ELE B7 Power Systems Engineering

Power System Components' Modeling

Power System Components

- The main components of a power system are generators, transformers and transmission lines.
- In this lecture, we shall discuss the models of these components that will be used subsequently in power system studies.

Section I: Transmission Lines

Transmission lines- CONSTRUCTION

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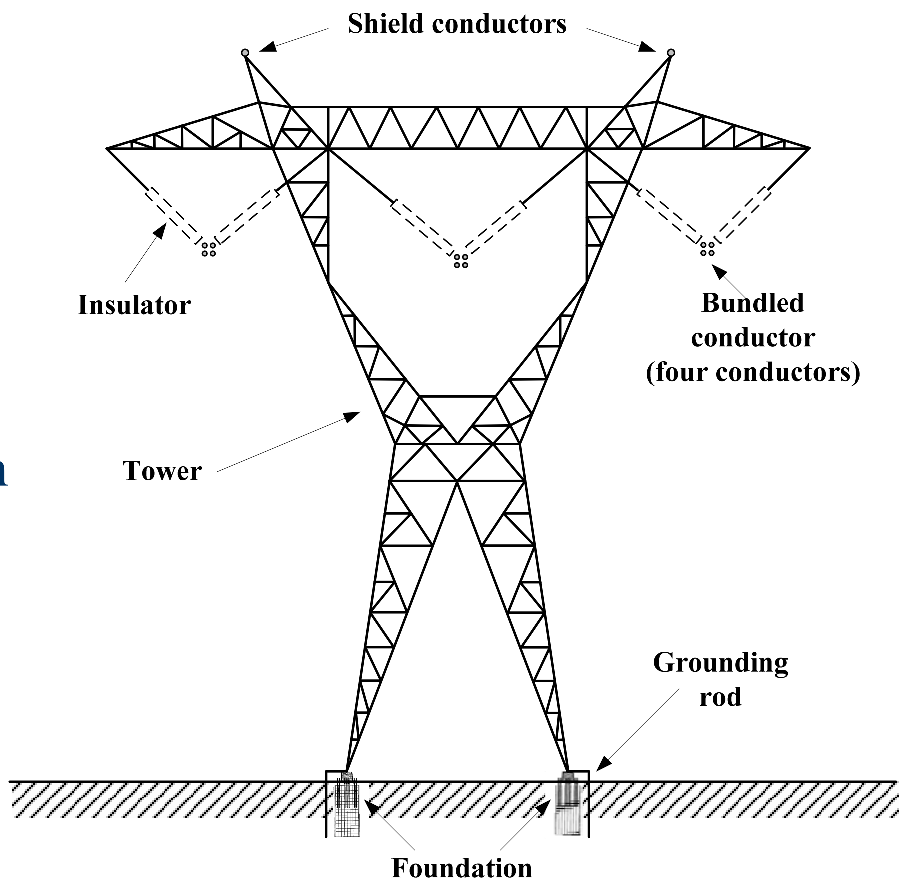
- **Three-phase conductors**, which carry the electric current;
- **Insulators**, which support and electrically isolate the conductors;
- **Tower**, which holds the insulators and conductors;
- **Foundation and grounding**; and
- **Optional shield conductors**, which protect against lightning



Transmission lines- VOLTAGE LEVELS

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- Overhead Transmission lines (OTL) are operating at different voltage levels:
  - Distribution: 6.3, 11, 13.8, 22, 33, 69 kV

    Supplies residential and commercial customers
  - Subtransmission: 69, 110, 132 kV

    Interconnection between substations and large industrial customers
  - Transmission: 132, 220, 400 kV

    Interconnection between substations, power plants
  - EHV transmission: 500, 735, 765 kV

    Interconnection between systems
  - UHV (experimental): 1200, 1500 kV

Transco 220 KV

Transmission Line- TYPES OF CONDUCTORS

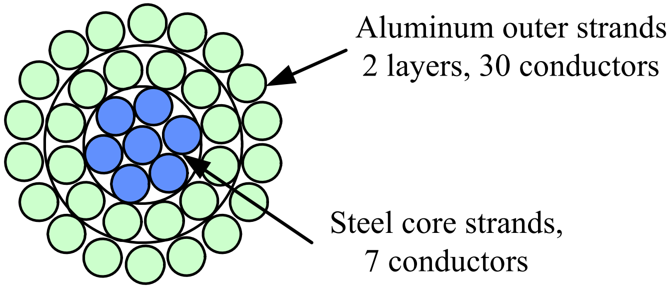
- Transmission line conductors can be made of copper or aluminum
- However, aluminum conductors have completely replaced copper for overhead lines because of the much lower cost and lighter weight of an aluminum conductor compared with a copper conductor of the same resistance.
- Symbols identifying different types of aluminum conductors are as follows:

AAC all-aluminum conductors

AAAC all-aluminum-alloy conductors

ACSR aluminum conductor, steel-reinforced

ACAR aluminum conductor, alloy-reinforced



ACSR

Transmission Line- PARAMETERS

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- A transmission line has four parameters :
  - Resistance,
  - Inductance,
  - Capacitance, and
  - Conductance.
- The conductance, exists between conductors or between conductors and the ground, accounts for the leakage current at the insulators of overhead lines and through the insulation of cables.
- Since leakage at insulators of overhead lines is negligible, the conductance between conductors of an overhead line is usually neglected.

Transmission Line Parameters-RESISTANCE

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- It is very well known that the dc resistance of a wire is given by:

$$R_{dc} = \frac{\rho l}{A}\ \Omega$$

where ρ is the resistivity of the wire in **Ω** - m, l is the length in meter and A is the cross sectional area in m^2

- The line resistance increases by:
  - Stranding
  - Temperature
  - Skin effect
- AC resistance higher than DC
- Accurate value from Tables

Transmission Line Parameters-RESISTANCE

Example:

A three-phase overhead transmission line is designed to deliver 190.5MVA at 220kV over a distance of 63km, such that the total transmission line loss is not to exceed 2.5% of the rated line MVA. Given the resistivity of the conductor material to be $2.84*10^{-8}$ ohm-meter. Determine the required conductor diameter and the conductor size. Neglecting power loss due to corona and insulator leakage and other reactive loss.

Solution:

The total transmission line loss is:

$$P_{Loss} = 190.5 * 2.5\% = 4.7625\ MW$$

$$I_L = \frac{190.5(MVA)}{\sqrt{3}(220kV)} = 500A$$

$$P_{Loss} = 3 * I_L^{\ 2} * R$$

$$R = \frac{4.7625MW}{3(500)^2} = 6.35\Omega$$

$$A = \frac{\rho * l}{R} = \frac{2.84\times10^{-8} * 63\times10^{3}}{6.35}$$

$$A = 2.81764\times10^{-4}\ m^2$$

$$d = 1.894\,cm = 0.7456in = 556{,}000cmil$$

$$1\ in = 1000\ mils$$
$$1\ cmils = sq\ mil$$

Transmission Line Parameters-INDUCTANCE

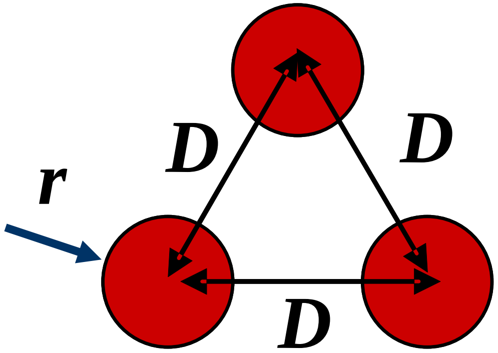
- The inductance per phase of 3-phase equilateral spaced solid conductors is given by:

$$L = 2\times10^{-7}(\ln\frac{D}{r'})\ \ H/m$$

where,

D is the distances between the conductor

$r' = r\ e^{-1/4}$, and r is the radius of the conductor



- For inequilaterally spaced conductors, the inductance per phase is:

$$L = 2\times10^{-7}\ln\frac{D_{eq}}{r'}\quad H/m$$

where,

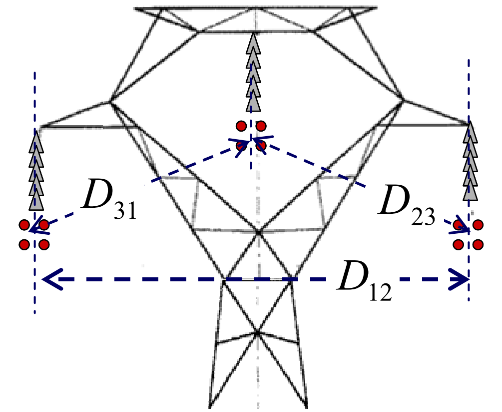
$$D_{eq} = \sqrt[3]{D_{12}D_{23}D_{31}}$$

