ELE B7
Power Systems Engineering

Power System Components' Modeling

Section III : Transformer Model

Power Transformers- CONSTRUCTION

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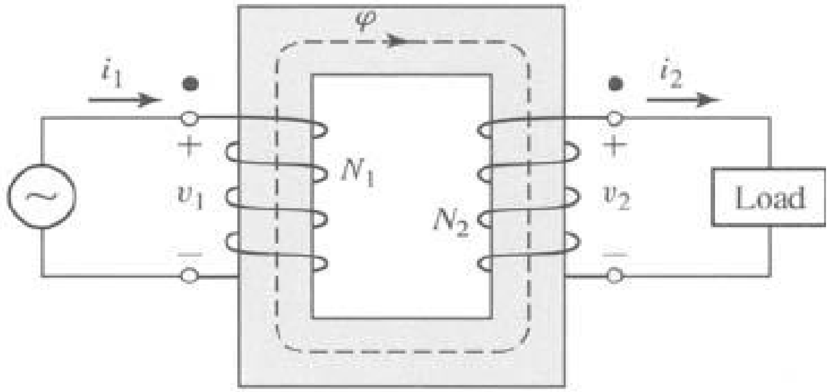
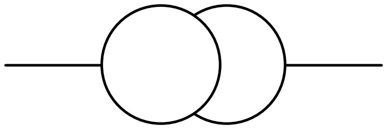
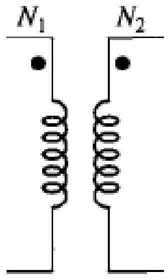
- **Primary windings,** connected to the alternating voltage source;
- **Secondary windings,** connected to the load;
- **Iron core,** which link the flux in both windings;



- The primary and secondary voltages are denoted by V_1 and V_2 respectively. The current entering the primary terminals is I_1.





Symbols

Power Transformers- IDEAL TRANSFORMER

- **Ideal transformer is characterized by:**
  - No real power loss;
  - No linkage flux;
  - Magnetic core has infinite permeability (μ).

$$v_1 = e_1 = N_1 \frac{d\Phi}{dt}$$

$$v_2 = e_2 = N_2 \frac{d\Phi}{dt}$$

  - Therefore

$$\frac{v_1}{v_2} = \frac{N_1}{N_2} = a$$

where, a is the turns ratio of the transformer

- **With no power loss:**

$$P_{in} = P_{out}$$

$$v_1 i_1 = v_2 i_2$$

- **Therefore**

$$\frac{i_1}{i_2} = \frac{N_2}{N_1} = \frac{1}{a}$$

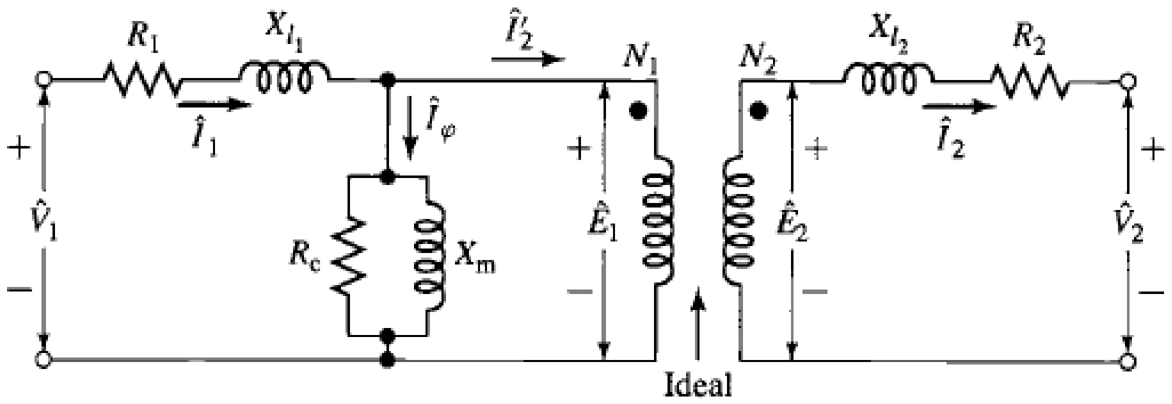
Power Transformers- REAL TRANSFORMER

- Real transformers
  - have losses
  - have leakage flux
  - have finite permeability of magnetic core

1. Real power losses
   - resistance in windings ($i^2 R$)
   - core losses due to eddy currents and hysteresis

Power Transformers- REAL TRANSFORMER

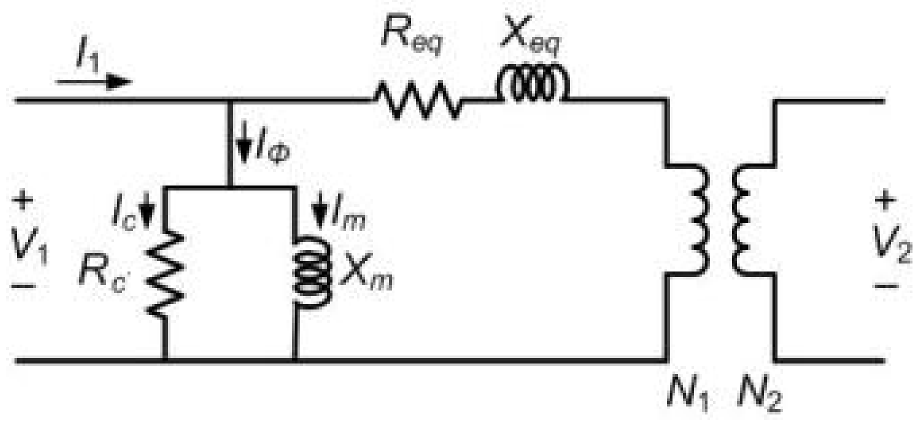
- The single-phase equivalent circuit of a real transformer is shown below.



