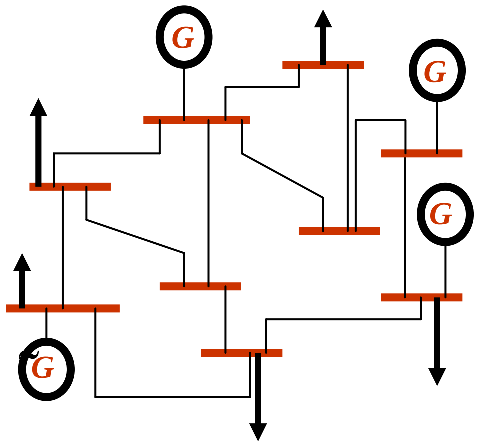
ELE B7 Power Systems Engineering

Power Flow- Introduction

Introduction to Load Flow Analysis

The power flow is the backbone of the power system *operation*, *analysis* and *design*. It is necessary for planning, operation, economic scheduling and exchange power between utilities.



The power flow is also required for many other applications such as *short-circuit* calculations, *transient stability* and *contingency analysis*.

For the network shown, there are some buses connected with the generators and other buses are connected to the loads.

The Real and Reactive power is known at each Load bus. The Generator Voltages are Also Specified at the generator buses.

The Transmission Lines interconnecting the buses have resistance and inductance. Therefore, the Electric Current flowing through the lines results in Electrical Losses.

The Generators in the System Must supply the Total Electrical Loads pulse the Electrical Losses.

There are some constrains should be considered while running the system:

1. The Generators **Must Operate** within their **Generation Capabilities**.
2. The Generators **Must Deliver** the required power at the **Desired Voltage** at the **Loads.**
3. There should be no bus voltage either above or below the specified **Voltage operating limits**.
4. There Should be no **Over-Loading** of equipment, including Transmission Lines and Transformers

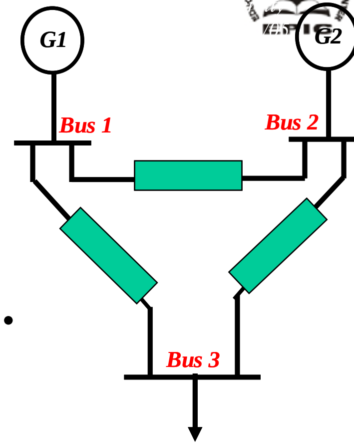
In Case of

An Equipment Over-Loaded Or Voltage-Limit Violation.

The **Generation Schedule** have to be adjusted and **Power Flow** in the transmission lines have to be **Re-routed** or **Capacitor Banks** have to be switched in order to bring the system into its **Normal Operating Conditions.**

To Satisfy all the previous requirement for a ***Reliable Power System Operation***, **Power Flow Study is a MUST**. The Power flow study is an essential part in power system Operation, Planning and Design.

Power Flow Concept



Consider the three-bus power system. Generators (*$G_1$and G_2*) are connected to the first two buses and an electric load is connected to the third bus.

The *real and reactive power demands* are known for the load bus (3). The *generator voltages* are also specified at bus 1 and bus 2.

The three transmission lines interconnecting the buses contain both *resistance and reactance*, thus currents flow through these lines results in *electrical-losses*.

The two generators (*G_1 and G_2*) must jointly supply the total *load requirements and the power losses* in the transmission lines.

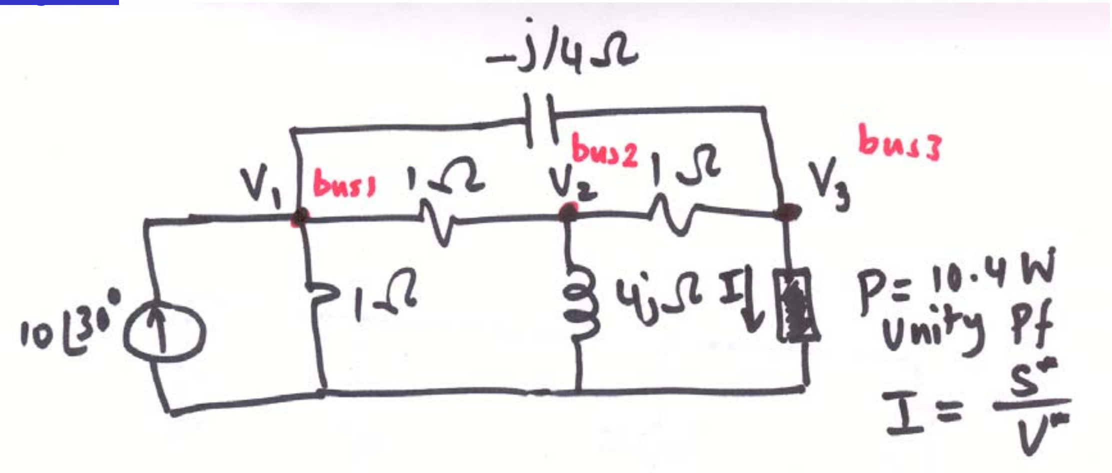
The generators are constrained to operate within their *power generation capabilities*.

The generators are also constrained to deliver the required *power at the desired voltage* at the customer loads.

In addition, there should be *no over-loading* of the power system equipments including transmission lines and transformers.

Furthermore, there should be no bus voltage either above or below specified values of the *bus voltage operating limits*.

Power Flow Analysis



Consider the above circuit, if all the components and loads are expressed in terms of constant power loads (i.e., in power systems, powers are known rather than currents), then the equation to be solved are given by

$$\left.\begin{aligned}[\mathbf{Y}][\mathbf{V}] &= [\mathbf{I}] \\ [\mathbf{I}] &= \left[\frac{\mathbf{S}^*}{\mathbf{V}^*}\right]\end{aligned}\right\}$$

It is clear that the set of equations are nonlinear and the solution (*bus voltages*) can only obtained by iterative techniques

Power Flow Analysis

It is the solution for the *static operating condition* of a power system.

The *node voltage method* is commonly used for the power system analysis. The formulation of the network equations results in *complex linear equations* in terms of node currents.

In power systems, *powers are known rather than currents*. Thus, resulting equations in terms of power become *non-linear* and must be solved by *iterative techniques*.

These *non-linear equations* are known as *power flow equations* or *load flow equations*.

The *power flow programs* compute the *voltage magnitude* and *phase angle* at each bus bar in the system *under steady-state operation condition*.

These programs use the bus-voltage data to compute the *power flow in the network* and the *power losses* for all equipment and transmission lines.

What are the power flow equations?

What do you expect to get by solving the power flow equations?

How do we benefit from the solution of the power flow equations?

