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Power Transformers

Power Transformers- INTRODUCTION

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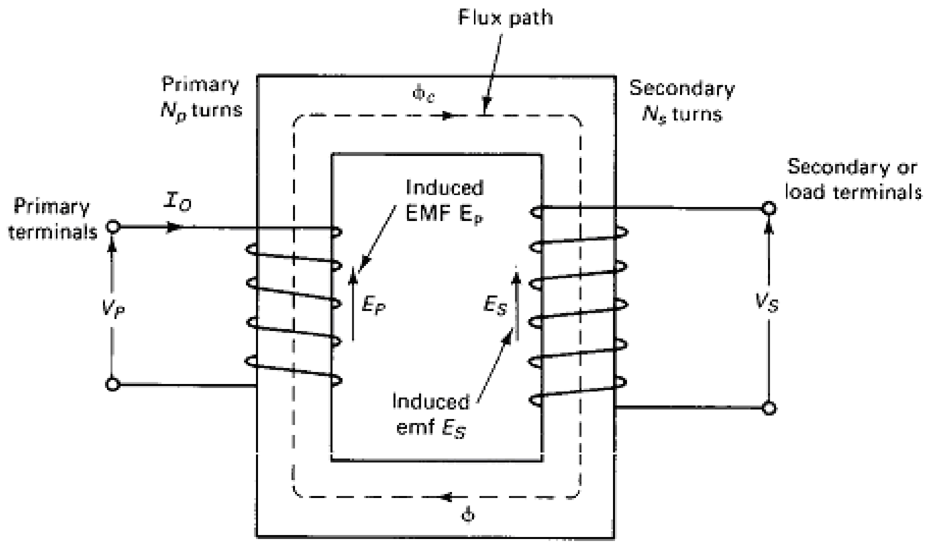
- A transformer is a device which transfers electrical energy (power) from one voltage level to another voltage level.

- Unlike in rotating machines, there no energy conversion.

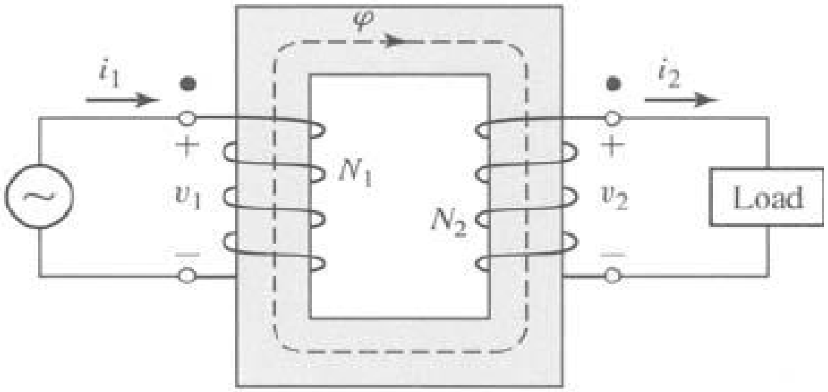
- A transformer is a static device an all currents and voltages are AC.

- The transfer of energy takes place through the magnetic field.

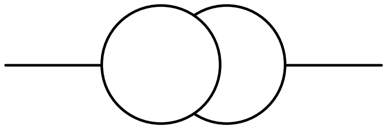


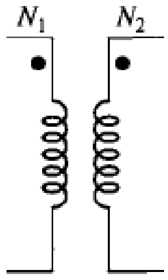
Power Transformers- CONSTRUCTION

- **Primary windings,** connected to the alternating voltage source;
- **Secondary windings,** connected to the load;
- **Iron core,** which link the flux in both windings;



- The primary and secondary voltages are denoted by V_1 and V_2 respectively. The current entering the primary terminals is I_1.





Symbols

Power Transformers- CONSTRUCTION

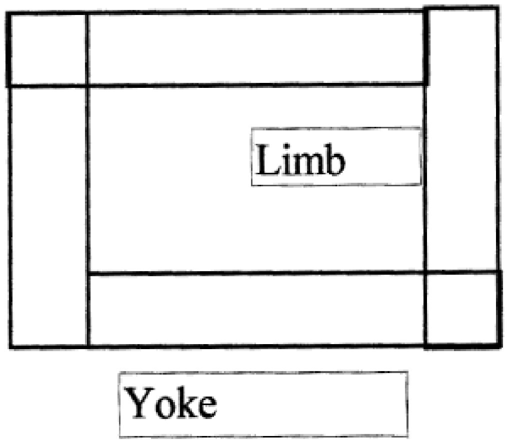
Transformer Construction

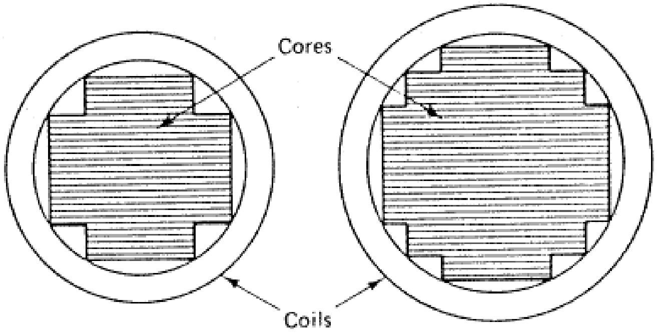
Iron Core

The iron core is made of thin laminated silicon steel (2-3 % silicon)

Pre-cut insulated sheets are cut or pressed in form and placed on the top of each other .

The sheets are overlap each others to avoid (reduce) air gaps.





Power Transformers- CONSTRUCTION

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Transformer Construction Winding

The winding is made of copper or aluminum conductor, insulated with paper or synthetic insulating material

The windings are manufactured in several layers, and insulation is placed between windings.

The primary and secondary windings are placed on top of each others but insulated by several layers of insulating sheets.

The windings are dried in vacuum and impregnated to eliminate moisture.