- The leakage inductances of the transformer is denoted by X_{l1} & X_{l1} and R_1 & R_2 are the transformer's winding resistances
- The core loss component is represented by R_c while the magnetizing reactance is denoted by X_m .

Power Transformers- Approximate Circuits

- Neglecting the voltage drop across the primary and secondary windings due to the excitation current.
- Therefore, the magnetization branch can be moved to either the primary or secondary terminals.
- The approximate equivalent circuit of the transformer referred to the primary side is shown below.

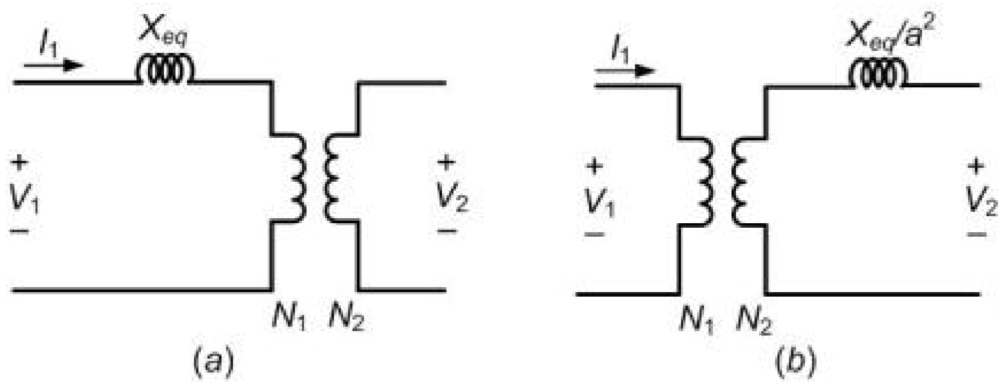


$$R_{eq} = R_1 + a^2 R_2$$

$$X_{eq} = X_{l_1} + a^2 X_{l_2}$$

Power Transformers- Approximate Circuits

- The impedance of the shunt branch is much larger compared to that of the series branch. Therefore we neglect R_c and X_m.
- R_{eq} is much smaller than X_{eq}. We can therefore neglect the series resistance. Therefore the transformer can be represented by the leakage reactance X_{eq}.



$a = N_1 / N_2$

Simplified equivalent circuit of a single-phase transformer: (a) when referred to the primary side and (b) when referred to the secondary side.

Power Transformers- EXAMPLE 1

A 25-kVA, 440/220-V, 60-Hz transformer has the following parameters:

$$R_1 = 0.16\ \Omega \qquad R_2 = 0.04\ \Omega \qquad R_{c1} = 270\ \Omega$$
$$X_1 = 0.32\ \Omega \qquad X_2 = 0.08\ \Omega \qquad X_{m1} = 100\ \Omega$$

The transformer delivers 20 kW at 0.8 power factor lagging to a load on the low-voltage side with 220 V across the load. Find the primary terminal voltage.

Power Transformers- EXAMPLE 1, Sol.

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The voltage across the load is taken as reference phasor; thus,

$$V_2 = 220\angle 0^\circ \text{ V}$$

The transformer turns ratio is $a = 440/220 = 2$.

For a load $P_2 = 20{,}000$ W at 0.8 power factor lagging, the secondary current is computed as follows:

$$I_2 = \frac{20{,}000}{(220)(0.8)}\angle -\cos^{-1} 0.8 = 113.64\angle -36.9^\circ \text{ A}$$

Power Transformers- EXAMPLE 1, Sol.

$$a\mathbf{V}_2 = 2(220\angle 0^\circ) = 440\angle 0^\circ \text{ V}$$

$$\mathbf{I}_2/a = (113.64\angle -36.9^\circ)/2 = 56.82\angle -36.9^\circ \text{ A}$$

$$a^2R_2 = (2)^2(0.04) = 0.16\ \Omega$$

$$a^2X_2 = (2)^2(0.08) = 0.32\ \Omega$$

$$\begin{aligned}\mathbf{E}_1 &= a\mathbf{V}_2 + (\mathbf{I}_2/a)(a^2R_2 + ja^2X_2)\\ &= 440\angle 0^\circ + (56.82\angle -36.9^\circ)(0.16 + j0.32)\\ &= 458.2 + j9.07 = 458.3\angle 1^\circ \text{ V}\end{aligned}$$

$$\mathbf{I}_c = \mathbf{E}_1/R_{c1} = (458.2 + j9.07)/270 = 1.7 + j0.03 \text{ A}$$

$$\mathbf{I}_m = \mathbf{E}_1/jX_{m1} = (458.2 + j9.07)/j100 = 0.09 - j4.58 \text{ A}$$

$$\mathbf{I}_e = \mathbf{I}_c + \mathbf{I}_m = 1.79 - j4.55 \text{ A}$$

Power Transformers- EXAMPLE 1, Sol.

Thus, the primary current and voltage are:

$$\mathbf{I}_1 = \mathbf{I}_e + \mathbf{I}_2/a$$
$$= (1.79 - j4.55) + (56.82 \angle -36.9°) = 61.04 \angle -39.3° \text{ A}$$

$$\mathbf{V}_1 = \mathbf{E}_1 + \mathbf{I}_1(R_1 + jX_1)$$
$$= (458.2 + j9.07) + (61.04 \angle -39.3°)(0.16 + j0.32)$$
$$= 478.1 + j18 = 478.4 \angle 2.2° \text{ V}$$

Power Transformers- EXAMPLE 2

A 150-kVA, 2400/240-V transformer has the following parameters referred to the primary side: $R_{e1} = 0.5\ \Omega$ and $X_{e1} = 1.5\ \Omega$. The shunt magnetizing impedance is very large and can be neglected. At full load, the transformer delivers rated kVA at 0.85 lagging power factor and the secondary voltage is 240 V. Calculate (a) the voltage regulation and (b) the efficiency assuming core losses amount to 600 W.