Transmission Line Parameters-INDUCTANCE

- The inductance per phase of 3-phase stranded conductors is:

$$L = 2\times10^{-7}\ln\frac{GMD}{GMR}\quad H/m$$

where,
- *GMD* is the Geometrical Mean Distance
- *GMR* is the Geometrical Mean Radius

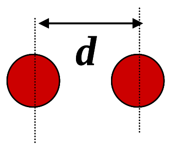


- The inductance per phase of 3-phase bundled conductors is:

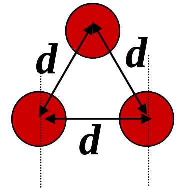
$$L = 2\times10^{-7}\ln\frac{GMD}{GMR}\quad H/m$$

$$GMD = D_{eq} = \sqrt[3]{D_{12}D_{23}D_{31}}$$

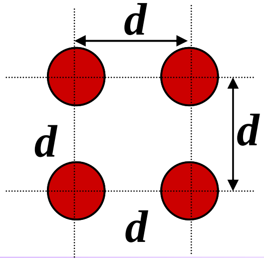
$$GMR = \sqrt[2]{D_s \times d}$$



$$GMR = \sqrt[3]{D_s \times d^2}$$



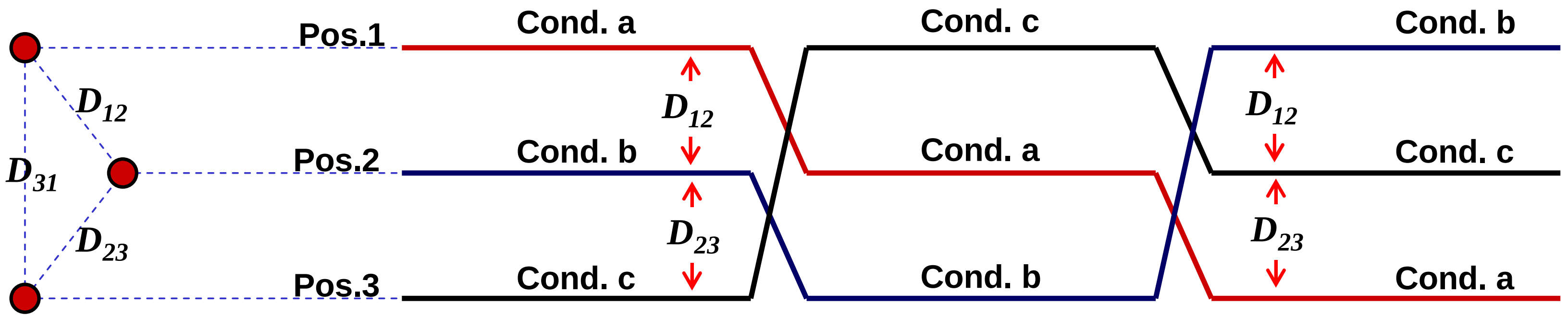
$$GMR = 1.091\sqrt[4]{D_s \times d^3}$$



- Transposition:

When the conductors of a three-phase line are not spaced equilaterally, the flux linkages and the inductance of each phase are not the same. A different inductance in each phase results in unbalance circuit.

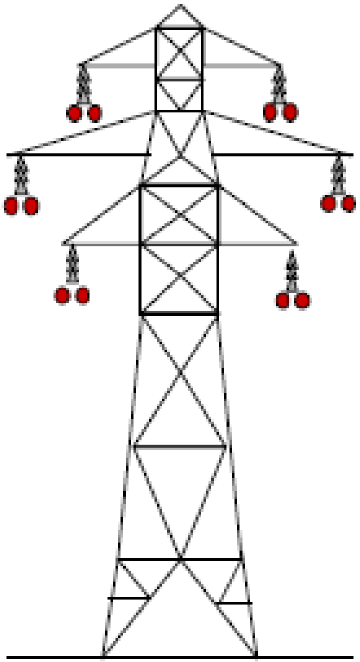
Balance of the three phases can be restored by exchanging the positions of the conductors at regular intervals along the line as shown below



Such an exchange of the conductor positions is called *transposition*.