Small transformer winding

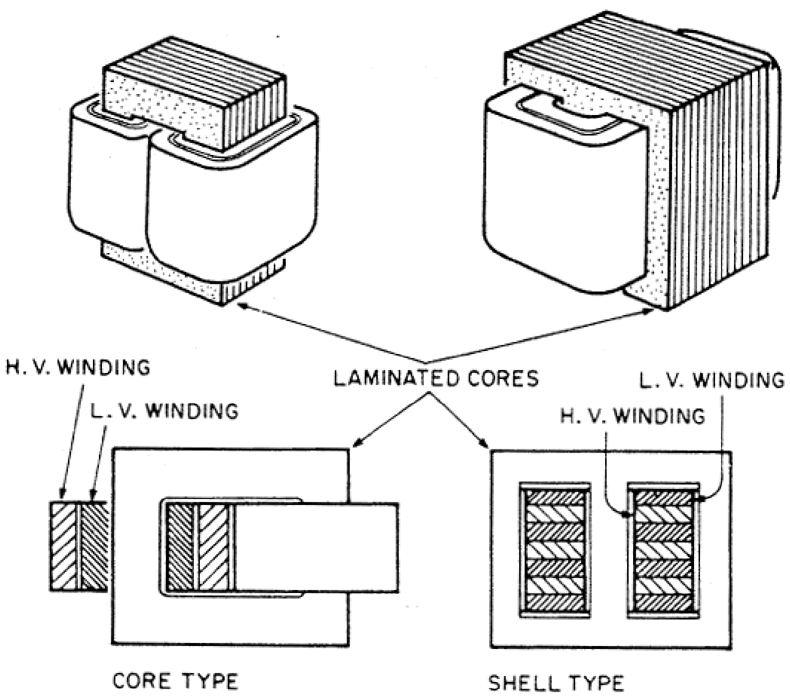

Figure 2-4 Transformer core types.

Power Transformers- CONSTRUCTION

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Transformer Construction

Iron Cores

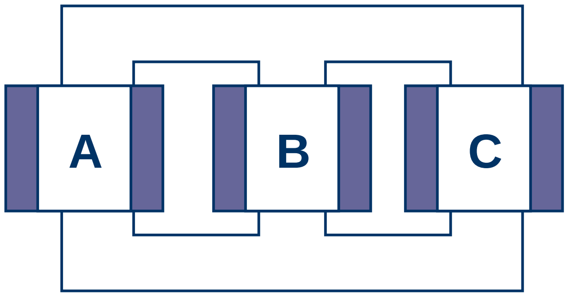
The three phase transformer iron core has three legs.

A phase winding is placed in each leg.

The high voltage and low voltage windings are placed on top of each other and insulated by layers or tubes.

Larger transformer use layered construction shown in the previous slides.

Three phase transformer iron core



Power Transformers- CONSTRUCTION

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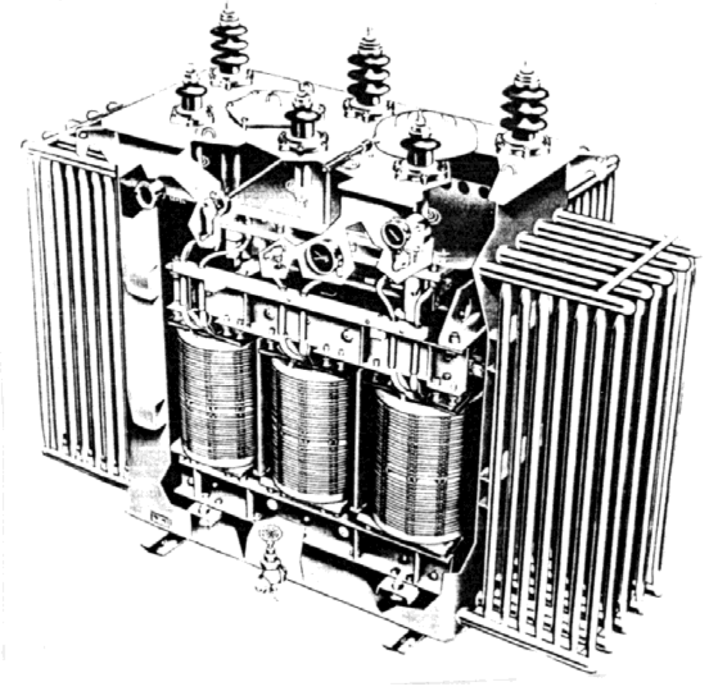
Transformer Construction

The dried and treated transformer is placed in a steel tank.

The tank is filled, under vacuum, with heated transformer oil.

The end of the windings are connected to bushings.

Three phase oil transformer



Power Transformers- CONSTRUCTION

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Transformer Construction

The transformer is equipped with cooling radiators which are cooled by forced ventilation.

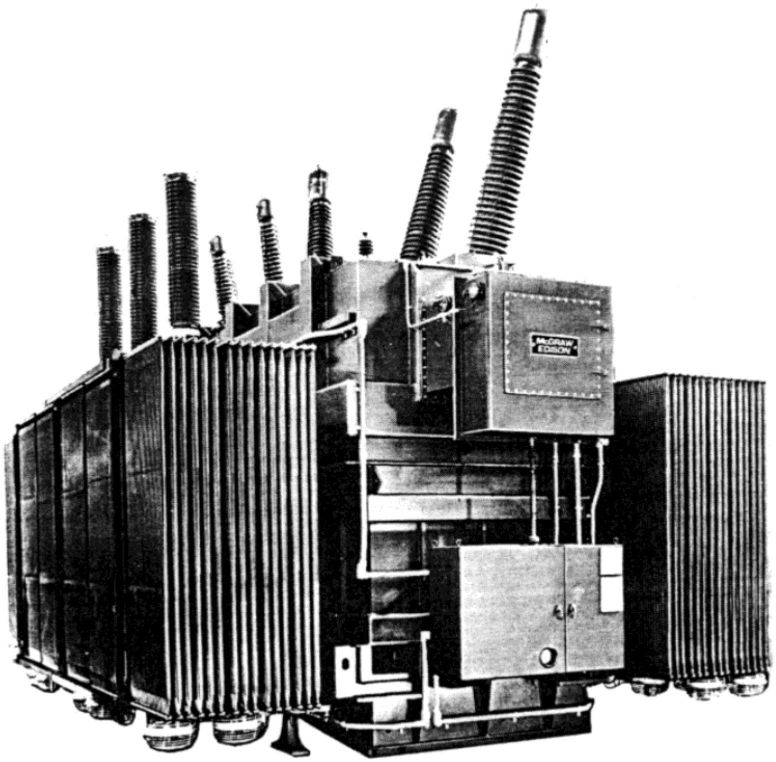
Cooling fans are installed under the radiators.