Solution:

a. The transformer turns ratio is

$$a = 2400/240 = 10$$

Take the secondary voltage $\mathbf{V}_2$ as the reference

$$\mathbf{V}_2 = 240\ \underline{/0^\circ}\ \text{V}$$

$$a\mathbf{V}_2 = 2400\ \underline{/0^\circ}\ \text{V}$$

Power Transformers- EXAMPLE 2, Sol.

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At rated load and 0.85 PF lagging:

$$\mathbf{I}_2 = (150{,}000/240)\,\angle{-\cos^{-1} 0.85^\circ} = 625\,\angle{-31.8^\circ}\text{ A}$$

$$\mathbf{I}_1 = \mathbf{I}_2/a = (625/10)\,\angle{-31.8^\circ} = 62.5\,\angle{-31.8^\circ}\text{ A}$$

The primary voltage is calculated as follows:

$$\begin{aligned}\mathbf{V}_1 &= a\mathbf{V}_2 + (\mathbf{I}_2/a)(R_{e1} + jX_{e1}) \\ &= 2400\,\angle{0^\circ} + (62.5\,\angle{-31.8^\circ})(0.5 + j1.5) = 2476.8\,\angle{1.5^\circ}\text{ V}\end{aligned}$$

Power Transformers- EXAMPLE 2, Sol.

$$\text{Voltage regulation} = \frac{V_1 - aV_2}{aV_2}\,100\%$$
$$= \frac{2476.8 - 2400}{2400}\,100\% = 3.2\%$$

b. At rated output,

$$P_{\text{output}} = (150{,}000)(0.85) = 127{,}500\text{ W}$$
$$P_{\text{cu}} = I_1^2 R_{e1} = (62.5)^2(0.5) = 1953\text{ W}$$
$$P_{\text{core}} = 600\text{ W}$$
$$P_{\text{input}} = P_{\text{output}} + \Sigma(\text{losses}) = 130{,}053\text{ W}$$

Therefore, the efficiency is found by using Eq. 4.51 as follows:

$$\eta = \frac{\text{power output}}{\text{power input}}\,100\%$$
$$= \frac{127{,}500}{130{,}053}\,100\% = 98\%$$

Per Unit Calculations

- A key problem in analyzing power systems is the large number of transformers.
  - It would be very difficult to continually have to refer impedances to the different sides of the transformers
- This problem is avoided by a normalization of all variables.
- This normalization is known as per unit analysis.

$$\text{quantity in per unit} = \frac{\text{actual quantity}}{\text{base value of quantity}}$$

Per Unit Conversion Procedure, 1ϕ

1. Pick a 1ϕ VA base for the entire system, S_B
2. Pick a voltage base for each different voltage level, V_B. Voltage bases are related by transformer turns ratios. Voltages are line to neutral.
3. Calculate the impedance base, $Z_B = (V_B)^2/S_B$
4. Calculate the current base, $I_B = V_B/Z_B$
5. Convert actual values to per unit

Note, per unit conversion on affects magnitudes, not the angles. Also, per unit quantities no longer have units (i.e., a voltage is 1.0 p.u., not 1 p.u. volts)

Three Phase Per Unit

Procedure is very similar to 1ϕ except we use a 3ϕ VA base, and use line to line voltage bases

1. Pick a 3ϕ VA base for the entire system, $S_B^{3\phi}$
2. Pick a voltage base for each different voltage level, V_B. Voltages are line to line.
3. Calculate the impedance base

$$Z_B = \frac{V_{B,LL}^2}{S_B^{3\phi}} = \frac{(\sqrt{3}\,V_{B,LN})^2}{3S_B^{1\phi}} = \frac{V_{B,LN}^2}{S_B^{1\phi}}$$

Exactly the same impedance bases as with single phase!

Three Phase Per Unit, cont'd

4. Calculate the current base, I_B

$$I_B^{3\phi} = \frac{S_B^{3\phi}}{\sqrt{3}\,V_{B,LL}} = \frac{3\,S_B^{1\phi}}{\sqrt{3}\,\sqrt{3}\,V_{B,LN}} = \frac{S_B^{1\phi}}{V_{B,LN}} = I_B^{1\phi}$$

Exactly the same current bases as with single phase!

