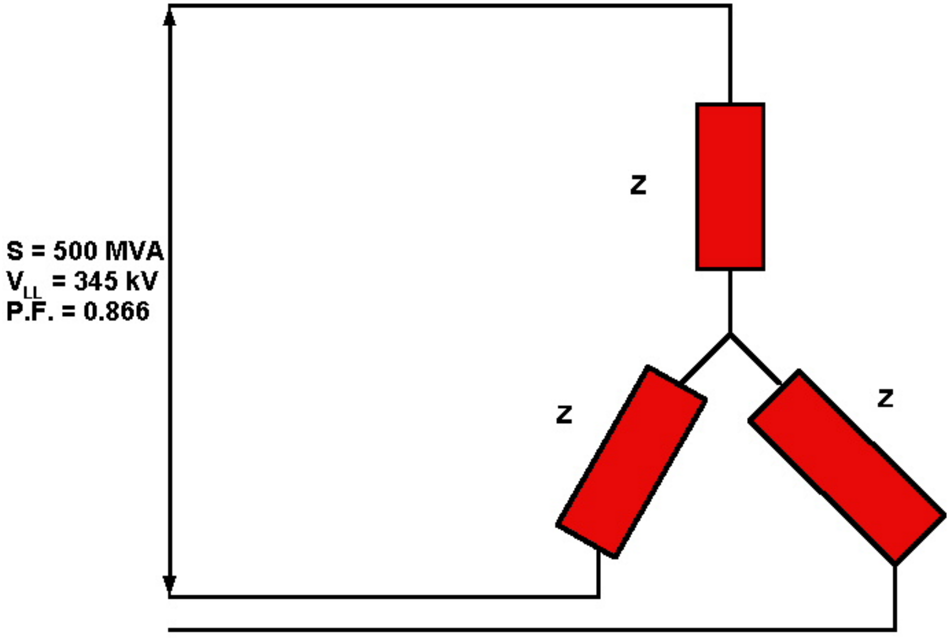
$$P_T = W_1 + W_2 = \sqrt{3} V_L I_L \cos(\theta)$$

Three-phase Circuits

Example 1

A 345 kV, three phase transmission line delivers 500 MVA, 0.866 power factor lagging, to a three phase load connected to its receiving end terminals. Assume the load is Y connected and the voltage at the receiving end is 345 kV, find:

- The load impedance per phase.
- The line and phase currents.
- The total real and reactive power.



$$(a)\ \ Z_{\phi} = \frac{V_{\phi}}{I_{\phi}}$$

$$V_{\phi} = \frac{345kV}{\sqrt{3}}\angle 0V, \left|\mathrm{I}_{\phi}\right| = \frac{S}{\sqrt{3}V_L} = \frac{500MVA}{\sqrt{3}345kV} = 836.7A$$