Transmission Line Parameters-INDUCTANCE

Example 1: **Find inductance of the transmission line**



The length of the conductor in kilometer is l	150 km
Diameter of the conductor	50 mm
Spacing between bundled sub-conductors	45 cm
Distance between phase a and b	3 m
Distance between phase b and c	3 m
Distance between phase c and a	6 m

$$L = 2\times 10^{-7} \ln \frac{D_{eq}}{GMR} \quad H/m$$

$$D_{eq} = \sqrt[3]{D_{ab} D_{ac} D_{bc}}$$

$D_{eq} = 3.7798$ *meters*

$$r' = r\, e^{-1/4}$$

$r' = 0.0195$ *meters*

$$GMR = \sqrt[2]{r'd}$$

$GMR = 0.094$ *meters*

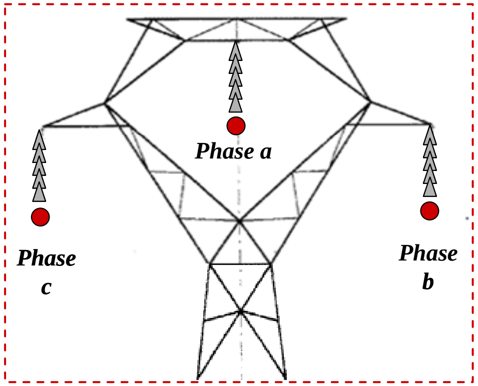
$\underline{L = 110.9\ mH}$ *(per phase for 150 km conductor)*

Transmission Line Parameters-CAPACITANCE

- Capacitance of a Three-Phase Transposed Line:

$$C_n = \frac{2\pi\varepsilon_o}{\ln(D_{eq}/r)} \quad F/m$$

$$GMD = D_{eq} = \sqrt[3]{D_{12}D_{23}D_{31}}$$



- Capacitance of Three-Phase Bundled Line:

$$C_n = \frac{2\pi\varepsilon_o}{\ln(D_{eq}/D_s^b)} \quad F/m$$

where,

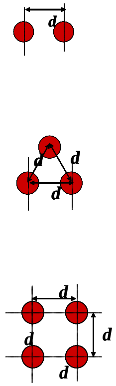
– $\varepsilon_0 = 8.85\times10^{-12}$

– r is the equivalent radius of the conductor

$$D_s^b = \sqrt[2]{r\times d}$$

$$D_s^b = \sqrt[3]{r\times d^2}$$

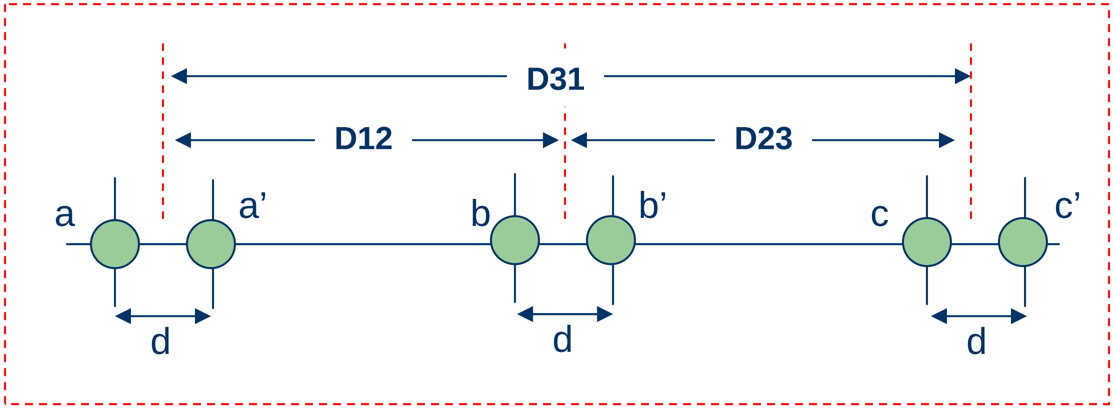
$$D_s^b = 1.091\sqrt[4]{r\times d^3}$$



Transmission Line Parameters-CAPACITANCE

EXAMPLE:

A 400kV, 60Hz, three phase with bundled conductor line, two sub-conductors per-phase as shown in the Figure. The center to center distance between adjacent phases is 12 m and distance between sub-conductors is 45 cm. The radius of each sub-conductor is 1.6 cm. Find the capacitance per phase per km.