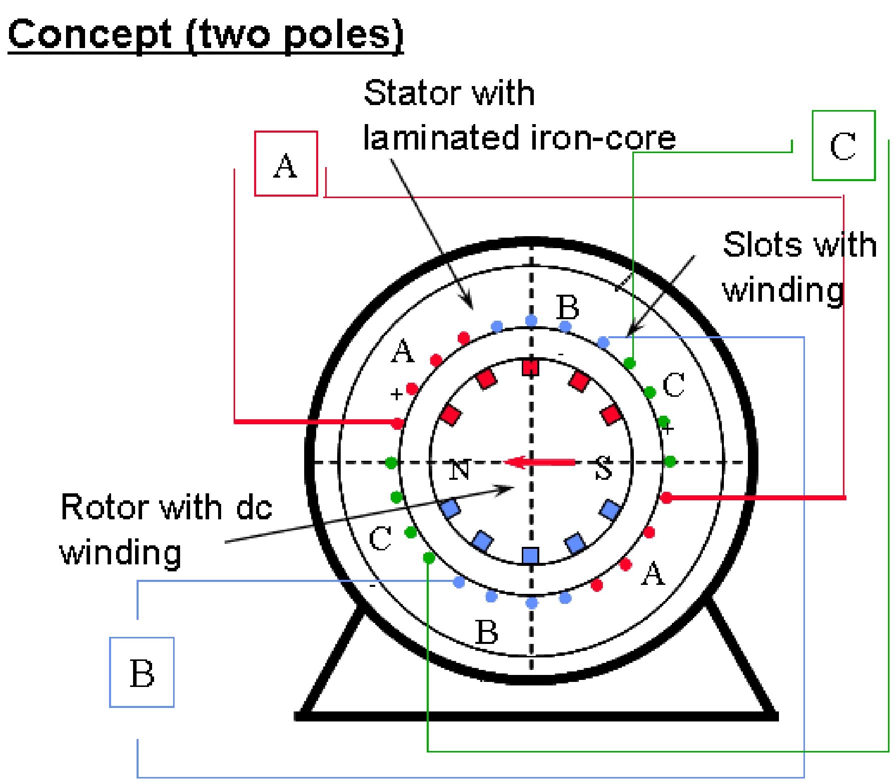
5. Convert actual values to per unit

Section IV: Synchronous Machine Model

Synchronous Generator- CONSTRUCTION

Round Rotor Machine

- The stator is a ring shaped laminated iron-core with slots.
  - Three phase windings are placed in the slots.
- Round solid iron rotor with slots.
  - A single winding is placed in the slots. DC current is supplied through slip rings.
- Characteristics:
  - High speed applications;
  - High power ratings (100s of MW)
  - Driving system: e.g., Steam turbines.



Synchronous Generator- CONSTRUCTION

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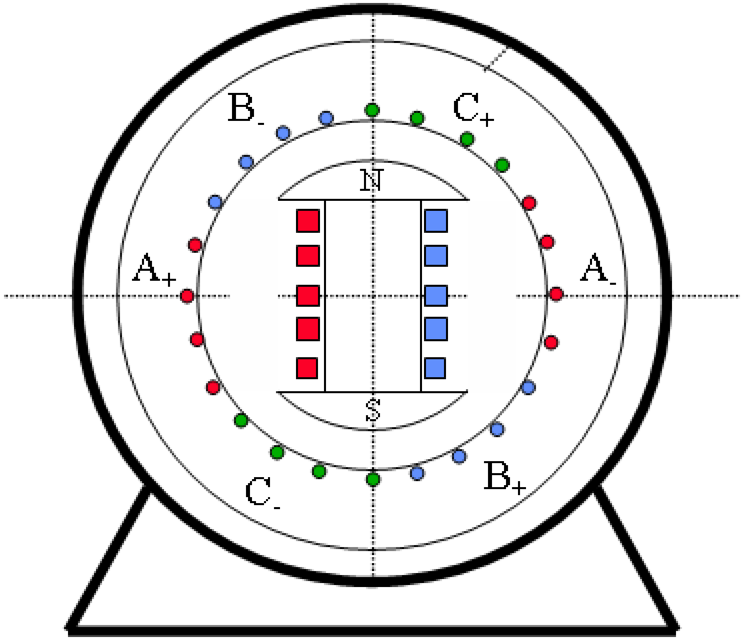
Salient Rotor Machine

- The stator has a laminated iron-core with slots and three phase windings placed in the slots.
- The rotor has salient poles excited by dc current.
- DC current is supplied to the rotor through slip-rings and brushes.
- Characteristics:
  - Low speed applications;
  - Low ratings (10s of MW)
  - Driving system: e.g., Hydro turbines.

- **Concept (two poles)**



Synchronous Generator- OPERATION

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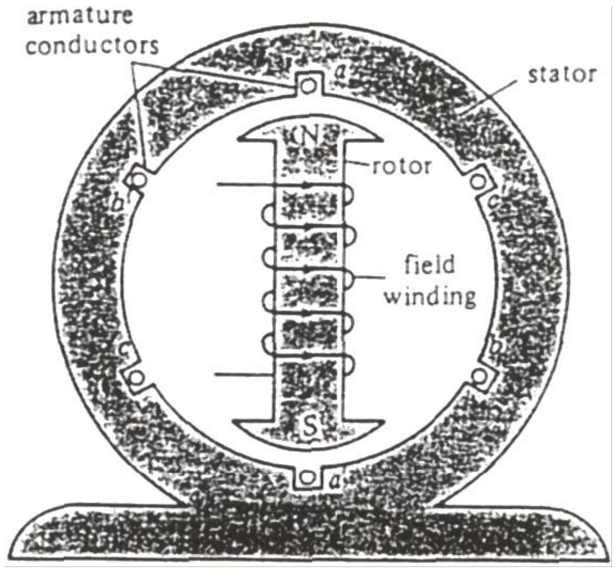
Principle of Operation

1) From an external source, the field winding is supplied with a DC current -> excitation.
2) Rotor (field) winding is mechanically turned (rotated) at synchronous speed.
3) The rotating magnetic field produced by the field current induces voltages in the outer stator (armature) winding. The frequency of these voltages is in synchronism with the rotor speed.



Operation Characteristics

- The frequency - speed relation is $f = (p / 120)\, n = p\, n / 120$

where:

p is the number of poles.

Typical rotor speeds are 3600 rpm for 2-pole, 1800 rpm for 4 pole and 450 rpm for 16 poles.

- The RMS value of the induced voltages of phase A is:

$$E_a = 4.44\, N\, BA\, f\,,\ (BA = \phi)$$

where:

N = number of turns, B= flux density, A = cross sectional area of the magnetic circuit, f = frequency, and ϕ = flux per pole

Round Rotor Type

1) DC current in the field winding produces the main flux, ϕ_f.
2) ϕ_f induces an emf, E_a, in the armature winding.
3) Depending on the load condition, the armature current I_A is established. In the following discussions, it is assumed to be a lagging power factor.
4) I_A produces its own flux due to armature reaction, E_{AR} is the induced emf by ϕ_{AR}.
5) The resulting phasor, $E_{resultant} = E_a + E_{AR}$ is the "true" induced emf that is available.

