ELEC B7

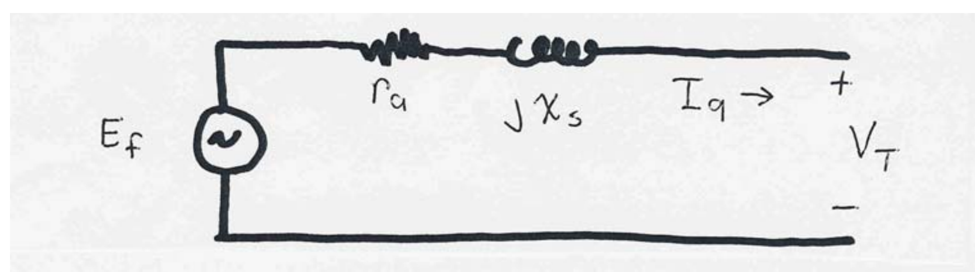
Load Flow

Load Models

- Ultimate goal is to supply loads with electricity at constant frequency and voltage
- Electrical characteristics of individual loads matter, but usually they can only be estimated
  - actual loads are constantly changing, consisting of a large number of individual devices
  - only limited network observability of load characteristics
- Two common models
  - constant power: $S_i = P_i + jQ_i$
  - constant impedance: $S_i = |V|^2 / Z_i$

Generator Models

- Generators are usually synchronous machines
- For generators we will use two different models:
  - a steady-state model, treating the generator as a constant power source operating at a fixed voltage; this model will be used for power flow and economic analysis
  - a short term model treating the generator as a constant voltage source behind a possibly time-varying reactance



Power Flow Analysis

- We now have the necessary models to start to develop the power system analysis tools
- The most common power system analysis tool is the power flow (also known sometimes as the load flow)
  - power flow determines how the power flows in a network
  - also used to determine all bus voltages and all currents
  - because of constant power models, power flow is a nonlinear analysis technique
  - power flow is a steady-state analysis tool

Linear versus Nonlinear Systems

A function **H** is linear if

$$\mathbf{H}(\alpha_1\mu_1 + \alpha_2\mu_2) = \alpha_1\mathbf{H}(\mu_1) + \alpha_2\mathbf{H}(\mu_2)$$

That is

1) the output is proportional to the input
2) the principle of superposition holds

Linear Example: $\mathbf{y} = \mathbf{H}(\mathbf{x}) = c\,\mathbf{x}$

$$\mathbf{y} = c(\mathbf{x}_1+\mathbf{x}_2) = c\mathbf{x}_1 + c\,\mathbf{x}_2$$

Nonlinear Example: $\mathbf{y} = \mathbf{H}(\mathbf{x}) = c\,\mathbf{x}^2$

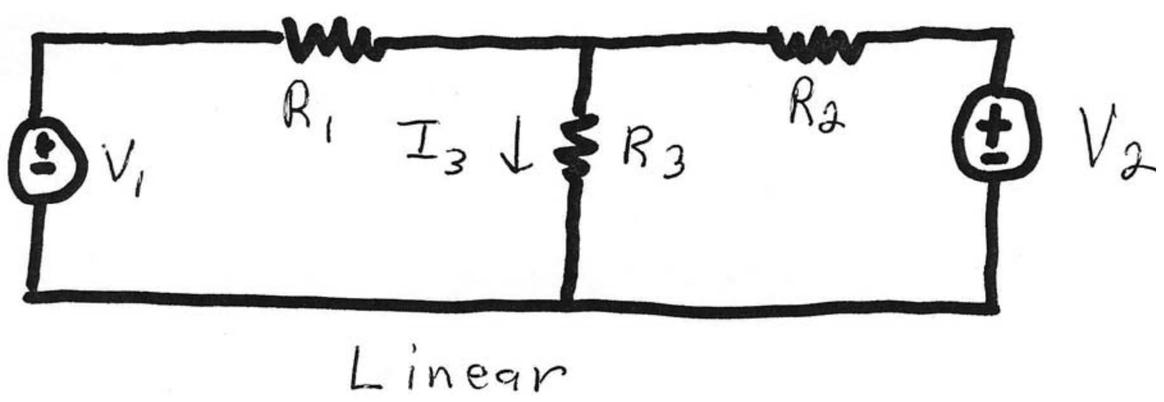
$$\mathbf{y} = c(\mathbf{x}_1+\mathbf{x}_2)^2 \neq (c\mathbf{x}_1)^2 + (c\,\mathbf{x}_2)^2$$