$$I_{\phi} = 836.7\angle -\cos^{-1}(0.866) = 836.7\angle -30$$

$$Z_{\phi} = 238\angle 30 = 206 + j119$$

$$(b)\ I_L = I_{\phi} = 836.7\angle -30$$

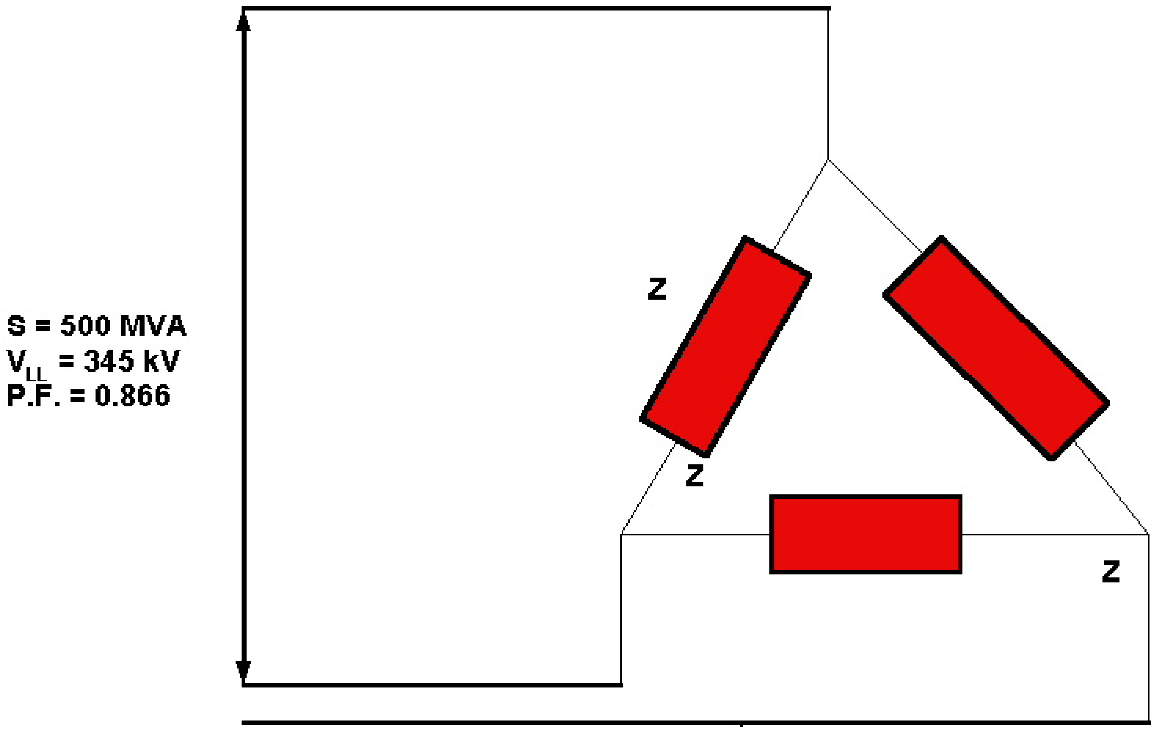
$$(c)\ P = \sqrt{3}V_L I_L \cos(\theta) = 433\,\mathrm{MW}$$

$$Q = \sqrt{3}V_L I_L \sin(\theta) = 249.9\,\mathrm{MVAR}$$

Three-phase Circuits

Example 2

Repeat example 2 assuming the load is Delta connected.



$$(a)\ V_\phi = V_L = 345kV\angle 0V,$$

$$\left|I_\phi\right| = \frac{S}{3V_L} = \frac{500MVA}{3*345kV} = 483.1A$$

$$I_\phi = 483.1\angle -\cos^{-1}(0.866) = 483.1\angle -30$$

$$Z_\phi = \frac{V_\phi}{I_\phi} = 714\angle 30 = 618.3 + j357$$

$$(b)\ I_L = \sqrt{3}I_\phi\angle -30 = 836.7\angle -60$$

$$(c)\ P = \sqrt{3}V_L I_L \cos(\theta) = 433\,\mathrm{MW}$$

$$Q = \sqrt{3}V_L I_L \sin(\theta) = 249.9\,\mathrm{MVAR}$$

Power factor correction

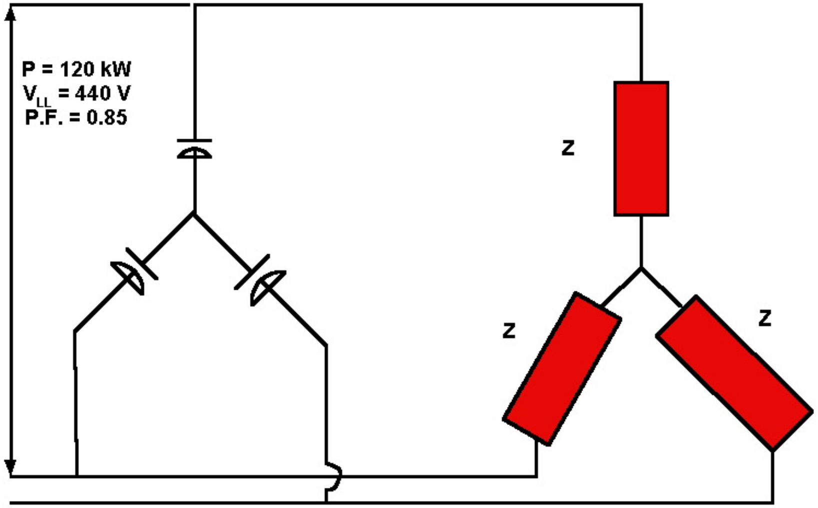
- Power factor (p.f.) correction is the process of making P.f. = 1.
- In order to correct the power factor in any system, a reactive (either inductive or capacitive) will be added to the load.
- If the load is inductive, then a capacitance is added.
  - Note: Correcting the P.f. WILL NOT affect the active power, why?