Synchronous Generator- Equivalent Circuit

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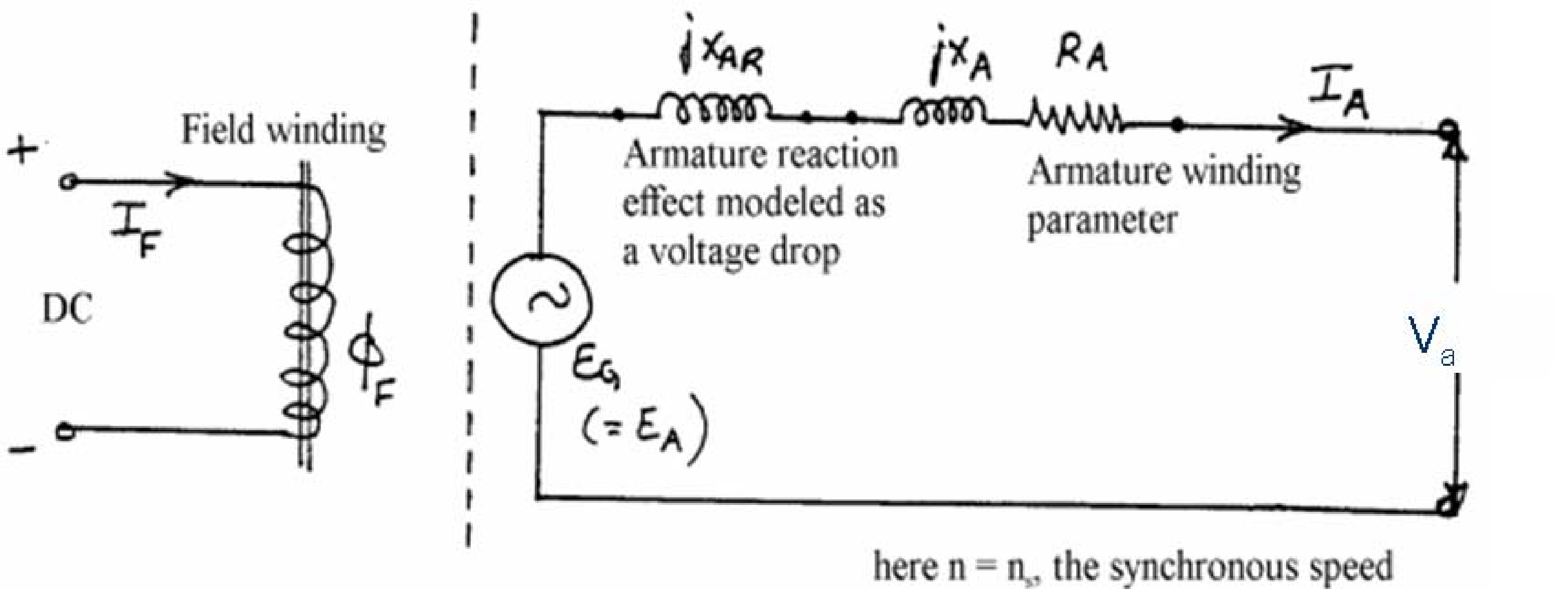
The per phase equivalent circuit for round rotor generator



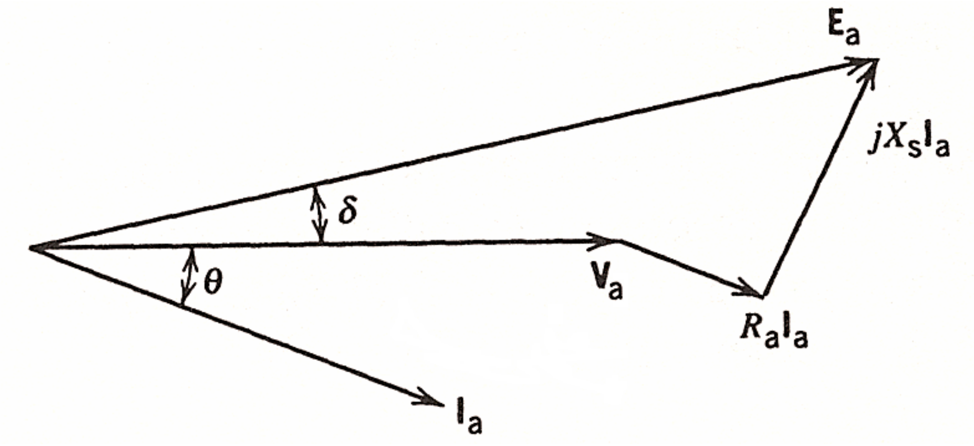
$$V_a = E_A - I_A jX_A - I_A jX_{AR} - I_A R_A = E_A - jX_s I_A - I_A R_A$$

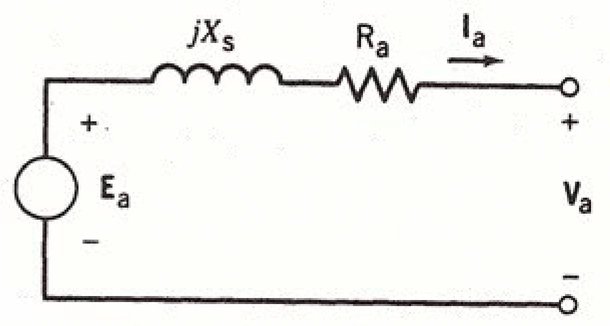
$$V_a = E_A - I_A\left(R_A + jX_s\right)$$

where, $(X_{AR} + X_A)$ = synchronous reactance, X_s.

Synchronous Generator- Equivalent Circuit

- The phasor diagram of a round rotor generator:





$$V_a = E_a - I_a(R_a + jX_s)$$

where

R_A: Armature winding resistance;

X_s: Synchronous reactance;

δ: Power angle of the synchronous generator.

Synchronous Generator- EXAMPLE 3

A 60-Hz, three-phase synchronous generator is observed to have a terminal voltage of 460 V (line-line) and a terminal current of 120 A at a power factor of 0.95 lagging. The field-current under this operating condition is 47 A. The machine synchronous reactance is equal to 1.68 Ω. Assume the armature resistance to be negligible.