Large bushings connect the windings to the electrical system.

The oil is circulated by pumps and forced through the radiators.

The oil temperature, pressure are monitored to predict transformer performance.

Large three phase oil transformer



Power Transformers- IDEAL TRANSFORMER

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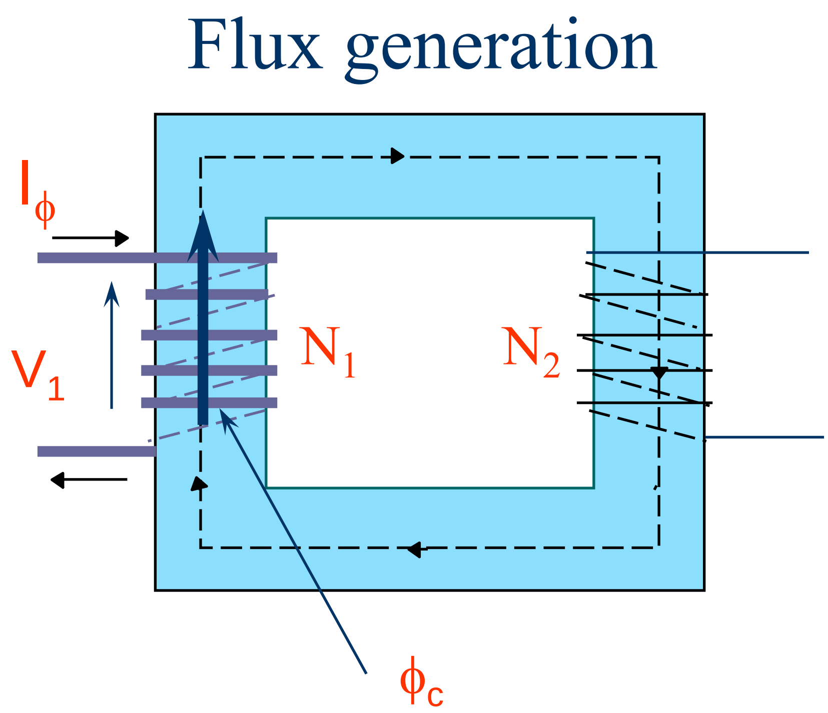
- **A component of the current, called the magnetization current I_m, is required to set-up the magnetic field (or the flux in the iron core, Φ_c).**
- **This flux which is a time-varying flux links both the primary and secondary windings. Accordingly, voltages (emfs) are induced in both windings.**
- **Since the iron core is exposed to AC current, the source should also supply a component of current called the core loss component, Ic, to account for hysteresis and eddy current losses.**
- **Total No-load current, $I\phi = Im + Ic$.**



- **Induced Voltages:**

- **The induced emf in primary winding is:**

$$E1 = 4.44\ N1\ \Phi_m\ f,$$

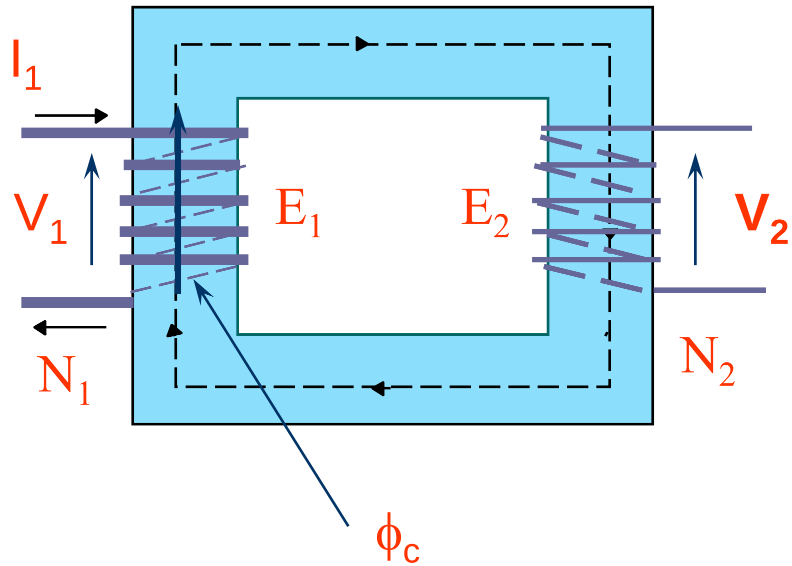
where N1 is the number of turns in primary winding, Φ_m, the maximum (peak) flux, and f the frequency of the supply voltage.

- **Similarly, the induced emf in secondary winding,**

$$E2 = 4.44\ N2\ \Phi_m\ f,$$

where N2 is the number of turns in secondary winding.