Linear Power System Elements

Resistors, inductors, capacitors, independent voltage sources and current sources are linear circuit elements

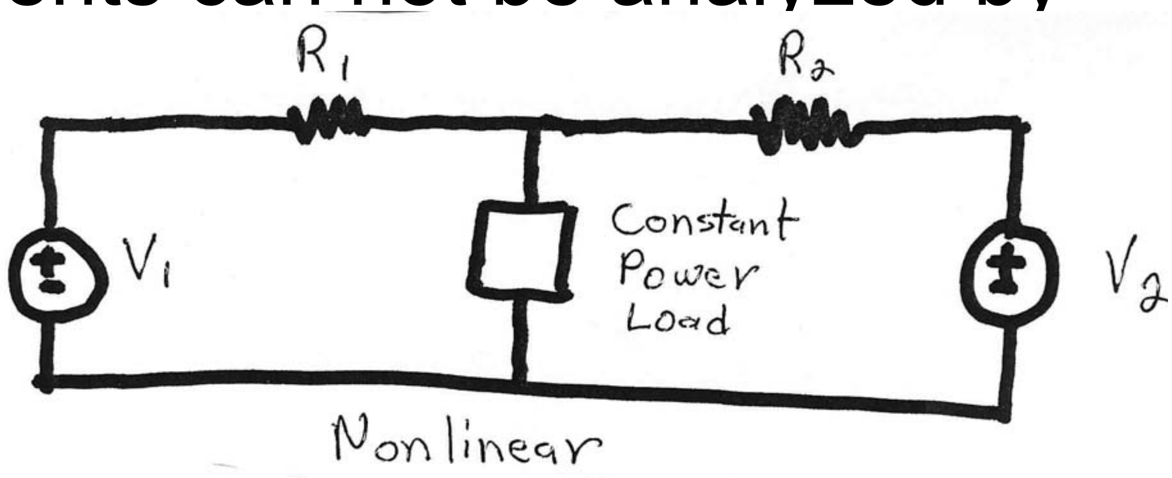
$$V = R\,I \qquad V = j\omega L\, I \qquad V = \frac{1}{j\omega C} I$$

Such systems may be analyzed by superposition



Nonlinear Power System Elements

- Constant power loads and generator injections are nonlinear and hence systems with these elements can not be analyzed by superposition



Nonlinear problems can be very difficult to solve, and usually require an iterative approach

Nonlinear Systems May Have Multiple Solutions or No Solution

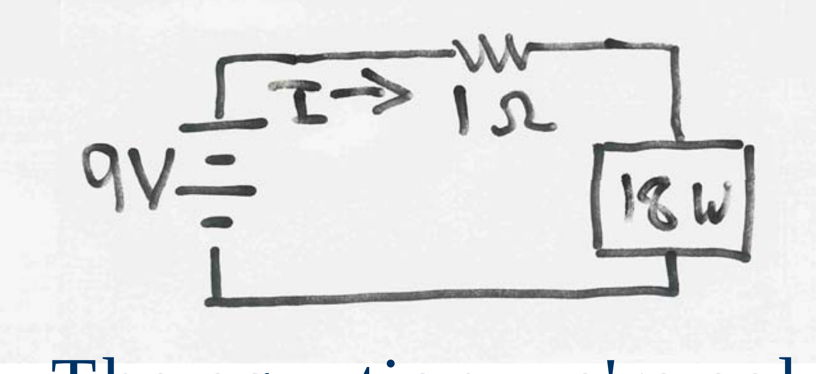
- Example 1: $x^2 - 2 = 0$ has solutions $x = \pm 1.414$
- Example 2: $x^2 + 2 = 0$ has no real solution