Calculate:

a) the generated voltage E_a in volts,

b) the mechanical power input to the generator in kW

Synchronous Generator- EXAMPLE 3-Sol.

Using the motor reference direction for the current and neglecting the armature resistance, the generated voltage can be found from the equivalent circuit:

$$E_a = V_a + jI_a X_s$$

$$\vec{V}_a = \frac{460}{\sqrt{3}} = 265.6 \angle 0$$

A lagging power factor of 0.95 corresponds to a power factor angle 18.2°

$$I_a = 120 \angle -18.2$$

Solution (*cont.*):

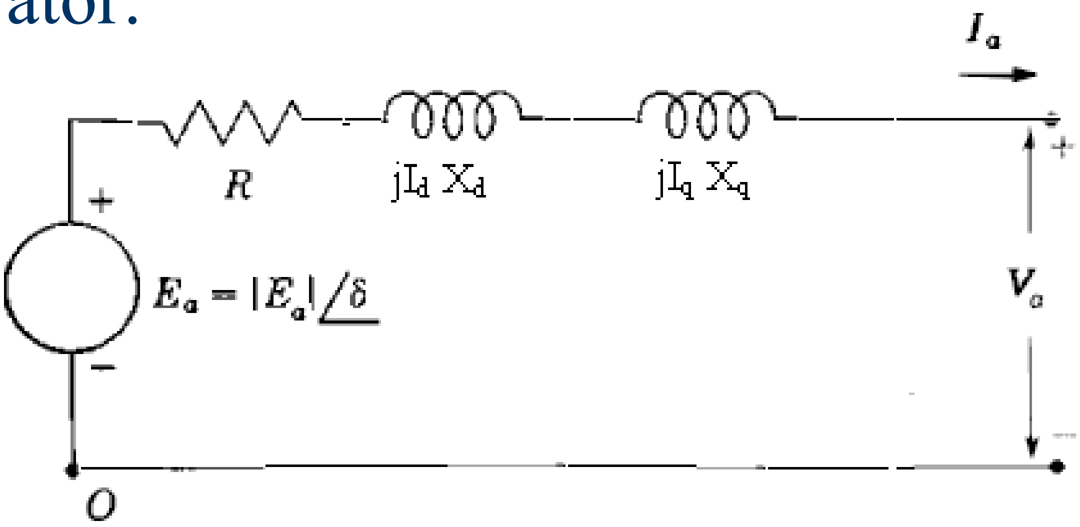
$$E_a = V_a + jI_a X_s$$
$$= 265.5\angle 0 + \left(120\angle -18.2 \times 1.68\angle 90\right)$$
$$= 380.2\angle 30.2 \quad V$$

b) The mechanical power input to the generator Pin can be found as three times the power input to phase a:

$$P_{in} = 3 \times E_a \times I_a \cos(\delta + \phi)$$
$$= 3 \times 380 \times 120 \times \cos(30.2 + 18.2)$$
$$= 90.825 \quad KW$$

Salient Rotor Type

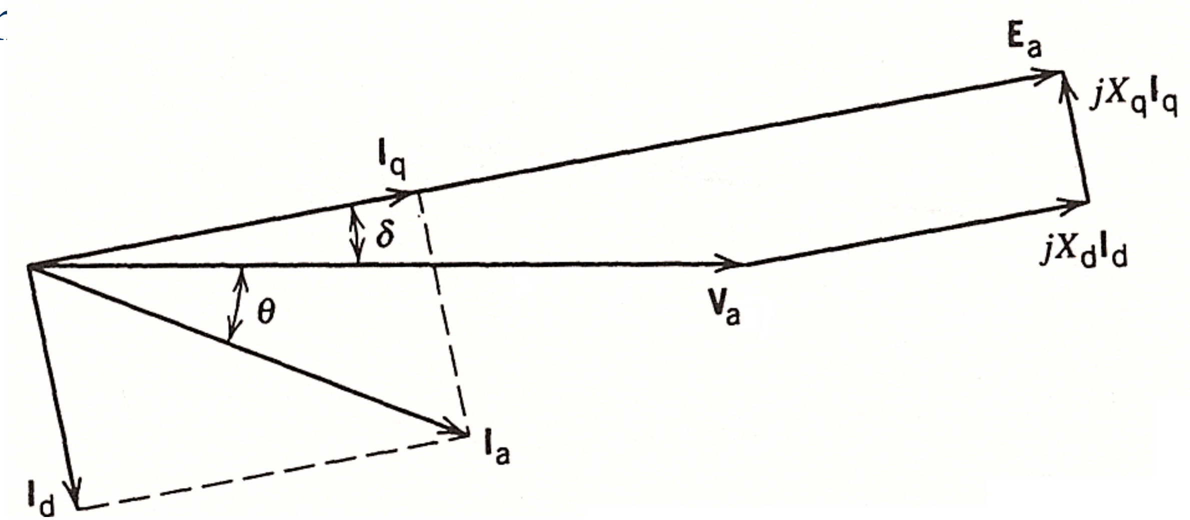
- The per phase equivalent circuit for salient rotor generator:



$$E_a = V_a + I_a R + jI_d X_d + jI_q X_q$$

Salient Rotor Type

- The phasor diagram of salient rotor generator (neglecting the arm



$$E_a = V_a + jI_d X_d + jI_q X_q$$

where

X_d: Direct axis reactance;

X_q: Quadrature axis reactance;