Bus Admittance Matrix or Y_{bus}

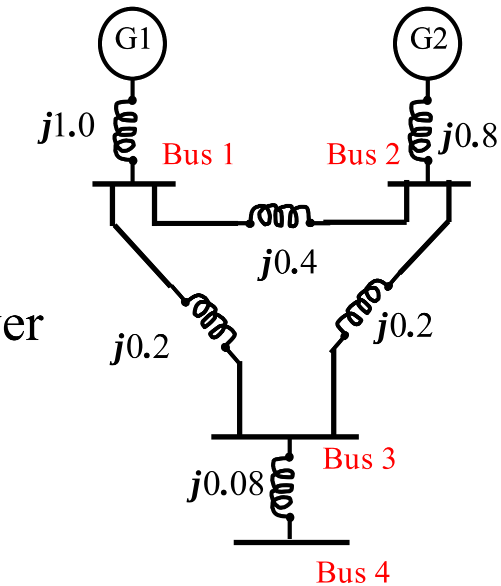
- First step in solving the power flow is to formulate the bus admittance matrix, often call the $\mathbf{Y}_{bus}$.
- The $\mathbf{Y}_{bus}$ gives the relationships between all the bus current injections, $\mathbf{I}$, and all the bus voltages, $\mathbf{V}$,

$$\mathbf{I} = \mathbf{Y}_{bus}\,\mathbf{V}$$

- The $\mathbf{Y}_{bus}$ is developed by applying KCL at each bus in the system to relate the bus current injections, the bus voltages, and the branch impedances and admittances

Y_{bus} Example

Formulate the *bus admittance matrix* for the network shown in the Figure. The *Impedance diagram* of the system is as indicated. Shunt elements are ignored.



Impedance diagram

Solution:

The *node voltage method* is commonly used for the power system analysis. Where,

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

Or

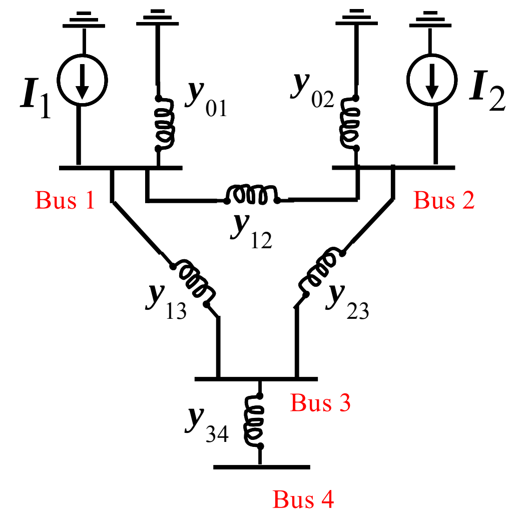
$$I_{bus} = [Y_{bus}]\, V_{bus}$$

The system can be represented in terms of its admittance elements as shown, where:

$$y_{ij} = \frac{1}{Z_{ij}}$$

$$y_{01} = \frac{1}{j1.0} = -j1.0 \qquad y_{12} = -j2.5$$

$$y_{02} = \frac{1}{j0.8} = -j1.25 \qquad y_{13} = y_{23} = -j5.0$$



Admittance diagram

Applying *KCL* at each node (bus), then

$$I_1 = (y_{01} + y_{12} + y_{13})V_1 - y_{12}V_2 - y_{13}V_3$$

$$I_2 = (y_{02} + y_{12} + y_{23})V_2 - y_{12}V_1 - y_{23}V_3$$

$$0 = (y_{31} + y_{32} + y_{34})V_3 - y_{31}V_1 - y_{32}V_2 - y_{34}V_4$$

$$0 = y_{34}V_4 - y_{34}V_3$$

defined:

$$Y_{11} = y_{01} + y_{12} + y_{13}$$

$$Y_{22} = y_{02} + y_{12} + y_{23}$$

$$Y_{33} = y_{31} + y_{32} + y_{34}$$

$$Y_{44} = y_{34}$$

$$Y_{42} = Y_{24} = 0$$

and

$$Y_{12} = Y_{21} = -y_{12}$$

$$Y_{13} = Y_{31} = -y_{13}$$

$$Y_{23} = Y_{32} = -y_{23}$$

$$Y_{34} = Y_{43} = -y_{34}$$

$$Y_{14} = Y_{41} = 0$$

Then, the Node Voltage Equation is:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

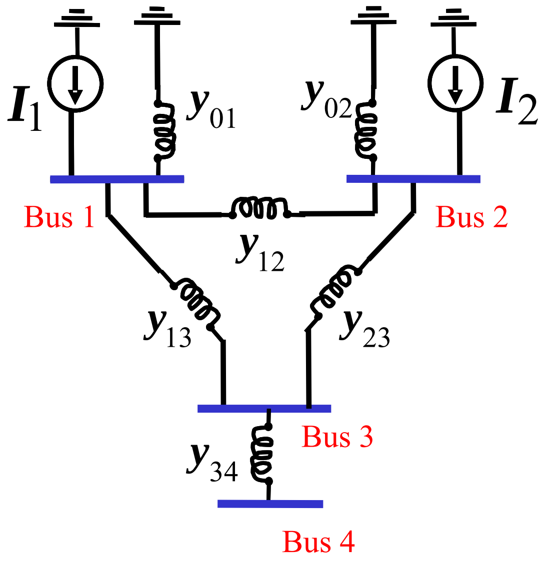
$$I_{bus} = [Y_{bus}]\, V_{bus} \qquad \text{Or} \qquad V_{bus} = [Y_{bus}^{-1}]\; I_{bus} = [Z_{bus}]\; I_{bus}$$

Substituting the values, then the bus admittance matrix of the network is:

$$Y_{bus} = \begin{bmatrix} -j8.5 & j2.5 & j5.0 & 0 \\ j2.5 & -j8.75 & j5.0 & 0 \\ j5.0 & j5.0 & -j22.5 & j12.5 \\ 0 & 0 & j12.5 & -j12.5 \end{bmatrix}$$

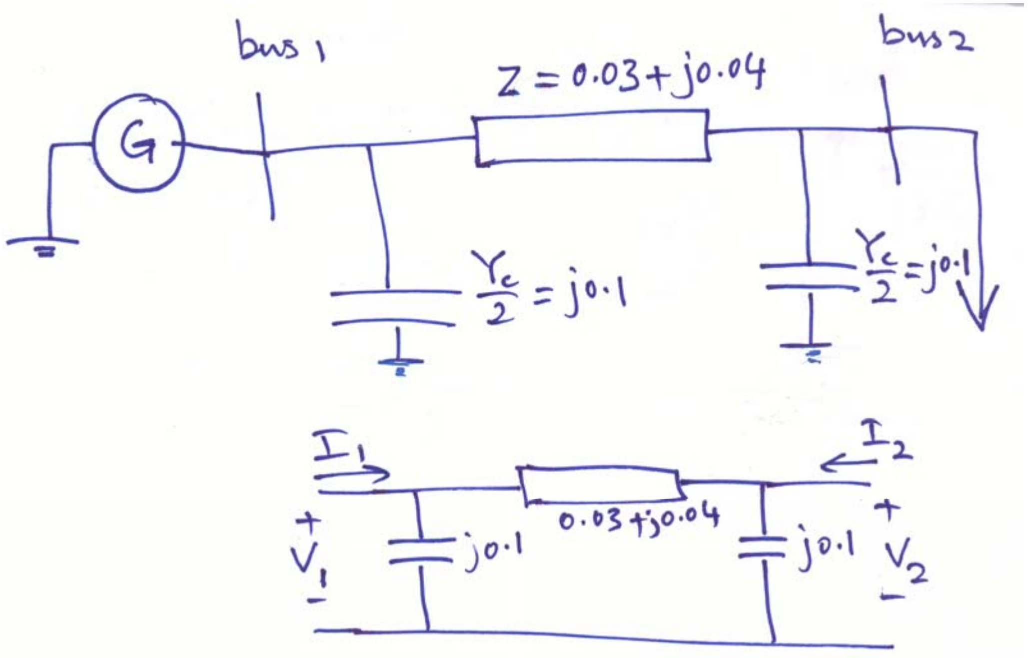
Y_{bus} General Form

- The diagonal terms, Y_{ii}, are the self admittance terms, equal to the sum of the admittances of all devices incident to bus i.
- The off-diagonal terms, Y_{ij}, are equal to the negative of the sum of the admittances joining the two buses.
- With large systems Y_{bus} is a sparse matrix (that is, most entries are zero)
- Shunt terms, such as with the π-line model, only affect the diagonal terms.