$f(x) = x^2 - 2$ $\qquad\qquad$ $f(x) = x^2 + 2$

two solutions where $f(x) = 0$ $\qquad\qquad$ no solution $f(x) = 0$

Multiple Solution Example 3

The dc system shown below has two solutions:



where the 18 watt load is a resistive load

The equation we're solving is

$$I^2 R_{Load} = \left(\frac{9 \text{ volts}}{1\Omega + R_{Load}} \right)^2 R_{Load} = 18 \text{ watts}$$

One solution is $R_{Load} = 2\Omega$

Other solution is $R_{Load} = 0.5\Omega$

Bus Admittance Matrix or Y_{bus}

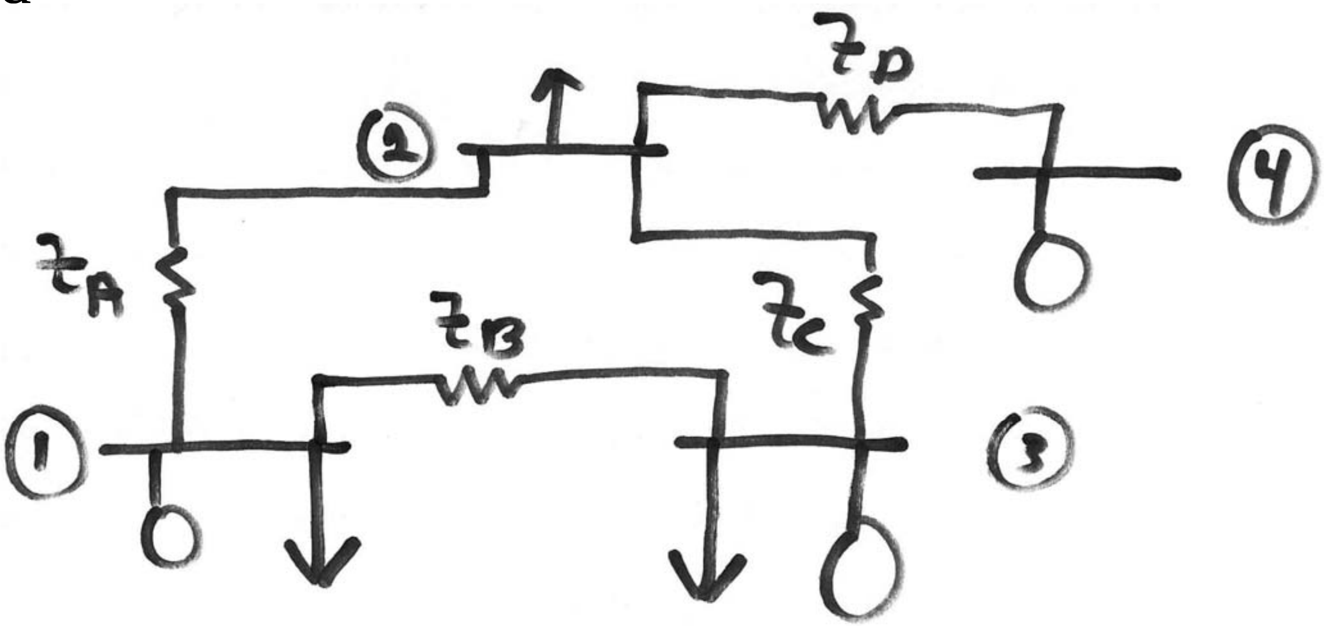
- First step in solving the power flow is to create what is known as the bus admittance matrix, often call the $\mathbf{Y}_{bus}$.
- The $\mathbf{Y}_{bus}$ gives the relationships between all the bus current injections, **I**, and all the bus voltages, **V**,

$$\mathbf{I} = \mathbf{Y}_{bus}\,\mathbf{V}$$

- The $\mathbf{Y}_{bus}$ is developed by applying KCL at each bus in the system to relate the bus current injections, the bus voltages, and the branch impedances and admittances

Y_{bus} Example

Determine the bus admittance matrix for the network shown below, assuming the current injection at each bus i is $I_i = I_{Gi} - I_{Di}$ where I_{Gi} is the current injection into the bus from the generator and I_{Di} is the current flowing into the load