Salient Rotor Type

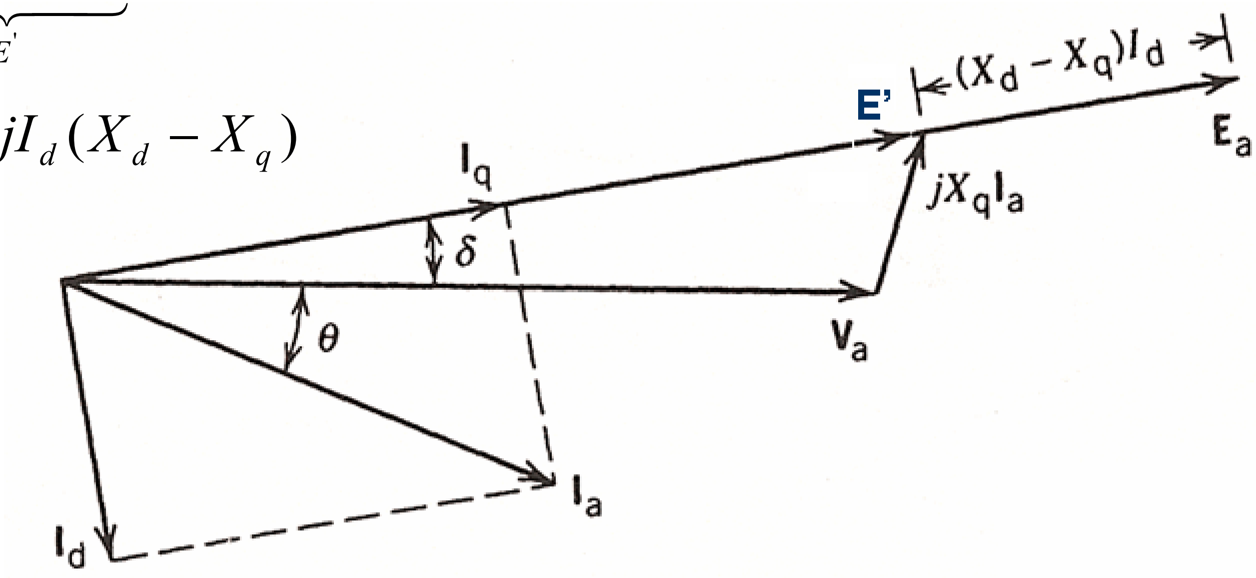
- The expression for $\mathbf{E_a}$ may be written as follows:

$$E_a = V_a + jI_d X_d + jI_q X_q + (jI_d X_q - jI_d X_q)$$

but, $\bar{I}_a = \bar{I}_d + \bar{I}_q$

$$\therefore E_a = \underbrace{V_a + jI_a X_q}_{E'} + jI_d (X_d - X_q)$$

$$\therefore E_a = E' + jI_d (X_d - X_q)$$



- The phase angle of **E'** = the phase angle of $\mathbf{E_a}$

Synchronous Generator- EXAMPLE 4

A 75-MVA, 13.8-kV, three-phase, eight-pole, 60-Hz salient-pole synchronous machine has the following d-axis and q-axis reactances: $X_d = 1.0$ pu and $X_q = 0.6$ pu. The synchronous generator is delivering rated MVA at rated voltage and 0.866 power factor lagging. Choose a power base of 75 MVA and a voltage base of 13.8 kV. Compute the excitation voltage E_a.

Solution The following calculations are performed in per unit using a power base of 75 MVA and a voltage base of 13.8 kV. The per-unit terminal voltage $\mathbf{V}_t$ is taken as reference phasor; thus

$$\mathbf{V}_t = 1.0\angle 0^\circ$$

At rated conditions and 0.866 PF lagging, the per-unit stator current is given by

$$\mathbf{I}_a = 1.0\angle -30^\circ$$

Synchronous Generator- EXAMPLE 4-Sol.

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$$\mathbf{E}' = 1.0\angle 0^\circ + (j0.6)(1.0\angle -30^\circ) = 1.40\angle 21.8^\circ \text{ pu}$$

Therefore, angle δ is equal to 21.8°.

The angle between $\mathbf{E}_a$ and $\mathbf{I}_a$ is found as follows:

$$(\delta + \theta) = 21.8^\circ + 30^\circ = 51.8^\circ$$

This angle is used to resolve $\mathbf{I}_a$ into its components:

$$\mathbf{I}_d = [I_a \sin(\delta + \theta)]\angle \delta - 90^\circ$$

$$= [1.0 \sin 51.8^\circ]\angle 21.8^\circ - 90^\circ = 0.786\angle -68.2^\circ \text{ pu}$$

$$\mathbf{I}_q = [I_a \cos(\delta + \theta)]\angle \delta$$

$$= [1.0 \cos 51.8^\circ]\angle 21.8^\circ = 0.618\angle 21.8^\circ \text{ pu}$$

Synchronous Generator- EXAMPLE 4-Sol.

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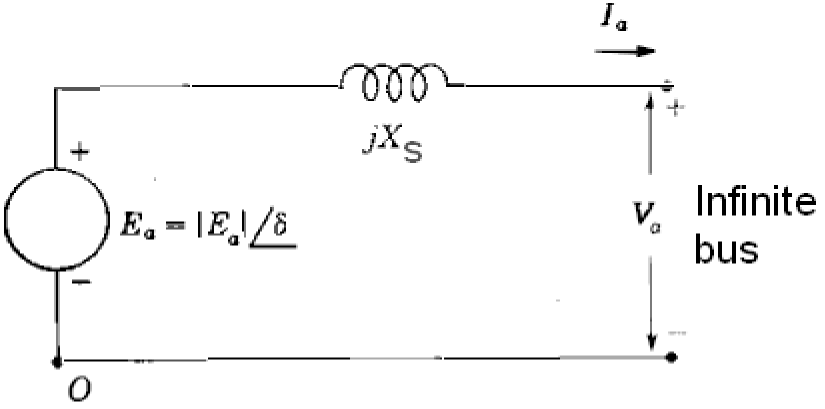
$$E_a = E' + (X_d - X_q)I_d$$
$$= 1.40 + (1.0 - 0.6)(0.786) = 1.714 \text{ pu}$$

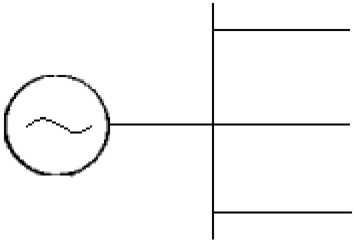
$$\mathbf{E}_a = E_a\angle\delta = 1.714\angle 21.8^\circ \text{ pu} = 23.6\angle 21.8^\circ \text{ kV}$$

Synchronous Generator- Power-Angle Cc's

- Power-Angle characteristics of a synchronous machine:
  - The maximum power that a synchronous machine can deliver is determined by the maximum torque that can be applied without loosing synchronism.
  - The expression of the power supplied will be expressed in terms of the machine parameters and the system that is connected o it.