Voltage generation



Power Transformers- IDEAL TRANSFORMER

- **Ideal transformer is characterized by:**
  - No real power loss;
  - No linkage flux;
  - Magnetic core has infinite permeability (μ).

$$v_1 = e_1 = N_1 \frac{d\Phi}{dt}$$

$$v_2 = e_2 = N_2 \frac{d\Phi}{dt}$$

  - Therefore

$$\frac{v_1}{v_2} = \frac{N_1}{N_2} = a$$

where, a is the turns ratio of the transformer

- **With no power loss:**

$$P_{in} = P_{out}$$

$$v_1 i_1 = v_2 i_2$$

- **Therefore**

$$\frac{i_1}{i_2} = \frac{N_2}{N_1} = \frac{1}{a}$$

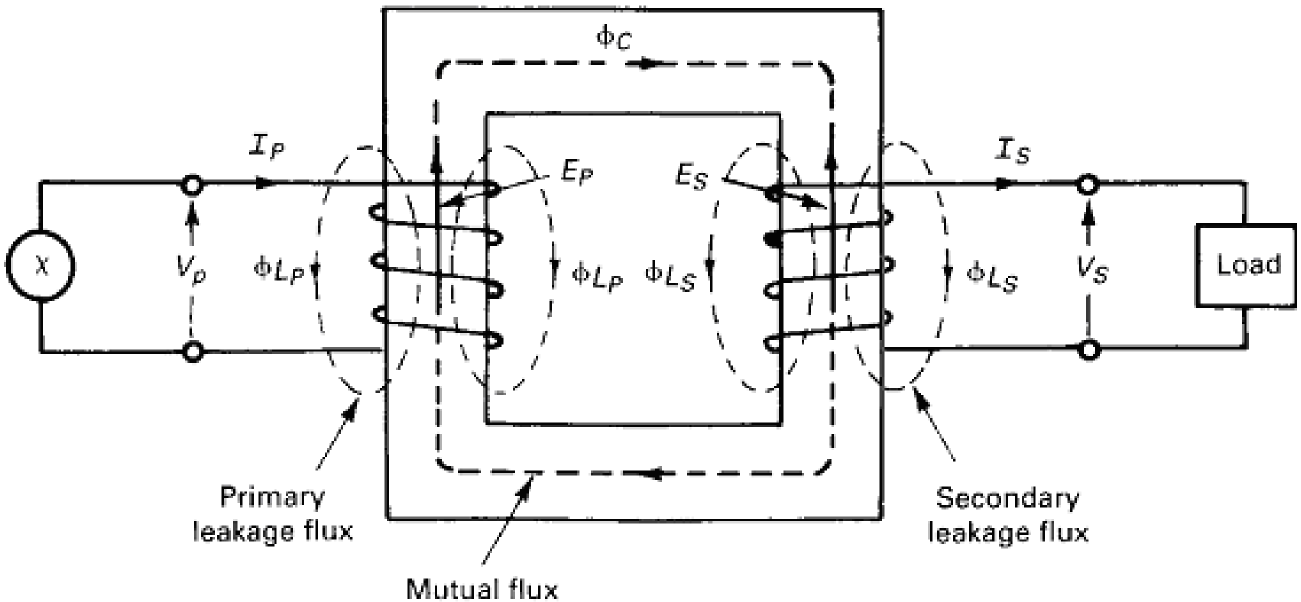
Power Transformers- REAL TRANSFORMER

- Real transformers
  - have losses
  - have leakage flux
  - have finite permeability of magnetic core

1. Real power losses
   - resistance in windings ($i^2 R$)
   - core losses due to eddy currents and hysteresis

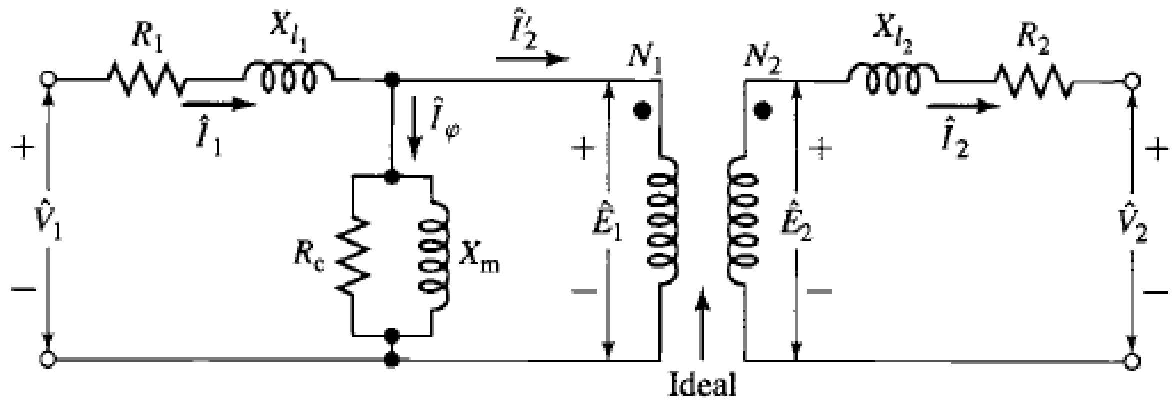
Power Transformers- REAL TRANSFORMER

- <u>Leakage Flux</u>: **Not all of the flux produced by the primary current links the winding, but there is leakage of some flux into air surrounding the primary. Similarly, not all of the flux produced by the secondary current (load current) links the secondary, rather there is loss of flux due to leakage. These effects are modelled as leakage reactance in the equivalent circuit representation.**



Power Transformers- REAL TRANSFORMER

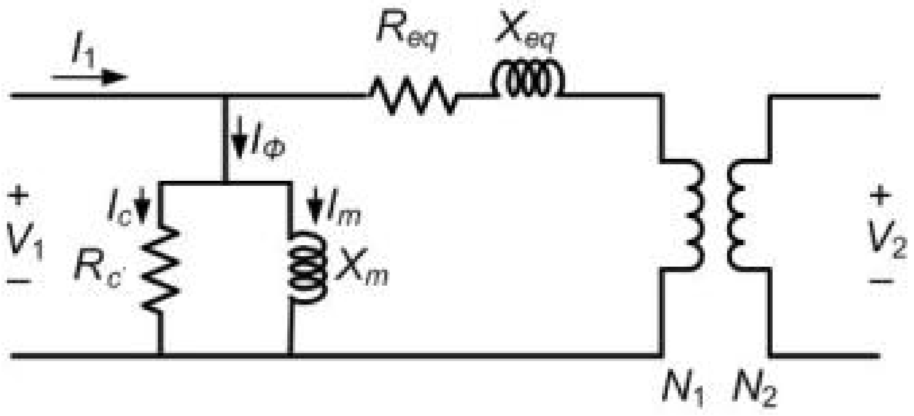
- The single-phase equivalent circuit of a real transformer is shown below.