$$\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} = \begin{bmatrix} (y_{01}+y_{12}+y_{13}) & (-y_{12}) & (-y_{13}) & (0) \\ (-y_{21}) & (y_{02}+y_{21}+y_{23}) & (-y_{23}) & (0) \\ (-y_{31}) & (-y_{32}) & (y_{31}+y_{32}+y_{43}) & (-y_{34}) \\ (0) & (0) & (-y_{43}) & (y_{43}) \end{bmatrix}$$

Two Bus System Example



$$I_1 = \frac{(V_1 - V_2)}{Z} + V_1 \frac{Y_c}{2} \qquad \frac{1}{0.03 + j0.04} = 12 - j16$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 12 - j15.9 & -12 + j16 \\ -12 + j16 & 12 - j15.9 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Using the Y_{bus}

If the voltages are known then we can solve for the current injections:

$$\mathbf{Y}_{bus}\mathbf{V} = \mathbf{I}$$

If the current injections are known then we can solve for the voltages:

$$\mathbf{Y}_{bus}^{-1}\mathbf{I} = \mathbf{V} = \mathbf{Z}_{bus}\mathbf{I}$$

where $\mathbf{Z}_{\text{bus}}$ is the bus impedance matrix

Solving for Bus Currents

For example, in previous case assume

$$\mathbf{V} = \begin{bmatrix} 1.0 \\ 0.8 - j0.2 \end{bmatrix}$$

Then

$$\begin{bmatrix} 12 - j15.9 & -12 + j16 \\ -12 + j16 & 12 - j15.9 \end{bmatrix} \begin{bmatrix} 1.0 \\ 0.8 - j0.2 \end{bmatrix} = \begin{bmatrix} 5.60 - j0.70 \\ -5.58 + j0.88 \end{bmatrix}$$

Therefore the power injected at bus 1 is

$$\mathrm{S}_1 = V_1 I_1^* = 1.0 \times (5.60 + j0.70) = 5.60 + j0.70$$

$$S_2 = V_2 I_2^* = (0.8 - j0.2) \times (-5.58 - j0.88) = -4.64 + j0.41$$

Solving for Bus Voltages

For example, in previous case assume

$$\mathbf{I} = \begin{bmatrix} 5.0 \\ -4.8 \end{bmatrix}$$

Then

$$\begin{bmatrix} 12 - j15.9 & -12 + j16 \\ -12 + j16 & 12 - j15.9 \end{bmatrix}^{-1} \begin{bmatrix} 5.0 \\ -4.8 \end{bmatrix} = \begin{bmatrix} 0.0738 - j0.902 \\ -0.0738 - j1.098 \end{bmatrix}$$

Therefore the power injected is

$$\mathrm{S}_1 = V_1 I_1^* = (0.0738 - j0.902) \times 5 = 0.37 - j4.51$$

$$S_2 = V_2 I_2^* = (-0.0738 - j1.098) \times (-4.8) = 0.35 + j5.27$$

Power Flow Analysis

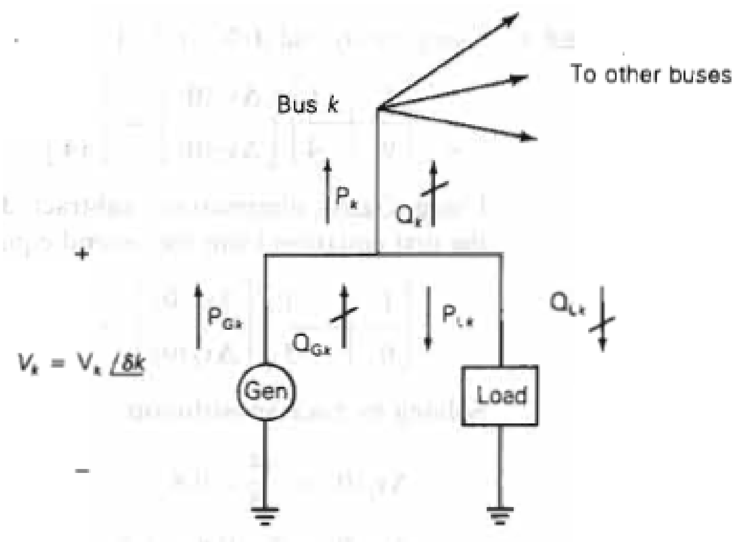
- When analyzing power systems we know neither the complex bus voltages nor the complex current injections
- Rather, we know the complex power being consumed by the load, and the power being injected by the generators plus their voltage magnitudes
- Therefore we can not directly use the Y_{bus} equations, but rather must use the power balance equations

$$\left.\begin{aligned} [\mathbf{Y}][\mathbf{V}] &= [\mathbf{I}] \\ [\mathbf{I}] &= \left[\frac{\mathbf{S}^*}{\mathbf{V}^*}\right] \end{aligned}\right\}$$

Power Flow Analysis , cont'd

Definitions:

The starting point is the single line diagram from which the input data can be obtained. The input data: Bus data, transmission line data and transformer data



As shown, 4 variables are associated with each bus *k*: the voltage magnitude V_k, phase angle δ_k , net power P_k and reactive power Q_k .

At each bus, two of these variables are specified as input data and the other two are unknowns to computed by the power flow analysis

Power Balance Equations

From KCL we know at each bus i in an n bus system the current injection, I_i, must be equal to the current that flows into the network

$$I_i = I_{Gi} - I_{Di} = \sum_{k=1}^{n} I_{ik}$$

Since $\mathbf{I} = \mathbf{Y}_{\text{bus}}\mathbf{V}$ we also know

$$I_i = I_{Gi} - I_{Di} = \sum_{k=1}^{n} Y_{ik} V_k$$

The network power injection is then $\text{S}_\text{i} = V_i I_i^*$

Power Balance Equations, cont'd

$$S_i = V_i I_i^* = V_i \left(\sum_{k=1}^{n} Y_{ik} V_k \right)^* = V_i \sum_{k=1}^{n} Y_{ik}^* V_k^*$$

This is an equation with complex numbers.