SOLUTION:

$$D_s^b = \sqrt{r \times d} = \sqrt{1.6 \times 45} = 8.485cm = 0.08485m$$

$$D_{eq} = \sqrt[3]{D_{ab} D_{ac} D_{bc}} = \sqrt[3]{12 * 12 * 24} = 15.119m$$

$$C_n = \frac{2\pi\varepsilon_o}{\ln(D_{eq} / D_s^b)} = 0.0107 \mu F / km$$

NOTE

Use r′ for L

Use r for C

Section II: Transmission Line Performance

Transmission line representation

- Series impedance

$$z = r + j\,x \quad \Omega / mi \qquad\qquad Z = z\,\ell = R + j\,X \quad \Omega$$

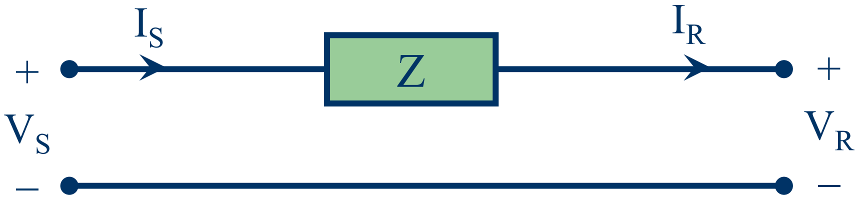
- Shunt admittance

$$y = g + j\,b \quad S / mi \qquad\qquad Y = y\,\ell = G + j\,B \quad S$$

- G is almost always ignored
- R is sometimes ignored for analysis purposes (loss-less line), but never in the real world!

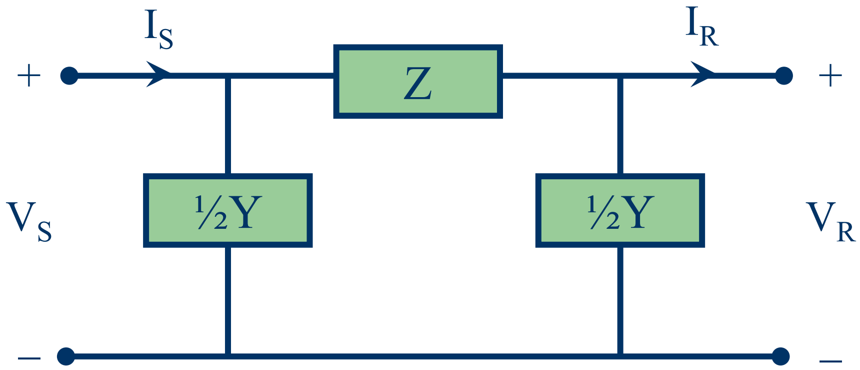
Transmission line models

Short line (up to 50 miles long)



Ignore shunt admittance

Medium-length line model (50-150 miles long)



Nominal π circuit

Two-Port Network- ABCD parameters

- ABCD parameters

$$V_S = AV_R + BI_R$$
$$I_S = CV_R + DI_R$$

or

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix} \quad (1)$$

- Applies to linear, passive, two-port networks
- A, B, C and D depend on transmission line parameters

ABCD Parameters – Example 1

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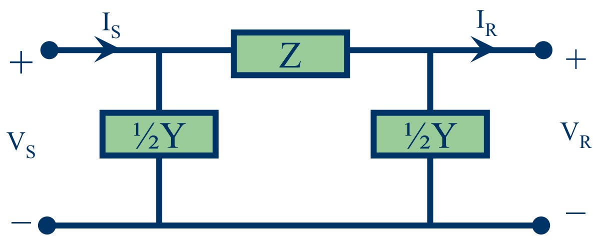
- Find the ABCD parameters for the nominal π circuit



KVL equation:

$$V_S = V_R + Z\left(I_R + \frac{Y\,V_R}{2}\right) \quad \Rightarrow \quad V_S = \left(1 + \frac{YZ}{2}\right)V_R + Z\,I_R \qquad (2)$$

KCL equation:

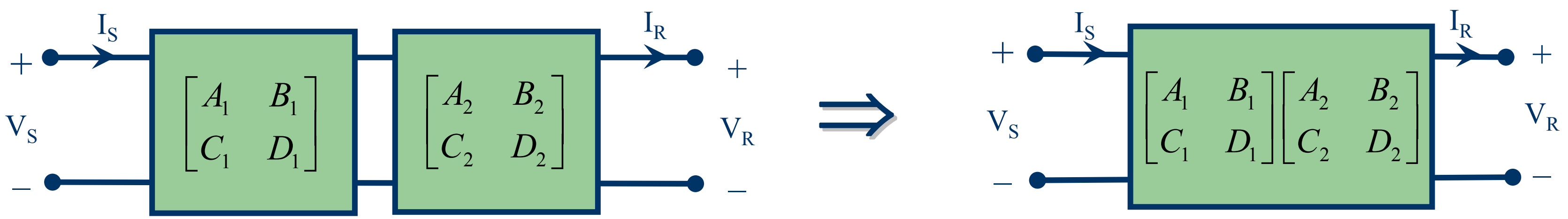
$$I_S = \frac{Y\,V_s}{2} + \frac{Y\,V_R}{2} + I_R \quad \Rightarrow \quad I_S = \left(Y + \frac{Y^2 Z}{4}\right)V_R + \left(1 + \frac{YZ}{2}\right)I_R \qquad (3)$$

By inspection,

$$A = D = 1 + \frac{YZ}{2}\ \text{pu}, \qquad B = Z\ \Omega, \qquad C = Y\left(1 + \frac{YZ}{4}\right)\ \text{S}$$

ABCD Parameters

- Series impedance Z: $A = D = 1, \quad B = Z \ \Omega, \quad C = 0$
- Shunt admittance Y: $A = D = 1, \quad B = 0, \quad C = Y \text{ S}$
- Why use ABCD?
  - Easier than circuit analysis for hand calculations
  - Easier to concatenate elements



ABCD Parameters – Example 2

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A 345 kV transmission line has z = 0.05946 + j0.5766 Ω/mi, y = 7.3874×10^{-3} S/mi and ℓ = 222 mi. Calculate:

a) ABCD parameters, assuming nominal π circuit model

$$Z = z\ell = 13.2 + j128\,\Omega, \quad Y = y\ell = j1.64 \times 10^{-3}\,\text{S}$$

$$A = D = 1 + \frac{YZ}{2} = 0.89511\angle 0.69°\ \text{pu}, \quad B = Z = 128.68\angle 84.11°\,\Omega$$

$$C = Y\left(1 + \frac{YZ}{4}\right) = 1.554 \times 10^{-3}\angle 90.33°\ \text{S}$$

b) Receiving-end no-load voltage

$$V_S = AV_R + BI_R \qquad I_R = 0$$

$$V_R = \frac{V_S}{A} = \frac{345/\sqrt{3}}{0.89511\angle 0.69°} = 222.5\angle -0.69°\ \text{kV}$$

$$\left|V_{RLL}\right| = 385.4\ \text{kV} \quad (12\%\ \text{above nominal})$$

The long transmission line

- Impedance and admittance parameters are distributed rather than lumped
- At 60 Hz, effects of distributed parameters are significant for long lines (>150 miles)
- Need a new model to accurately represent “long” transmission lines
  1. Derive exact model for a generic transmission line represented as a two-port network (ABCD matrix)
  2. Determine relationship to nominal π model

The long transmission line

- The exact network equations in ABCD format:

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} \cosh(\gamma \ell) & Z_c \sinh(\gamma \ell) \\ \dfrac{\sinh(\gamma \ell)}{Z_c} & \cosh(\gamma \ell) \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

where,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cosh(\gamma \ell) & Z_c \sinh(\gamma \ell) \\ \dfrac{\sinh(\gamma \ell)}{Z_c} & \cosh(\gamma \ell) \end{bmatrix}$$

$$\gamma = \sqrt{zy} \ \ \mathrm{m}^{-1} \quad \text{(propagation constant)}$$

$$Z_c = \sqrt{\frac{z}{y}} \ \ \Omega \quad \text{(characteristic impedance)}$$

The long transmission line

- Relationship to nominal π circuit model:

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} \cosh(\gamma\ell) & Z_c \sinh(\gamma\ell) \\ \dfrac{\sinh(\gamma\ell)}{Z_c} & \cosh(\gamma\ell) \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

Exact network equations in ABCD format

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1 + \dfrac{Y'Z'}{2} & Z' \\ Y'\left(1 + \dfrac{Y'Z'}{4}\right) & 1 + \dfrac{Y'Z'}{2} \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$$Z' = Z_c \sinh(\gamma\ell), \qquad \frac{Y'}{2} = \frac{\tanh(\gamma\ell/2)}{Z_c}$$