- The leakage inductances of the transformer is denoted by X_{l1} & X_{l1} and R_1 & R_2 are the transformer's winding resistances
- The core loss component is represented by R_c while the magnetizing reactance is denoted by X_m .

Power Transformers- Approximate Circuits

- Neglecting the voltage drop across the primary and secondary windings due to the excitation current.
- Therefore, the magnetization branch can be moved to either the primary or secondary terminals.
- The approximate equivalent circuit of the transformer referred to the primary side is shown below.



$$R_{eq} = R_1 + a^2 R_2$$

$$X_{eq} = X_{l_1} + a^2 X_{l_2}$$

Power Transformers- Approximate Circuits

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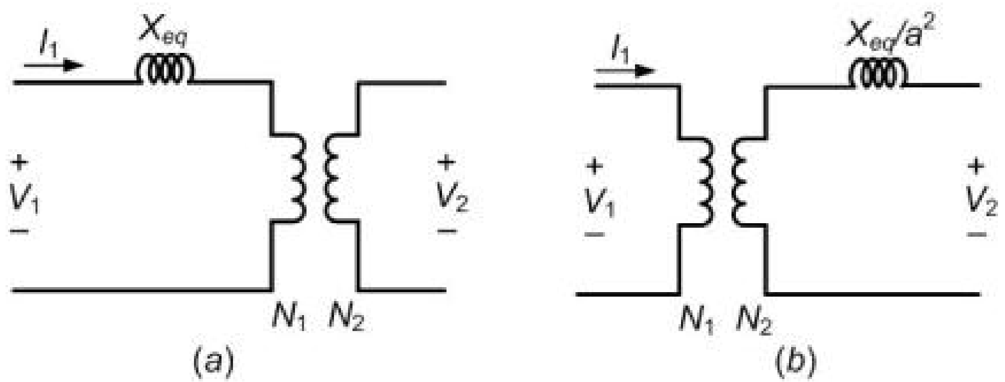
- The impedance of the shunt branch is much larger compared to that of the series branch. Therefore we neglect R_c and X_m.
- R_{eq} is much smaller than X_{eq}. We can therefore neglect the series resistance. Therefore the transformer can be represented by the leakage reactance X_{eq}.



$a = N_1 / N_2$

Simplified equivalent circuit of a single-phase transformer: (a) when referred to the primary side and (b) when referred to the secondary side.

Power Transformers- **EXAMPLE 1**

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A 25-kVA, 440/220-V, 60-Hz transformer has the following parameters:

$$R_1 = 0.16\ \Omega \qquad R_2 = 0.04\ \Omega \qquad R_{c1} = 270\ \Omega$$
$$X_1 = 0.32\ \Omega \qquad X_2 = 0.08\ \Omega \qquad X_{m1} = 100\ \Omega$$

The transformer delivers 20 kW at 0.8 power factor lagging to a load on the low-voltage side with 220 V across the load. Find the primary terminal voltage.

Power Transformers- EXAMPLE 1, Sol.

The voltage across the load is taken as reference phasor; thus,

$$V_2 = 220\angle 0^\circ \text{ V}$$

The transformer turns ratio is $a = 440/220 = 2$.

For a load $P_2 = 20{,}000$ W at 0.8 power factor lagging, the secondary current is computed as follows:

$$I_2 = \frac{20{,}000}{(220)(0.8)}\angle -\cos^{-1} 0.8 = 113.64\angle -36.9^\circ \text{ A}$$

Power Transformers- **EXAMPLE 1, Sol.**

$$a\mathbf{V}_2 = 2(220\angle 0^\circ) = 440\angle 0^\circ \text{ V}$$

$$\mathbf{I}_2/a = (113.64\angle -36.9^\circ)/2 = 56.82\angle -36.9^\circ \text{ A}$$

$$a^2R_2 = (2)^2(0.04) = 0.16\ \Omega$$

$$a^2X_2 = (2)^2(0.08) = 0.32\ \Omega$$

$$\begin{aligned}\mathbf{E}_1 &= a\mathbf{V}_2 + (\mathbf{I}_2/a)(a^2R_2 + ja^2X_2)\\ &= 440\angle 0^\circ + (56.82\angle -36.9^\circ)(0.16 + j0.32)\\ &= 458.2 + j9.07 = 458.3\angle 1^\circ \text{ V}\end{aligned}$$

$$\mathbf{I}_c = \mathbf{E}_1/R_{c1} = (458.2 + j9.07)/270 = 1.7 + j0.03 \text{ A}$$

$$\mathbf{I}_m = \mathbf{E}_1/jX_{m1} = (458.2 + j9.07)/j100 = 0.09 - j4.58 \text{ A}$$

$$\mathbf{I}_e = \mathbf{I}_c + \mathbf{I}_m = 1.79 - j4.55 \text{ A}$$

Power Transformers- EXAMPLE 1, Sol.

Thus, the primary current and voltage are:

$$\mathbf{I}_1 = \mathbf{I}_e + \mathbf{I}_2/a$$
$$= (1.79 - j4.55) + (56.82\angle -36.9^\circ) = 61.04\angle -39.3^\circ \text{ A}$$

$$\mathbf{V}_1 = \mathbf{E}_1 + \mathbf{I}_1(R_1 + jX_1)$$
$$= (458.2 + j9.07) + (61.04\angle -39.3^\circ)(0.16 + j0.32)$$
$$= 478.1 + j18 = 478.4\angle 2.2^\circ \text{ V}$$

Voltage Regulation

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$$\% \, VR = (V_{NL} - V_{FL}) \times 100 \, / \, V_{FL}$$

$$= (V_p - aV_s) \times 100 \, / \, aV_s$$

$$= (V_p - V'_s) \times 100 \, / \, V'_s$$

Note: The primary side voltage is always adjusted to meet the load changes; hence, V'_s and V_s are kept constant. There is no source on the secondary side

Efficiency

As always, efficiency is defined as power output to power input ratio.

$$\eta = P_{out}/P_{in} \times 100\ \%$$

$$P_{in} = P_{out} + P_{core} + P_{copper}$$

P_{copper} represents the copper losses in primary and secondary windings. There are no rotational losses.