Sometimes we would like an equivalent set of real power equations. These can be derived by defining

$$Y_{ik} = G_{ik} + jB_{ik}$$

$$V_i = |V_i| e^{j\theta_i} = |V_i| \angle \theta_i$$

$$\theta_{ik} = \theta_i - \theta_k$$

Recall $e^{j\theta} = \cos\theta + j\sin\theta$

Real Power Balance Equations

$$\mathrm{S_i} \;=\; P_i + jQ_i = V_i \sum_{k=1}^{n} Y_{ik}^{*} V_k^{*} = \sum_{k=1}^{n} |V_i||V_k|\, e^{j\theta_{ik}} (G_{ik} - jB_{ik})$$

$$= \sum_{k=1}^{n} |V_i||V_k| (\cos\theta_{ik} + j\sin\theta_{ik})(G_{ik} - jB_{ik})$$

Resolving into the real and imaginary parts

$$\mathrm{P_i} \;=\; \sum_{k=1}^{n} |V_i||V_k| (G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}) = P_{Gi} - P_{Di}$$

$$\mathrm{Q_i} \;=\; \sum_{k=1}^{n} |V_i||V_k| (G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}) = Q_{Gi} - Q_{Di}$$

Power Flow Requires Iterative Solution

In the power flow we assume we know S_i and the $\mathbf{Y}_{bus}$. We would like to solve for the V's. The problem is the below equation has no closed form solution:

$$\mathrm{S_i} = V_i I_i^* = V_i \left(\sum_{k=1}^{n} Y_{ik} V_k \right)^* = V_i \sum_{k=1}^{n} Y_{ik}^* V_k^*$$

Rather, we must pursue an iterative approach.

Gauss Iteration

There are a number of different iterative methods we can use. We'll consider two: Gauss and Newton.

With the Gauss method we need to rewrite our equation in an implicit form: $x = h(x)$

To iterate we first make an initial guess of x, $x^{(0)}$, and then iteratively solve $x^{(v+1)} = h(x^{(v)})$ until we find a "fixed point", $\hat{x}$, such that $\hat{x} = h(\hat{x})$.

Gauss Iteration Example

Example: Solve $x - \sqrt{x} - 1 = 0$

$$x^{(v+1)} = 1 + \sqrt{x^{(v)}}$$

Let k = 0 and arbitrarily guess $x^{(0)} = 1$ and solve

k	$x^{(v)}$	k	$x^{(v)}$
0	1	5	2.61185
1	2	6	2.61612
2	2.41421	7	2.61744
3	2.55538	8	2.61785
4	2.59805	9	2.61798

Stopping Criteria

A key problem to address is when to stop the iteration. With the Guass iteration we stop when

$$\left|\Delta x^{(v)}\right| < \varepsilon \qquad \text{with } \Delta x^{(v)} = x^{(v+1)} - x^{(v)}$$

If x is a scalar this is clear, but if x is a vector we need to generalize the absolute value by using a norm

$$\left\|\Delta x^{(v)}\right\|_j < \varepsilon$$

Two common norms are the Euclidean & infinity

$$\left\|\Delta \mathbf{x}\right\|_2 = \sqrt{\sum_{i=1}^{n} \Delta x_i^2} \qquad \left\|\Delta \mathbf{x}\right\|_\infty = \max_i \left|\Delta \mathrm{x}_\mathrm{i}\right|$$

Gauss Power Flow

We first need to put the equation in the correct form

$$\mathrm{S}_{\mathrm{i}} = V_i I_i^* = V_i \left(\sum_{k=1}^{n} Y_{ik} V_k \right)^* = V_i \sum_{k=1}^{n} Y_{ik}^* V_k^*$$

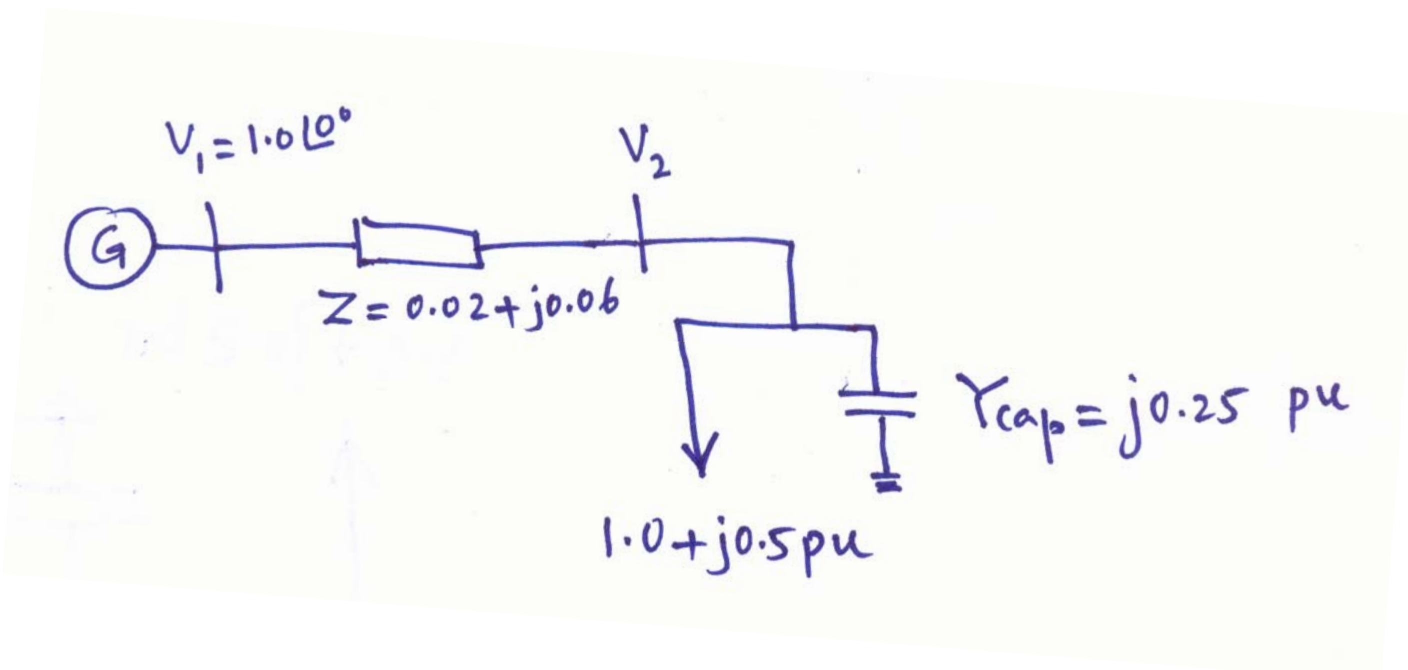
$$\mathrm{S}_i^* = V_i^* I_i = V_i^* \sum_{k=1}^{n} Y_{ik} V_k = V_i^* \sum_{k=1}^{n} Y_{ik} V_k$$

$$\frac{\mathrm{S}_i^*}{V_i^*} = \sum_{k=1}^{n} Y_{ik} V_k = Y_{ii} V_i + \sum_{k=1, k \neq i}^{n} Y_{ik} V_k$$

$$V_i = \frac{1}{Y_{ii}} \left(\frac{\mathrm{S}_i^*}{V_i^*} - \sum_{k=1, k \neq i}^{n} Y_{ik} V_k \right)$$

Gauss Two Bus Power Flow Example

A 100 MW, 50 Mvar load is connected to a generator through a line with z = 0.02 + j0.06 p.u. and line charging of 5 Mvar on each end (100 MVA base). Also, there is a 25 Mvar capacitor at bus 2. If the generator voltage is 1.0 p.u., what is V_2?



Gauss Two Bus Example, cont'd

The unknown is the complex load voltage, V_2.