Y_{bus} Example, cont'd

By KCL at bus 1 we have

$$I_1 \triangleq I_{G1} - I_{D1}$$

$$I_1 = I_{12} + I_{13} = \frac{V_1 - V_2}{Z_A} + \frac{V_1 - V_3}{Z_B}$$

$$I_1 = (V_1 - V_2)Y_A + (V_1 - V_3)Y_B \qquad (\text{with } \mathrm{Y_j} = \frac{1}{Z_j})$$

$$= (Y_A + Y_B)V_1 - Y_A V_2 - Y_B V_3$$

Similarly

$$I_2 = I_{21} + I_{23} + I_{24}$$

$$= -Y_A V_1 + (Y_A + Y_C + Y_D)V_2 - Y_C V_3 - Y_D V_4$$

Y_{bus} Example, cont'd

We can get similar relationships for buses 3 and 4

The results can then be expressed in matrix form

$$\mathbf{I} = \mathbf{Y}_{bus}\mathbf{V}$$

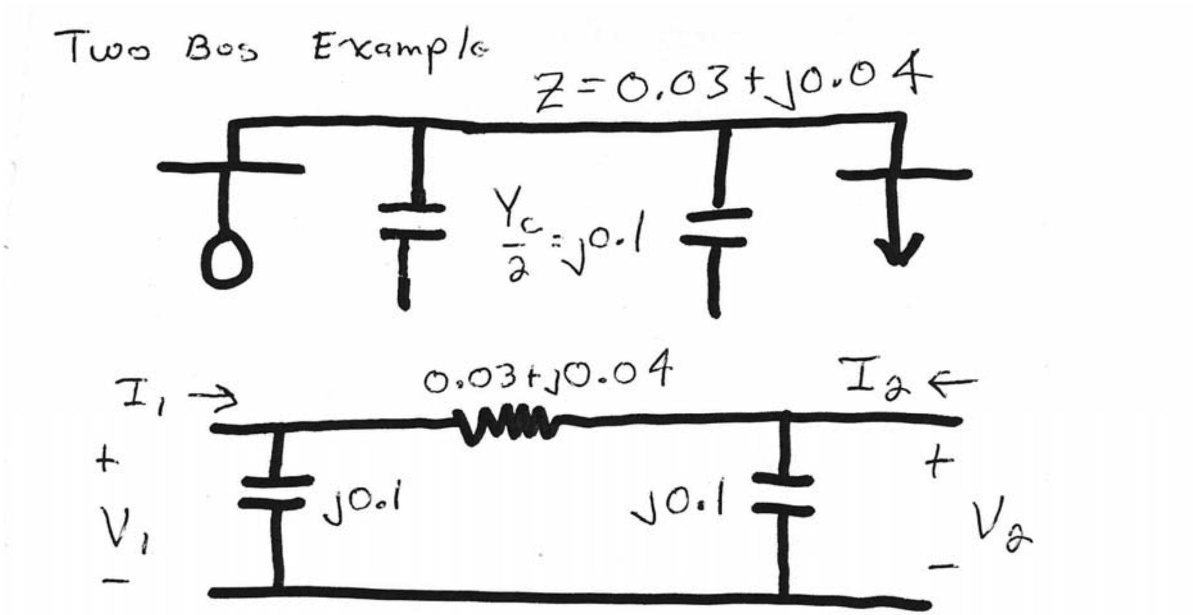
$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} Y_A + Y_B & -Y_A & -Y_B & 0 \\ -Y_A & Y_A + Y_C + Y_D & -Y_C & -Y_D \\ -Y_B & -Y_C & Y_B + Y_C & 0 \\ 0 & -Y_D & 0 & Y_D \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

For a system with n buses, Y_{bus} is an n by n symmetric matrix (i.e., one where $A_{ij} = A_{ji}$)

Y_{bus} General Form

- The diagonal terms, Y_{ii}, are the self admittance terms, equal to the sum of the admittances of all devices incident to bus i.
- The off-diagonal terms, Y_{ij}, are equal to the negative of the sum of the admittances joining the two buses.
- With large systems Y_{bus} is a sparse matrix (that is, most entries are zero)
- Shunt terms, such as with the π line model, only affect the diagonal terms.