Three-phase Circuits

Example 3

A 3-phase load draws 120 kW at a power factor of 0.85 lagging from a 440 V bus. In parallel with this load, a three phase capacitor bank that is rated 50 kVAR is inserted, find:

- The line current without the capacitor bank.
- The line current with the capacitor bank.
- The P.F. without the capacitor bank.
- The P.F. with the capacitor bank.



$$(a)\ P = \sqrt{3} V_L I_L \cos(\theta)$$

$$|I_L| = \frac{120 * 10^3}{\sqrt{3}\,440\,(0.85)} = 185.25\,A$$

$$I_L = 185.25 \angle -\cos^{-1}(0.85) = 185.2 \angle -31.8,$$

$$(b)\ Q_{capacitor} = \sqrt{3} V_L I_L \sin(\theta)$$

Since we have pure capacitor load

$$\sin(\theta) = 1$$

$$Q_{capacitor} = \sqrt{3} V_L I_{L(capacitor)}$$

$$|I_{L(capacitor)}| = \frac{50 * 10^3}{\sqrt{3}\,440} = 65.6\,A$$

$$I_{L(capacitor)} = 65.6 \angle 90$$

$$I_{L(new)} = I_L + I_{L(capacitor)} = 160.6 \angle -11.49$$

$$(c)\ P.\mathrm{F.}\ (\text{no capacitor}) = 0.85$$

$$(d)\ P.\mathrm{F.}\ (\text{with capacitor}) = \cos(11.49) = 0.98$$

Home Work

In the circuit of Figure 1 below, $\mathbf{Z}_1 = 1\angle 60°\ \Omega$ and $\mathbf{Z}_2 = 5\angle 36.9°\ \Omega$. The line-to-line voltage is 208 V. What is the magnitude of the line current?

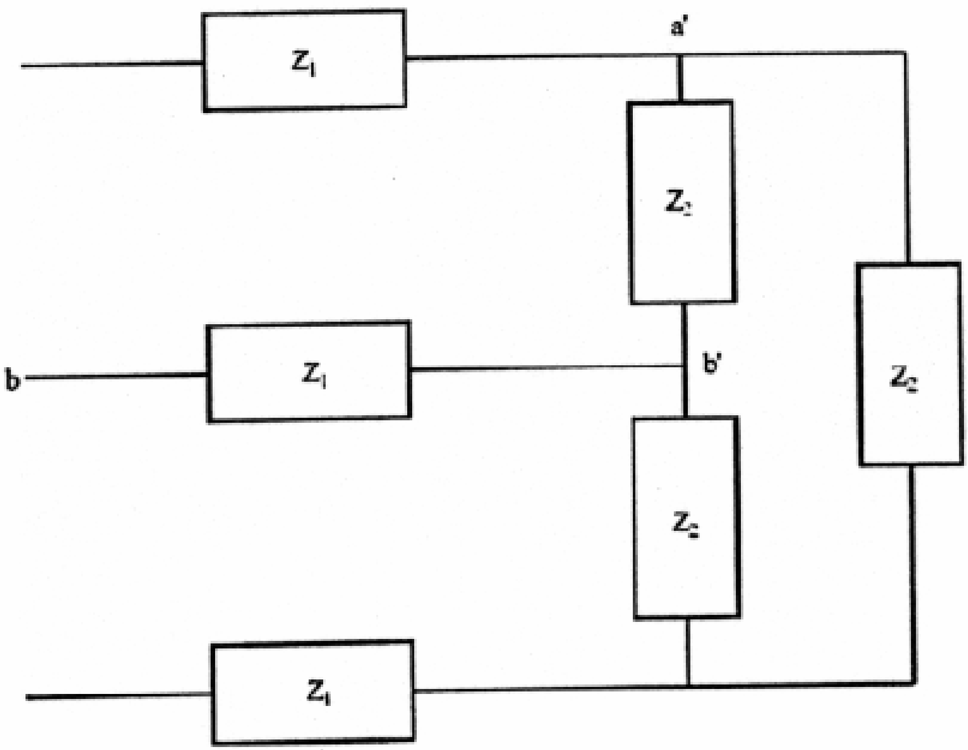


Figure 1

Home Work

a. The figure below represents a small factory supplied by a balanced 60 Hz, 600 V, 3ϕ source. Determine the wattmeter readings, W1 and W2, for the system as shown. What is the power factor?

b. If the induction motor is replaced by a synchronous motor that can operate at 0.8 pf leading while supplying the same power as the induction motor, determine the new wattmeter readings. What is the new power factor?

c. With the synchronous motor in place, it is determined that the power factor is below 0.9, and the factory will be penalized. How many kVARs of capacitor are needed to improve the pf to 0.9?