To determine V_2 we need to know the $\mathbf{Y}_{\text{bus}}$.

$$\frac{1}{0.02 + j0.06} = \ 5 - j15$$

$$\text{Hence } \mathbf{Y}_{\text{bus}} = \begin{bmatrix} 5 - j14.95 & -5 + j15 \\ -5 + j15 & 5 - j14.70 \end{bmatrix}$$

$$(\text{Note } B_{22} = \ - j15 \ + j0.05 + j0.25)$$

Gauss Two Bus Example, cont'd

$$V_2 = \frac{1}{Y_{22}}\left(\frac{S_2^*}{V_2^*} - \sum_{k=1,k\neq i}^{n} Y_{ik}V_k\right)$$

$$V_2 = \frac{1}{5 - j14.70}\left(\frac{-1+j0.5}{V_2^*} - (-5 + j15)(1.0\angle 0)\right)$$

Guess $V_2^{(0)} = 1.0\angle 0$ (this is known as a flat start)

v	$V_2^{(v)}$	v	$V_2^{(v)}$
0	$1.000 + j0.000$	3	$0.9622 - j0.0556$
1	$0.9671 - j0.0568$	4	$0.9622 - j0.0556$
2	$0.9624 - j0.0553$		

Gauss Two Bus Example, cont'd

$$V_2 = 0.9622 - j0.0556 = 0.9638\angle -3.3°$$

Once the voltages are known all other values can be determined, such as the generator powers and the line flows

$$S_1^* = V_1^*(Y_{11}V_1 + Y_{12}V_2) = 1.023 - j0.239$$

In actual units $P_1 = 102.3$ MW, $Q_1 = 23.9$ Mvar

The capacitor is supplying $|V_2|^2\, 25 = 23.2$ Mvar

Slack Bus

- In previous example we specified S_2 and V_1 and then solved for S_1 and V_2.
- We can not arbitrarily specify S at all buses because total generation must equal total load + total losses
- We also need an angle reference bus.
- To solve these problems we define one bus as the "slack" bus. This bus has a fixed voltage magnitude and angle, and a varying real/reactive power injection.

Gauss with Many Bus Systems

With multiple bus systems we could calculate new V_i's as follows:

$$V_i^{(v+1)} = \frac{1}{Y_{ii}}\left(\frac{S_i^*}{V_i^{(v)*}} - \sum_{k=1,k\neq i}^{n} Y_{ik}V_k^{(v)}\right)$$

$$= h_i(V_1^{(v)}, V_2^{(v)}, \ldots, V_n^{(v)})$$

But after we've determined $V_i^{(v+1)}$ we have a better estimate of its voltage , so it makes sense to use this new value. This approach is known as the Gauss-Seidel iteration.

Gauss-Seidel Iteration

Immediately use the new voltage estimates:

$$V_2^{(v+1)} = h_2(V_1, V_2^{(v)}, V_3^{(v)}, \ldots, V_n^{(v)})$$

$$V_3^{(v+1)} = h_2(V_1, V_2^{(v+1)}, V_3^{(v)}, \ldots, V_n^{(v)})$$

$$V_4^{(v+1)} = h_2(V_1, V_2^{(v+1)}, V_3^{(v+1)}, V_4^{(v)} \ldots, V_n^{(v)})$$

$$\vdots$$

$$V_n^{(v+1)} = h_2(V_1, V_2^{(v+1)}, V_3^{(v+1)}, V_4^{(v+1)} \ldots, V_n^{(v)})$$

The Gauss-Seidel works better than the Gauss, and is actually easier to implement. It is used instead of Gauss.

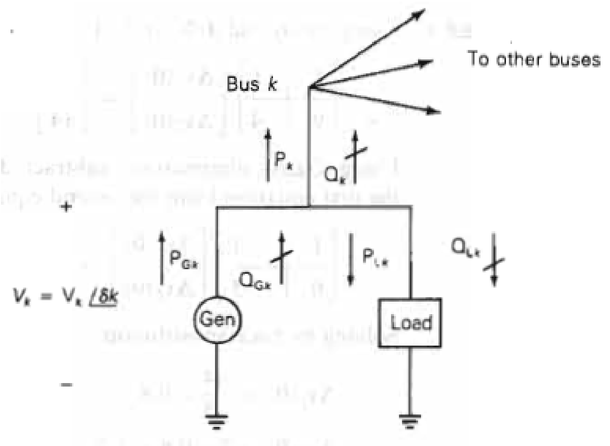
Three Types of Power Flow Buses

- There are three main types of power flow buses
  - Load (PQ) at which P/Q are fixed; iteration solves for voltage magnitude and angle.
  - Slack at which the voltage magnitude and angle are fixed; iteration solves for P/Q injections
  - Generator (PV) at which P and |V| are fixed; iteration solves for voltage angle and Q injection
    - special coding is needed to include PV buses in the Gauss-Seidel iteration

Three Types of Power Flow Buses

Each bus *i* can be categorized into one of the following:

1. Load Bus: Input data: P_k and Q_k
   Output (solution): V_k and δ_k
   Load Bus with no generation: $P_k = -P_{Lk}$
   $Q_k = -Q_{Lk}$ (inductive) ; $Q_k = +Q_{Lk}$ (capacitive)

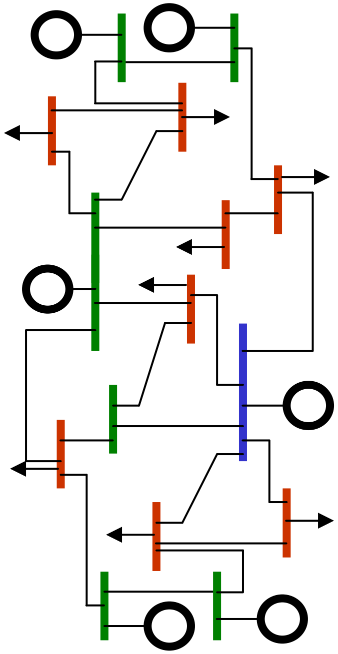


2. Swing (Reference) Bus: Only one swing bus (bus #1)
   input data: $V_1 \angle \delta_1 = 1.0 \angle 0^o$
   Output (solution): P_1 and Q_1

3. Voltage Controlled Bus: Generators, Switched shunt capacitors, static var systems
   Input data: P_k, V_k, Q_{kGmax} and Q_{kGmin}
   Output (solution): Q_k and δ_k

Inclusion of PV Buses in G-S

Assuming a power system has *n* buses, then; one bus will be considered as a slack bus and the other buses are load buses (PQ-buses) and voltage controlled buses (PV-buses). Let the system buses be numbered as:



$$i = 1 \qquad Salck\ bus$$

$$i = 2, 3, \ldots\ldots, m \qquad PV - buses$$

$$i = m+1, m+2, \ldots\ldots, n \qquad PQ - buses$$

For the voltage controlled buses,

$$P_i \ \ and\ |V_i|\ are\ known \quad \& \quad Q_i\ \ and\ \theta_i\ are\ unknown$$

$$|\boldsymbol{V_i}| = |\boldsymbol{V_i}|_{Specified}$$

$$\boldsymbol{Q_{i,min}} \langle\ \boldsymbol{Q_i}\ \langle\ \boldsymbol{Q_{i,max}}$$

The second requirement for the voltage controlled bus may be violated if the bus voltage becomes too high or too small. It is to be noted that we can control the bus voltage by controlling the bus reactive power.