Two Bus System Example



$$I_1 = \frac{(V_1 - V_2)}{Z} + V_1 \frac{Y_c}{2} \qquad \frac{1}{0.03 + j0.04} = 12 - j16$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 12 - j15.9 & -12 + j16 \\ -12 + j16 & 12 - j15.9 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Using the Y_{bus}

If the voltages are known then we can solve for the current injections:

$$\mathbf{Y}_{bus}\mathbf{V} = \mathbf{I}$$

If the current injections are known then we can solve for the voltages:

$$\mathbf{Y}_{bus}^{-1}\mathbf{I} = \mathbf{V} = \mathbf{Z}_{bus}\mathbf{I}$$

where $\mathbf{Z}_{\text{bus}}$ is the bus impedance matrix

Solving for Bus Currents

For example, in previous case assume

$$\mathbf{V} = \begin{bmatrix} 1.0 \\ 0.8 - j0.2 \end{bmatrix}$$

Then

$$\begin{bmatrix} 12 - j15.9 & -12 + j16 \\ -12 + j16 & 12 - j15.9 \end{bmatrix} \begin{bmatrix} 1.0 \\ 0.8 - j0.2 \end{bmatrix} = \begin{bmatrix} 5.60 - j0.70 \\ -5.58 + j0.88 \end{bmatrix}$$

Therefore the power injected at bus 1 is

$$\mathrm{S}_1 = V_1 I_1^* = 1.0 \times (5.60 + j0.70) = 5.60 + j0.70$$

$$S_2 = V_2 I_2^* = (0.8 - j0.2) \times (-5.58 - j0.88) = -4.64 + j0.41$$

Solving for Bus Voltages

For example, in previous case assume

$$\mathbf{I} = \begin{bmatrix} 5.0 \\ -4.8 \end{bmatrix}$$

Then

$$\begin{bmatrix} 12 - j15.9 & -12 + j16 \\ -12 + j16 & 12 - j15.9 \end{bmatrix}^{-1} \begin{bmatrix} 5.0 \\ -4.8 \end{bmatrix} = \begin{bmatrix} 0.0738 - j0.902 \\ -0.0738 - j1.098 \end{bmatrix}$$

Therefore the power injected is

$$\mathrm{S}_1 = V_1 I_1^* = (0.0738 - j0.902) \times 5 = 0.37 - j4.51$$

$$S_2 = V_2 I_2^* = (-0.0738 - j1.098) \times (-4.8) = 0.35 + j5.27$$

Power Flow Analysis

- When analyzing power systems we know neither the complex bus voltages nor the complex current injections
- Rather, we know the complex power being consumed by the load, and the power being injected by the generators plus their voltage magnitudes
- Therefore we can not directly use the Y_{bus} equations, but rather must use the power balance equations

Power Balance Equations

From KCL we know at each bus i in an n bus system the current injection, I_i, must be equal to the current that flows into the network

$$I_i = I_{Gi} - I_{Di} = \sum_{k=1}^{n} I_{ik}$$

Since $\mathbf{I} = \mathbf{Y}_{\text{bus}}\mathbf{V}$ we also know

$$I_i = I_{Gi} - I_{Di} = \sum_{k=1}^{n} Y_{ik} V_k$$

The network power injection is then $\mathrm{S}_\mathrm{i} = V_i I_i^*$

Power Balance Equations, cont'd

$$S_i = V_i I_i^* = V_i \left(\sum_{k=1}^{n} Y_{ik} V_k \right)^* = V_i \sum_{k=1}^{n} Y_{ik}^* V_k^*$$

This is an equation with complex numbers.

Sometimes we would like an equivalent set of real power equations. These can be derived by defining

$$Y_{ik} \triangleq G_{ik} + jB_{ik}$$

$$V_i \triangleq |V_i| e^{j\theta_i} = |V_i| \angle \theta_i$$

$$\theta_{ik} \triangleq \theta_i - \theta_k$$

Recall $\mathrm{e}^{\mathrm{j}\theta} = \cos\theta + j\sin\theta$

Real Power Balance Equations

$$\mathrm{S_i} \;=\; P_i + jQ_i = V_i \sum_{k=1}^{n} Y_{ik}^{*} V_k^{*} = \sum_{k=1}^{n} |V_i||V_k| \, e^{j\theta_{ik}} (G_{ik} - jB_{ik})$$

$$= \sum_{k=1}^{n} |V_i||V_k| (\cos\theta_{ik} + j\sin\theta_{ik})(G_{ik} - jB_{ik})$$