Power Transformers- EXAMPLE 2

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A 150-kVA, 2400/240-V transformer has the following parameters referred to the primary side: $R_{e1} = 0.5\ \Omega$ and $X_{e1} = 1.5\ \Omega$. The shunt magnetizing impedance is very large and can be neglected. At full load, the transformer delivers rated kVA at 0.85 lagging power factor and the secondary voltage is 240 V. Calculate (a) the voltage regulation and (b) the efficiency assuming core losses amount to 600 W.

Solution:

a. The transformer turns ratio is

$$a = 2400/240 = 10$$

Take the secondary voltage $\mathbf{V}_2$ as the reference

$$\mathbf{V}_2 = 240\angle 0^\circ\ \text{V}$$

$$a\mathbf{V}_2 = 2400\angle 0^\circ\ \text{V}$$

Power Transformers- EXAMPLE 2, Sol.

At rated load and 0.85 PF lagging:

$$\mathbf{I}_2 = (150{,}000/240)\,\angle{-\cos^{-1} 0.85°} = 625\,\angle{-31.8°}\ \text{A}$$

$$\mathbf{I}_1 = \mathbf{I}_2/a = (625/10)\,\angle{-31.8°} = 62.5\,\angle{-31.8°}\ \text{A}$$

The primary voltage is calculated as follows:

$$\mathbf{V}_1 = a\mathbf{V}_2 + (\mathbf{I}_2/a)(R_{e1} + jX_{e1})$$

$$= 2400\,\angle{0°} + (62.5\,\angle{-31.8°})(0.5 + j1.5) = 2476.8\,\angle{1.5°}\ \text{V}$$

Power Transformers- EXAMPLE 2, Sol.

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$$\text{Voltage regulation} = \frac{V_1 - aV_2}{aV_2} 100\%$$

$$= \frac{2476.8 - 2400}{2400} 100\% = 3.2\%$$

b. At rated output,

$$P_{\text{output}} = (150{,}000)(0.85) = 127{,}500 \text{ W}$$

$$P_{\text{cu}} = I_1^2 R_{e1} = (62.5)^2(0.5) = 1953 \text{ W}$$

$$P_{\text{core}} = 600 \text{ W}$$

$$P_{\text{input}} = P_{\text{output}} + \Sigma(\text{losses}) = 130{,}053 \text{ W}$$

Therefore, the efficiency is found by using Eq. 4.51 as follows:

$$\eta = \frac{\text{power output}}{\text{power input}} 100\%$$

$$= \frac{127{,}500}{130{,}053} 100\% = 98\%$$

Equivalent circuit parameters

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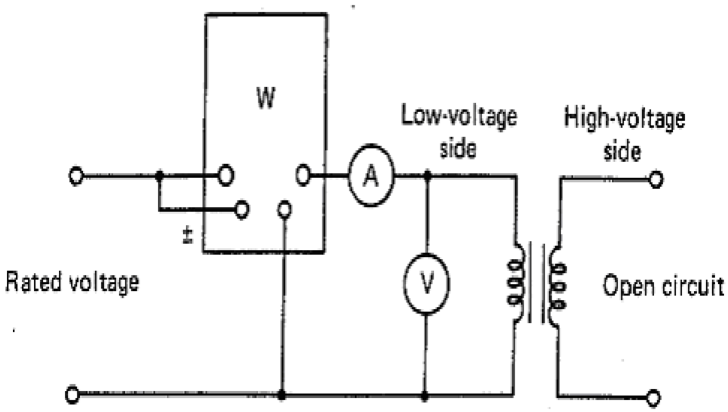
- **Open Circuit Test:**

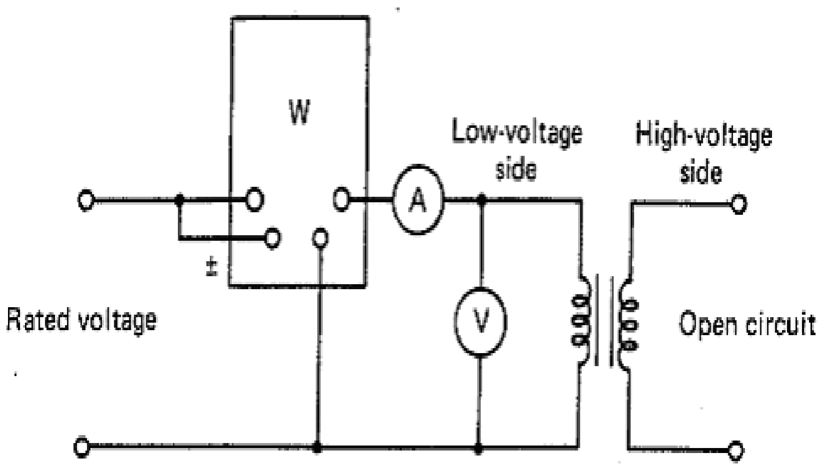
  Secondary (normally the HV winding) is open, that means there is no load across secondary terminals; hence there is no current in the secondary.
- Winding losses are negligible, and the source mainly supplies the core losses, P_{core}.
- **Parameters obtained:** Test is done at rated voltage with secondary open. So, the ammeter reads the no-load current, I_o; the wattmeter reads the core losses, and the voltmeter reads the applied primary voltage.