Therefore, during any iteration, if *the PV-bus reactive power* violates its limits then set it according to the following rule.

$$Q_{i,min} < Q_i < Q_{i,max}$$

$$Q_i > Q_{i,max} \quad set \quad Q_i = Q_{i,max}$$

$$Q_i < Q_{i,min} \quad set \quad Q_i = Q_{i,min}$$

And *treat this bus as PQ-bus*.

NOTE

For PQ – bus

P_i *and* Q_i *are known*

& $|V_i|$ *and* θ_i *are unknown*

Load flow solution when PV buses are present

a. Find Q_i

To solve for V_i at PV buses, we must first make a guess of Q_i

$$S_i^* = V_i^* \sum_{k=1}^{n} Y_{ik} V_k = V_i^* (Y_{i1}\ V_1\ + Y_{i2}\ V_2\ + \ldots\ldots + Y_{ii}\ V_i\ + \ldots Y_{in}\ V_n\ ) = P_i\ + jQ_i$$

$$Q_i^{(v)} = -\operatorname{Im}\left[V_i^{(v)*} \sum_{k=1}^{n} Y_{ik} V_k^{(v)} \right]$$

in the iteration, we use $S_i^{(v)} = P_i\ + jQ_i^{(v)}$

b. Check Q_i^{v+1} *to see if it is within the limits*

$$Q_{i,\min}\ \langle\ Q_i\ \langle\ Q_{i,\max}$$

***Case 1: If the reactive power limits are not violated*, calculate** V_i^{v+1}

$$V_i^{(v+1)} = \frac{1}{Y_{ii}} \left[\frac{S_i^{(v)*}}{V_i^{(v)*}} - \sum_{k=i,k\neq 1}^{n} Y_{ik}\ V_k^{(v)} \right]$$

- *Use the most updated value of Q_i to calculate S_i.*
- *New Voltage magnitude and angle are obtained*

Use $|V_i|_{speci}$ *and* θ_i^{v+1} **For the *PV-bus voltage*.**

Reset the magnitude $$|V_i^{v+1}| = |V_i|_{Speci}$$

Voltage magnitude is known for PV bus, therefore the new calculated magnitude will not be used.

$$V_i^{v+1} = |V_i|_{Speci} \angle \theta_i^{v+1}$$

Only the calculated angle will be updated and used.

***Case 2: If the reactive power limits are violated*,**

$$Q_i^{v+1} \rangle\ Q_{i,\max} \quad set \quad Q_i^{v+1} = Q_{i,\max}$$

Or

$$Q_i^{v+1} \langle\ Q_{i,\min} \quad set \quad Q_i^{v+1} = Q_{i,\min}$$

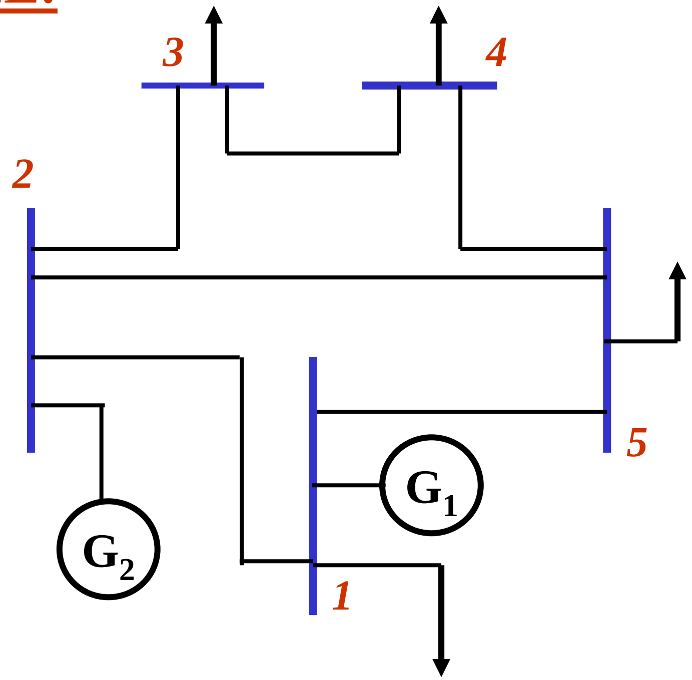
Consider this bus as a *PQ-Bus*, calculate bus voltage V_i^{v+1}

$$V_i^{(v+1)} = \frac{1}{Y_{ii}} \left[\frac{S_i^{(v)*}}{V_i^{(v)*}} - \sum_{k=i,k\neq 1}^{n} Y_{ik} V_k^{(v)} \right]$$

$$V_i^{v+1} = |V_i^{v+1}| \angle \theta_i^{v+1}$$

The PV-bus becomes PQ-bus and both Voltage magnitude and angle are calculated and used

EXAMPLE:



Line Data for the 5 buses Network

From Bus	To Bus	R	X
1	2	0.0500	0.1500
2	3	0.0500	0.1500
2	4	0.0500	0.1500
3	4	0.0500	0.1500
1	5	0.0500	0.1500
4	5	0.0500	0.1500

Each line has an impedance of 0.05+j0.15

The shunt admittance is neglected

Bus Data for the the 5 buses Network Before load flow solution

Bus No.	Bus code	Volt Mag.	Volt Angle	Load MW	Load MVAR	Gen. MW	Gen. MVAR	Q Min.	Q Max.	Inject MVAR
1	Slack	1.0200	0	100	50	?	?	0	0	0
2	PV	1.0200	?	0	0	200	?	20	60	0
3	PQ	?	?	50	20	0	0	0	0	0
4	PQ	?	?	50	20	0	0	0	0	0
5	PQ	?	?	50	20	0	0	0	0	0

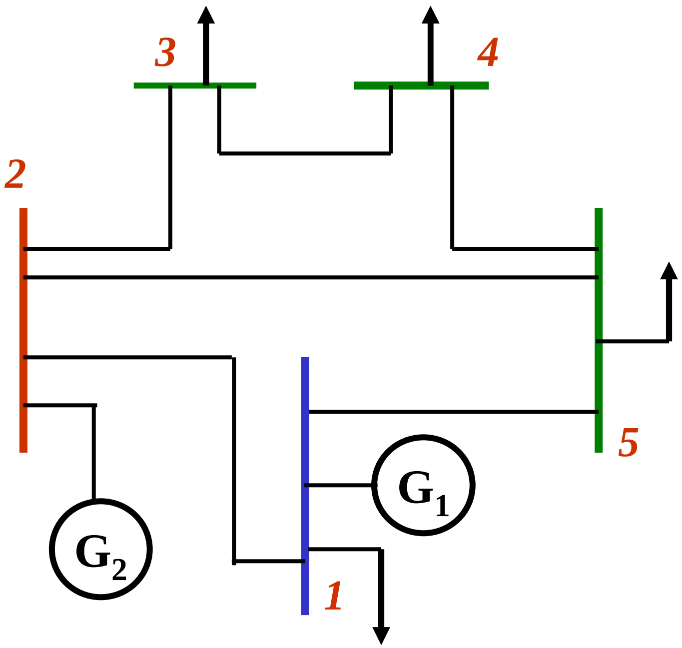
For the ‘5’ bus system

Construct the bus admittance matrix Y_{bus}

$$Find\ \ Q_2, \delta_2, V_3\ , V_4\ \ and\ V_5$$

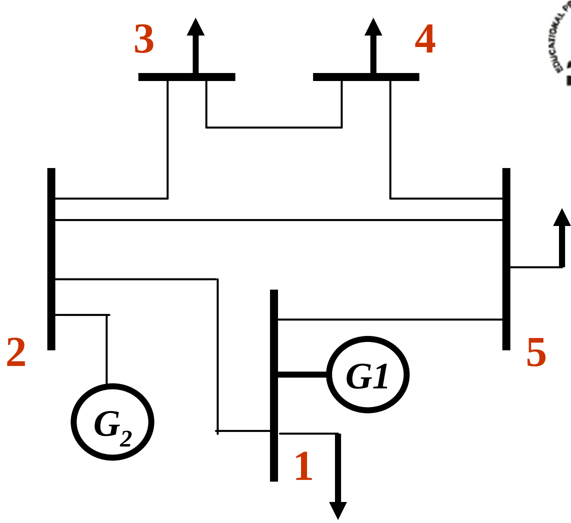
$$Q_{max} = 0.6\ pu$$

$$Q_{min} = 0.2\ pu$$



SOLUTION:

Y_{bus} Construction



$$y = \frac{1}{z} = \frac{1}{0.05 + j0.15} = 2 - j6$$

$$Y_{11} = y_{12} + y_{15} = 4 - j12$$

$$Y_{22} = y_{21} + y_{23} + y_{25} = 6 - j18$$

$$Y_{33} = y_{32} + y_{34} = 4 - j12$$

$$Y_{44} = y_{43} + y_{45} = 4 - j12$$

$$Y_{55} = y_{51} + y_{52} + y_{54} = 6 - j18$$

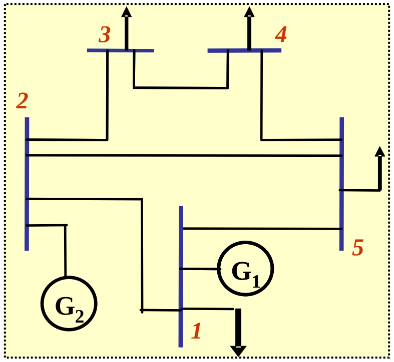
$$Y_{12} = -y_{12} = -2 + j6$$

$$Y_{15} = -y_{15} = -2 + j6$$

$$Y_{13} = Y_{14} = 0$$

$$Y_{bus} = \begin{bmatrix} 4.0 - J12.0 & -2.0 + J6.0 & 0 & 0 & -2.0 + J6.0 \\ -2.0 + J6.0 & 6.0 - J18.0 & -2.0 + J6.0 & 0 & -2.0 + J6.0 \\ 0 & -2.0 + J6.0 & 4.0 - J12.0 & -2.0 + J6.0 & 0 \\ 0 & 0 & -2.0 + J6.0 & 4.0 - J12.0 & -2.0 + J6.0 \\ -2.0 + J6.0 & -2.0 + J6.0 & 0 & -2.0 + J6.0 & 6.0 - J18.0 \end{bmatrix}$$

The net scheduled power injected at each bus is:



Bus No.	Bus code	Volt Mag.	Volt Angle	Load MW	Load MVAR	Gen. MW	Gen. MVAR	Q Min.	Q Max.	Inject MVAR
1	Slack	1.0200	0	100	50	?	?	0	0	0
2	PV	1.0200	?	0	0	200	?	20	60	0
3	PQ	?	?	50	20	0	0	0	0	0
4	PQ	?	?	50	20	0	0	0	0	0
5	PQ	?	?	50	20	0	0	0	0	0

$$S_{1,sch} = (P_{1,g} - P_{1,d}) + j(Q_{1,g} - Q_{1,d})$$

$$S_{1,sch} = (P_{1,g} - 1.0) + j(Q_{1,g} - 0.5)$$

$$S_{2,sch} = (P_{2,g} - P_{2,d}) + j(Q_{2,g} - Q_{2,d})$$

$$S_{2,sch} = (2.0 - 0) + j(Q_{2,g} - 0)$$

$$S_{3,sch} = (0 - 0.5) + j(0 - 0.2)$$

$$S_{3,sch} = -0.5 - j0.2$$

$$S_{4,sch} = -0.5 - j0.2$$

$$S_{5,sch} = -0.5 - j0.2$$

The known values are:

$$V_1 = 1.02\angle 0^o$$

$$|V_2|_{spec} = 1.02$$

The bus admittance matrix is

$$\begin{bmatrix} 4.0 - J12.0 & -2.0 + J6.0 & 0 & 0 & -2.0 + J6.0 \\ -2.0 + J6.0 & 6.0 - J18.0 & -2.0 + J6.0 & 0 & -2.0 + J6.0 \\ 0 & -2.0 + J6.0 & 4.0 - J12.0 & -2.0 + J6.0 & 0 \\ 0 & 0 & -2.0 + J6.0 & 4.0 - J12.0 & -2.0 + J6.0 \\ -2.0 + J6.0 & -2.0 + J6.0 & 0 & -2.0 + J6.0 & 6.0 - J18.0 \end{bmatrix}$$

$Q_{2,min} = 0.2$ **and** $Q_{2,max} = 0.6$

Using GS method, *select the initial values for the unknowns* as:

$V_3^o = V_4^o = V_5^o = 1\angle 0^o$ **and** $\delta_2^o = 0$

Start the first iteration

Bus 2 is PV Bus

Check Q_2 is within the limits $\quad Q_{2,min} \langle \; Q_2 \; \langle \; Q_{2,max}$

$$P_i - jQ_i = V_i^*(Y_{i1}V_1 + Y_{i2}V_2 + ...+ Y_{ii}V_i + ...Y_{in}V_n)$$

$$Q_2^1 = -Im\{V_2^*(Y_{21}V_1 + Y_{22}V_2^o + Y_{23}V_3^o + Y_{24}V_4^o + Y_{25}V_5^o)\}$$

$$Q_2^1 = 0.2448$$

$$Q_{2,min} \langle \; Q_2 \; \langle \; Q_{2,max} \qquad i.e.; \quad 0.20 \; \langle \; 0.2448 \; \langle \; 0.6$$

The reactive power limits are not violated,

Calculate:

$$V_2^1 = \frac{K_2}{(V_2^o)^*} - \left[L_{21} V_1 + L_{23} V_3^o + L_{24} V_4^o + L_{25} V_5^o \right]$$

The values for K_i and L_{ip} are computed once in the beginning and used in every iteration.

$$K_2 = \frac{P_2 - jQ_2}{Y_{22}} \qquad L_{21} = \frac{Y_{21}}{Y_{22}} \qquad L_{23} = \frac{Y_{23}}{Y_{22}} \qquad L_{24} = \frac{Y_{24}}{Y_{22}} \qquad L_{25} = \frac{Y_{25}}{Y_{22}}$$

$$S_{2,sch} = 2.0 + j0.2448$$

$K_2 = 0.0456 + j0.0959$ $\quad L_{21} = -0.3333$ $\quad L_{23} = -0.3333$ $\quad L_{24} = 0.0$ $\quad L_{25} = -0.3333$

$$V_2^1 = 1.0555\angle 5.1113^o$$

Reset the magnitude

$$|V_2^1| = |V_2|_{Speci} = 1.02$$

Therefore,

$$\delta_2^1 = 5.1113^o$$

$$V_2^1 = 1.02\angle 5.1113^o$$

Voltage magnitude is known and fixed for a PV bus, therefore the new calculated magnitude will not be used.