Resolving into the real and imaginary parts

$$\mathrm{P_i} \;=\; \sum_{k=1}^{n} |V_i||V_k| (G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}) = P_{Gi} - P_{Di}$$

$$\mathrm{Q_i} \;=\; \sum_{k=1}^{n} |V_i||V_k| (G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}) = Q_{Gi} - Q_{Di}$$

Slack Bus

- We can not arbitrarily specify S at all buses because total generation must equal total load + total losses
- We also need an angle reference bus.
- To solve these problems we define one bus as the "slack" bus. This bus has a fixed voltage magnitude and angle, and a varying real/reactive power injection.

Three Types of Power Flow Buses

- There are three main types of power flow buses
  - Load (PQ) at which P/Q are fixed; iteration solves for voltage magnitude and angle.
  - Slack at which the voltage magnitude and angle are fixed; iteration solves for P/Q injections
  - Generator (PV) at which P and |V| are fixed; iteration solves for voltage angle and Q injection

Generator Reactive Power Limits

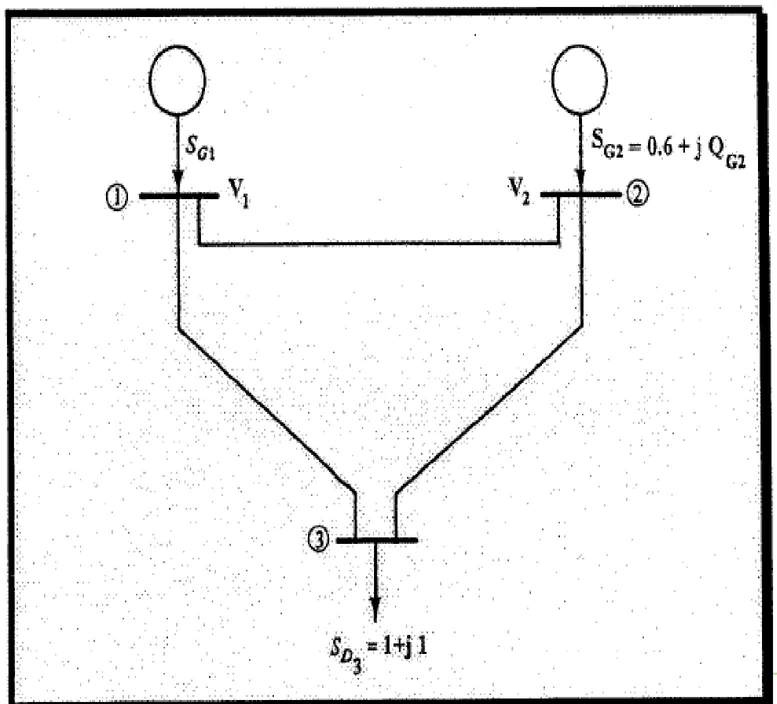
- The reactive power output of generators varies to maintain the terminal voltage; on a real generator this is done by the exciter
- To maintain higher voltages requires more reactive power
- Generators have reactive power limits, which are dependent upon the generator's MW output
- These limits must be considered during the power flow solution.

Home Work

Consider the system shown in the single-line diagram of Figure 2, where all line admittances are identical and have the same value of $Y_L = -j5$.

b- Write the bus admittance matrix of the system Y. What are the primary unknowns for the power flow problem for the system? [5 points]

c- Write the power flow equations assuming that bus 1 is the slack bus whose voltage is unity and whose angle is zero. Bus 2 is maintained at a voltage of 1.025 [5 points]



Home Work

Consider the system shown in the single-line diagram of Figure 2, where the line admittance between bus 1 and 2 is the same as that between bus 1 and 3 as: $Y_L = 4 - j5$. It is required to:

b- Find the voltage V_2 and its phase angle exactly given that $S_{D2} = 0.8 + j0.6$. [5 points]

c- Find the voltage V_3 and its phase angle exactly given that $S_{D3} = 0.4 + j0.3$ [5 points]

d- Find the value of S_{G1} and the generator power factor. [5 points]