Open circuit test

Equivalent circuit parameters

- Wattmeter reading = $P_{oc} = P_{core}$
- Hence, $R_{c\ (LV)} = V^2_{(LV)}/P_{oc}$
- Note: The open circuit test was done by energizing the LV (low voltage) side with secondary (HV) open.
- Once, $R_{c(LV)}$ is known, X_m can be found as follows.
- $I_{c(LV)} = V_{(LV)}/R_{c(LV)}$
- But, Ammeter reading = I_o.
- Therefore, $\mathbf{I}_{m(LV)} = \mathbf{I}_o - \mathbf{I}_{c(LV)}$
- $X_m = V_{(LV)}/I_{m(LV)}$



Open circuit test

Equivalent circuit parameters

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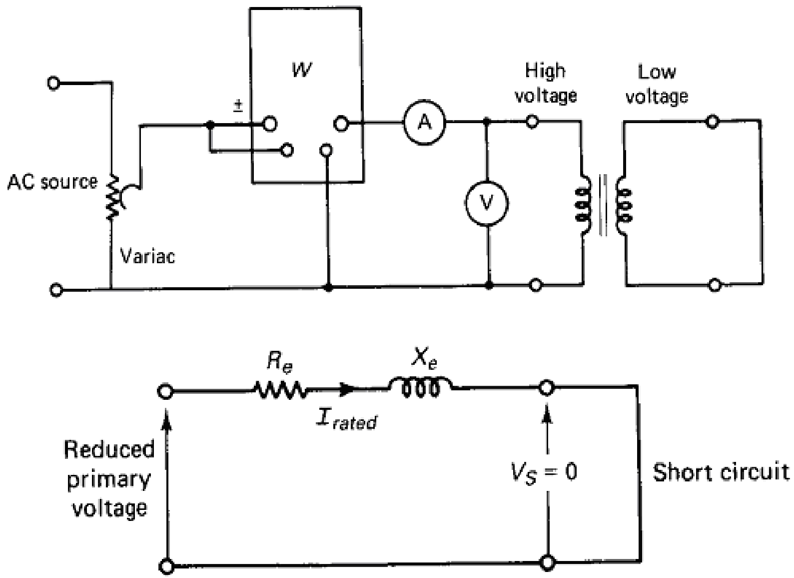
- **Short Circuit Test:**
- Secondary (normally the LV winding) is shorted, that means there is no voltage across secondary terminals; but a large current flows in the secondary.

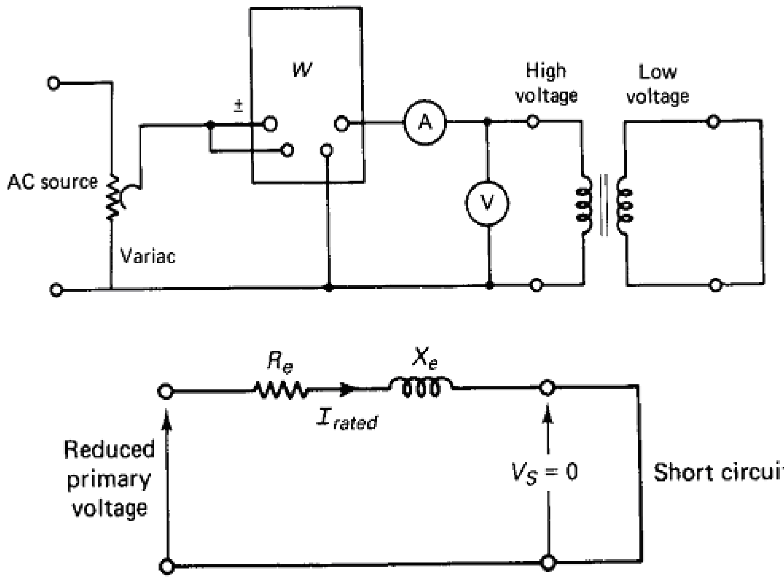
- **Parameters obtained:** Test is done at reduced voltage (about 5% of rated voltage) with full-load current in the secondary. So, the ammeter reads the full-load current, Ip; the wattmeter read the winding losses, and the voltmeter reads the applied primary voltage.



Short- circuit test

Equivalent circuit parameters

- Core losses are negligible as the applied voltage is << rated voltage.
- $Rep = P_{sc}/I^2_{sc}$
- But, $Z_{ep}(HV) = V_{sc}(HV)/I_{sc}$ hence, $X_{ep}(HV)$ can be obtained



Short- circuit test

Circuit parameters- EXAMPLE 3

a 50 kVA, 2400/240 V transformer has the following test data:

	Voltage (v)	Current (A)	Power (W)
Short circuit test	55	20.8	600
Open circuit test	240	5.0	450

Calculate:

1. The voltage regulation and efficiency when the transformer is connected to a load that takes 156 A at 220 V and 0.8 power factor lagging.

Circuit parameters- **EXAMPLE 3, Sol.**

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Find the transformer parameters :-

$$Y_2 = \frac{I_{oc}}{V_{oc}} = \frac{5}{240} = 0.2083$$

$$-\theta_2 = -\cos^{-1}\left(\frac{P_{oc}}{V_{oc}\,I_{oc}}\right) = -67.98$$

$$Y_2 = .02083 \angle -67.98 = .007081 - j.0193$$

$$\Rightarrow R_{c_2} = 128\,\Omega \quad \& \quad X_{m_2} = 51.8$$

Since a O.C. test was conducted from the secondary side ⇒ all calculated values are referred to the sec. side.

$I_{sc} = 20.8 = I_{rated}$ in the primary side

⇒ S.C. test was conducted from the primary side.

$$R_{e_1} = \frac{P_{sc}}{I_{sc}^2} = \frac{600}{(20.8)^2} = 1.387\,\Omega$$

$$Z_{e_1} = \frac{V_{sc}}{I_{sc}} = \frac{55}{20.8} = 2.644\,\Omega$$

$$\Rightarrow X_{e_1} = \sqrt{Z_{e_1}^2 - R_{e_1}^2} = 2.25\,\Omega$$

Circuit parameters- EXAMPLE 3, Sol.