Equivalent π circuit equations
(Z' and Y' are the series impedance and shunt admittance of the equivalent π circuit that models the terminal behavior exactly)

- In terms of the "usual" Z = zℓ and Y = yℓ:

$$Z' = Z\,\frac{\sinh(\gamma\ell)}{\gamma\ell} \qquad \textit{and} \qquad \frac{Y'}{2} = \frac{Y}{2}\,\frac{\tanh(\gamma\ell/2)}{\gamma\ell/2}$$

The long transmission line – Example 3

- For the line in Example 2, calculate:

a. Propagation constant and characteristic impedance

$$\gamma = \sqrt{zy} = 2.069 \times 10^{-3} \angle 87.06^\circ \ m^{-1}, \qquad Z_c = \sqrt{\frac{z}{y}} = 280.11 \angle -2.94^\circ$$

b. Equivalent π circuit parameters

$$Z' = Z \frac{\sinh(\gamma \ell)}{\gamma \ell} = 1.2422 \angle 84.32^\circ \quad (4\% \text{ lower})$$

$$\frac{Y'}{2} = \frac{Y}{2} \frac{\tanh(\gamma \ell / 2)}{\gamma \ell / 2} = 0.8346 \times 10^{-3} \angle 89.9^\circ$$

$$Y' = 1.6693 \times 10^{-3} \angle 89.9^\circ \quad (2\% \text{ higher})$$

Loadability

- Power transfer capability of a transmission line may be limited by any one of the following:

1. Conductor temperature & sag requirements
2. Voltage profile
3. Stability considerations
4. Real power losses
5. Reservation requirements

Temperature / sag limitations

- Operate within conductor or insulator temperature rating
  - Heat gain (I^2R, other heat sources)
  - Heat dissipation (wind, conduit)
  - Summer vs. winter ratings; continuous vs. emergency ratings
- Meet minimum sag requirements
  - Heat causes conductors to stretch, which reduces ground clearance for overhead lines
- For most short transmission lines, temperature / sag limitations dictate transfer capability

Voltage profile

- Voltage regulation (typically, VR ≤ 10%)

$$VR\,(\%) = \frac{\left|V_{R\,NL}\right| - \left|V_{R\,FL}\right|}{\left|V_{R\,FL}\right|} \times 100, \qquad V_{R\,NL} = \frac{V_S}{A}$$

- Operating range

$$0.95 \le V_R \le 1.05 \text{ pu} \qquad \textit{and} \qquad 0.95 \le V_S \le 1.05 \text{ pu}$$

- Voltage profile is a consideration for all lines

Stability considerations

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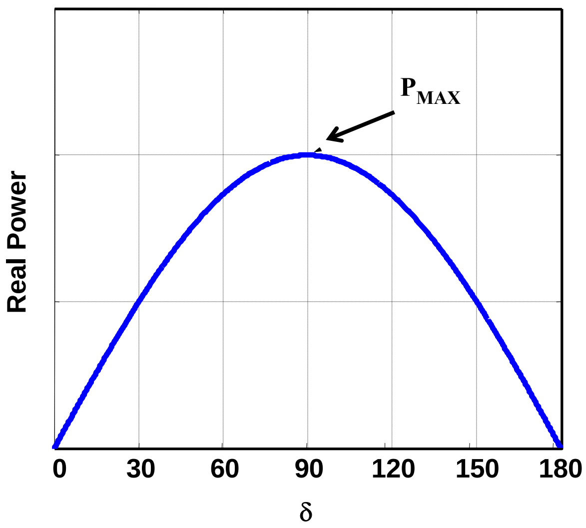
- Steady-state stability limit

For a loss-less 3-phase line (R=G=0), ignoring distributed effects:

$$P = P_S = P_R = \frac{V_R V_S}{X} \sin\delta, \quad \delta = \theta_{V_s} - \theta_{V_R}$$

$$P_{MAX} = \frac{V_R V_S}{X} \quad or \quad P_{MAX\ 3\phi} = 3\frac{V_R V_S}{X}$$



  - For $\delta > 90^o$, synchronism between sending and receiving end cannot be maintained
  - Lines are operated at $\delta < 35^o$ to prevent transient instability during system disturbances

- Long lines are typically stability-limited