Bus 3 is PQ Bus

$$V_3^1 = \frac{K_3}{(V_3^o)^*} - \left[L_{31} V_1 + L_{32} V_2^1 + L_{34} V_4^o + L_{35} V_5^o\right]$$

$$K_3 = \frac{P_3 - jQ_3}{Y_{33}} \qquad L_{31} = \frac{Y_{31}}{Y_{33}} \qquad L_{32} = \frac{Y_{32}}{Y_{33}} \qquad L_{34} = \frac{Y_{34}}{Y_{33}} \qquad L_{35} = \frac{Y_{35}}{Y_{33}}$$

$K_3 =$ -0.0275 - j0.0325 $L_{31} =$ 0.0 $L_{32} =$ -0.5000 $L_{34} =$ -0.5000 $L_{35} =$ 0.0

$$V_3^1 = 0.9806\angle 0.7559^o$$

Bus 4 is PQ Bus

$$V_4^1 = \frac{K_4}{(V_4^o)^*} - \left[L_{41} V_1 + L_{42} V_2^1 + L_{43} V_3^1 + L_{45} V_5^o\right]$$

$$K_4 = \frac{P_4 - jQ_4}{Y_{44}} \qquad L_{41} = \frac{Y_{41}}{Y_{44}} \qquad L_{42} = \frac{Y_{42}}{Y_{44}} \qquad L_{43} = \frac{Y_{43}}{Y_{44}} \qquad L_{45} = \frac{Y_{45}}{Y_{44}}$$

$K_4 =$ -0.0275 - j0.0325 $L_{41} =$ 0.0 $L_{42} =$ 0.0 $L_{43} =$ -0.5000 $L_{45} =$ -0.5000

$$V_4^1 = 0.9631\angle -1.5489^o$$

Bus 5 is PQ Bus

$$V_5^1 = \frac{K_5}{(V_5^o)^*} - \left[L_{51} V_1 + L_{52} V_2^1 + L_{53} V_3^3 + L_{54} V_4^1\right]$$

$K_5 = -0.0183 - 0.0217i$ $L_{51} = -0.3333$ $L_{52} = -0.3333$ $L_{53} = 0.0$ $L_{54} = -0.3333$

$$V_5^1 = 0.9812\angle -0.0031^o$$

Start the second iteration

Bus 2 is PV Bus

Check Q_2 is within the limits $0.2 \langle\ Q_2 \langle\ 0.6$

$$Q_2^2 = -Im\{V_2^{1*}(Y_{21} V_1 + Y_{22} V_2^1 + Y_{23} V_3^1 + Y_{24} V_4^1 + Y_{25} V_5^1)\}$$

$$Q_2^2 = 0.0290$$

The reactive power limits are violated

$Q_2 \langle\ Q_{i,min}$ set $Q_2 = Q_{i,min} = 0.2$

And treat this bus as PQ-bus $S_{2,sch} = 2.0 + j0.2$

Use the most updated value of Q_2 to calculate the constant K_2

All Buses 2, 3, 4 and 5 are PQ Buses. Find the bus voltages using GS method

Accelerated G-S Convergence

Previously in the Gauss-Seidel method we were calculating each value x as

$$x^{(v+1)} = h(x^{(v)})$$

To accelerate convergence we can rewrite this as

$$x^{(v+1)} = x^{(v)} + h(x^{(v)}) - x^{(v)}$$

Now introduce acceleration parameter α

$$x^{(v+1)} = x^{(v)} + \alpha(h(x^{(v)}) - x^{(v)})$$

With $\alpha = 1$ this is identical to standard gauss-seidel.

Larger values of α may result in faster convergence.

Accelerated Convergence, cont'd

Consider the previous example: $x - \sqrt{x} - 1 = 0$

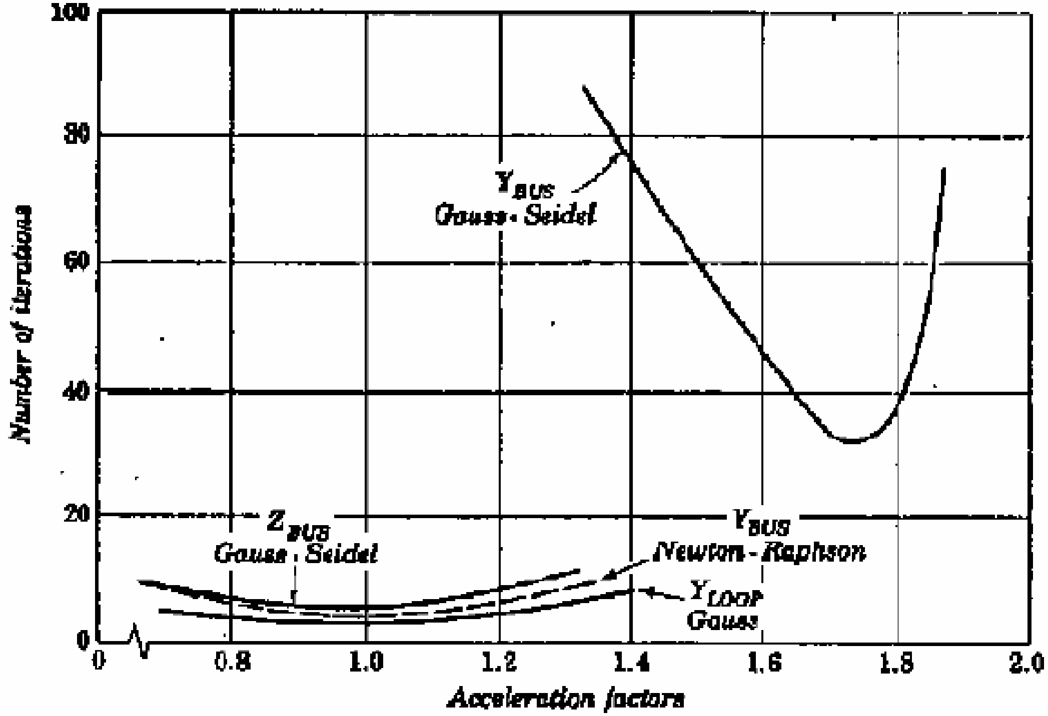
$$x^{(v+1)} = x^{(v)} + \alpha(1 + \sqrt{x^{(v)}} - x^{(v)})$$

Comparison of results with different values of α

k	$\alpha = 1$	$\alpha = 1.2$	$\alpha = 1.5$	$\alpha = 2$
0	1	1	1	1
1	2	2.20	2.5	3
2	2.4142	2.5399	2.6217	2.464
3	2.5554	2.6045	2.6179	2.675
4	2.5981	2.6157	2.6180	2.596
5	2.6118	2.6176	2.6180	2.626

Accelerated Convergence, cont'd

The Effect of Acceleration Factor



Adequate Values of the Acceleration Factor:

$$1.5 \leq \alpha \leq 1.7$$

Gauss-Seidel Advantages

- Each iteration is relatively fast (computational order is proportional to number of branches + number of buses in the system
- Relatively easy to program

Gauss-Seidel Disadvantages

- Tends to converge relatively slowly, although this can be improved with acceleration
- Has tendency to miss solutions, particularly on large systems
- Tends to diverge on cases with negative branch reactances (common with compensated lines)
- Need to program using complex numbers