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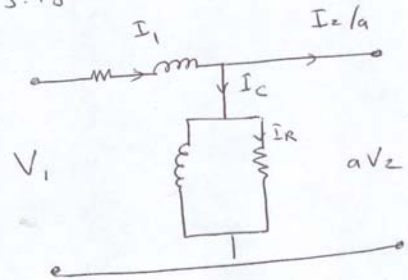
⟹ Refer every thing to primary:

$$R_{c1} = a^2 R_{c1} = \left(\frac{2400}{240}\right)^2 128 = 12.8\,K\Omega$$

$$X_{m1} = \left(\frac{2400}{240}\right)^2 (51.8) = 5.18\,K\Omega$$



(a) $I_2 = 156 \angle -\cos^{-1}(0.8)$

$$= 156 \angle -36.9$$

$$\frac{I_2}{a} = 15.6 \angle -36.9$$

$$I_c = aV_2 \left[\frac{1}{R_{c1}} + \frac{1}{jX_{m1}}\right]$$

$$= (220)(10)\left[\frac{1}{12.8\times10^3} + \frac{1}{j5.18\times10^3}\right]$$

$$= 0.172 - j\,0.425 =$$

$$I_1 = I_2/a + I_c = 15.99 \angle -37.72$$

$$V_1 = I_1 Z_{e1} + aV_2 = 2239.6 \angle 0.38$$

$$V.R. = \frac{|V_1| - |aV_2|}{|aV_2|} = \frac{2239.6 - 2200}{2200} = 1.8\,\%$$

$$P_{core} = I_R^2 R_{c1} = (0.172)^2 \times 12{,}800 = 378.69\,W$$

$$P_{cu} = I_1^2 R_{e1} = (15.99)^2 \times 1.387 = 354.62\,W$$

Circuit parameters- EXAMPLE 3, Sol.

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$$\eta = \frac{P_{out}}{P_{in}}$$

$$= \frac{P_{out}}{P_{out} + \text{losses}}$$

$$P_{out} = (220)(156)(0.8) = 27,456$$

$$\Rightarrow \eta = \frac{27,456}{27,456 + 378.69 + 354.62} = 97.4\%$$

or

$$P_{in} = V_1 I_1 \cos\theta$$

$$= (2239)(15.99)\cos(0.38 + 37.72)$$

$$= 28173.5$$

$$\Rightarrow \eta = \frac{27456}{28173.5} = 97.4\%$$

Home work

Q1 A 10 kVA, 2200/220 V, 60 Hz, single-phase transformer provides the following test results:

	Voltage	Current	Power input
Open-circuit test	220 V	2.5 A	100 W
Short-circuit test	150 V	4.55 A	215 W

a. Indicate on which side the measurements were taken, and the reasons for your choice.

b. Determine the approximate parameters of the transformer referred to the high-voltage winding.

b. The transformer is used as a step-up transformer. If the load is 7 kW at 0.707 pf lagging (attached to the high-voltage winding), and rated voltage appears across the load, what is the voltage at the primary? What is the percent voltage regulation (PVR) for this load?

Home work

Q2:

A 100 kVA, 60 Hz, 7200/480 V, single-phase transformer has the following parameters:

$R_{HV} = 3.06\ \Omega$	$X_{HV} = 6.05\ \Omega$	$X_{m,HV} = 17{,}809\ \Omega$
$R_{LV} = 0.014\ \Omega$	$X_{LV} = 0.027\ \Omega$	$R_{c,HV} = 71{,}400\ \Omega$

The transformer load draws rated current at 480 V and 0.75 power factor lagging. Sketch the *approximate* equivalent circuit (indicating whether you are showing the HV side of the LV side), and determine:

a. the transformer equivalent impedance, referred to the HV side;
b. the load current referred to the HV side;
c. the source voltage; and,
d. the excitation current (referred to the HV side).

Per Unit Calculations

- A key problem in analyzing power systems is the large number of transformers.
  - It would be very difficult to continually have to refer impedances to the different sides of the transformers
- This problem is avoided by a normalization of all variables.
- This normalization is known as per unit analysis.

$$\text{quantity in per unit} = \frac{\text{actual quantity}}{\text{base value of quantity}}$$

Per Unit Conversion Procedure, 1ϕ

1. Pick a 1ϕ VA base for the entire system, S_B
2. Pick a voltage base for each different voltage level, V_B. Voltage bases are related by transformer turns ratios. Voltages are line to neutral.
3. Calculate the impedance base, $Z_B = (V_B)^2/S_B$
4. Calculate the current base, $I_B = V_B/Z_B$
5. Convert actual values to per unit

Note, per unit conversion on affects magnitudes, not the angles. Also, per unit quantities no longer have units (i.e., a voltage is 1.0 p.u., not 1 p.u. volts)

Three Phase Per Unit

Procedure is very similar to 1ϕ except we use a 3ϕ VA base, and use line to line voltage bases

1. Pick a 3ϕ VA base for the entire system, $S_B^{3\phi}$
2. Pick a voltage base for each different voltage level, V_B. Voltages are line to line.
3. Calculate the impedance base

$$Z_B = \frac{V_{B,LL}^2}{S_B^{3\phi}} = \frac{(\sqrt{3}\,V_{B,LN})^2}{3S_B^{1\phi}} = \frac{V_{B,LN}^2}{S_B^{1\phi}}$$

Exactly the same impedance bases as with single phase!

Three Phase Per Unit, cont'd

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4. Calculate the current base, I_B

$$I_B^{3\phi} = \frac{S_B^{3\phi}}{\sqrt{3}\,V_{B,LL}} = \frac{3\,S_B^{1\phi}}{\sqrt{3}\,\sqrt{3}\,V_{B,LN}} = \frac{S_B^{1\phi}}{V_{B,LN}} = I_B^{1\phi}$$

Exactly the same current bases as with single phase!

5. Convert actual values to per unit