• Round rotor synchronous machine:





Neglecting Ra and taking V_a as a reference phasor,

$$\vec{V}_a = V_a \angle 0 \qquad \& \qquad \vec{E}_a = E_a \angle \delta$$

$$\therefore I_a = \frac{\vec{E}_a - \vec{V}_a}{jX_s}$$

- **Round rotor synchronous machine:**
  - **The complex power is given by:**

$$\vec{S} = P + jQ = 3\,\vec{V}_a\,\bar{I}_a^*$$

$$= 3\,V_a\angle 0\left(\frac{(E_a\angle -\delta) - (V_a\angle 0)}{-\,jX_s}\right)$$

$$= 3\left(\left(\frac{E_a V_a}{X_s}\angle -\delta + 90\right) - j\frac{V_a^2}{X_s}\right)$$

$$\therefore\; P = 3\,\frac{E_a V_a}{X_s}\sin\delta$$

and

$$Q = 3\left(\left(\frac{E_a V_a}{X_s}\cos\delta\right) - \frac{V_a^2}{X_s}\right)$$

Synchronous Generator- Power-Angle Cc's

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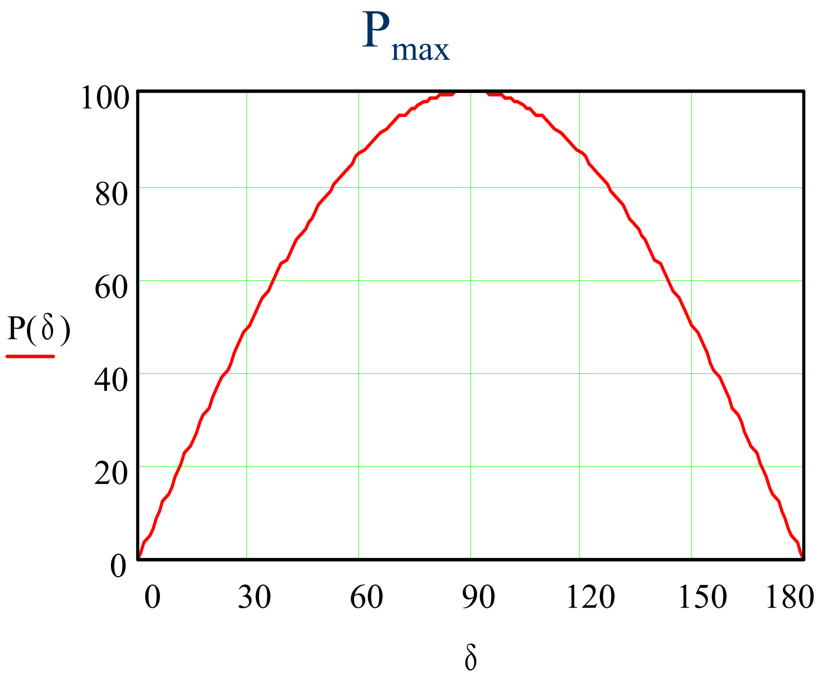
Power angle Characteristics

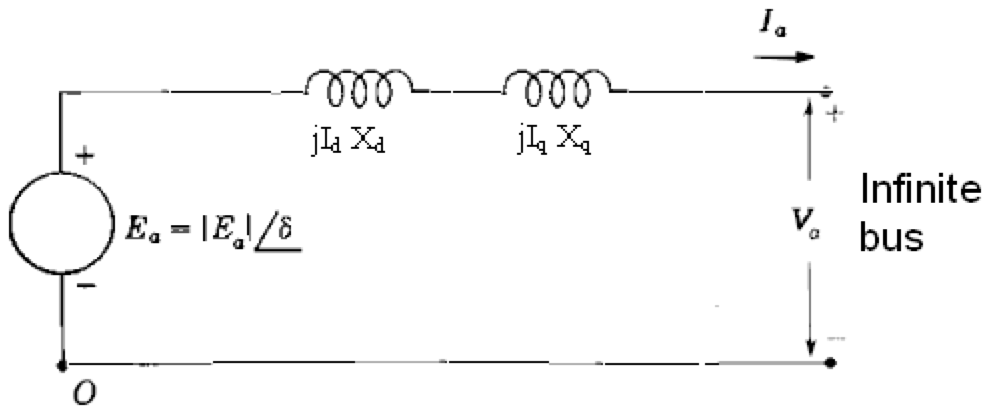
- The P(δ) curve shows that the increase of power increases the angle between the induced voltage and the terminal voltage.
- The power is maximum when $\delta = 90^o$
- The further increase of input power forces the generator out of synchronism. This generates large current and mechanical forces.
- The maximum power is the static stability limit of the system.

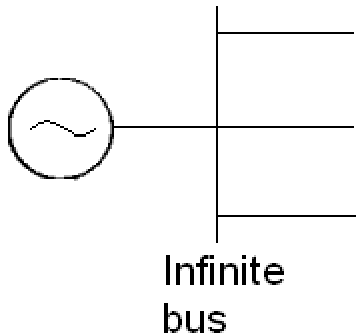
Round Rotor Machine



Synchronous Generator- Power-Angle Cc's

- **Salient rotor synchronous machine:**



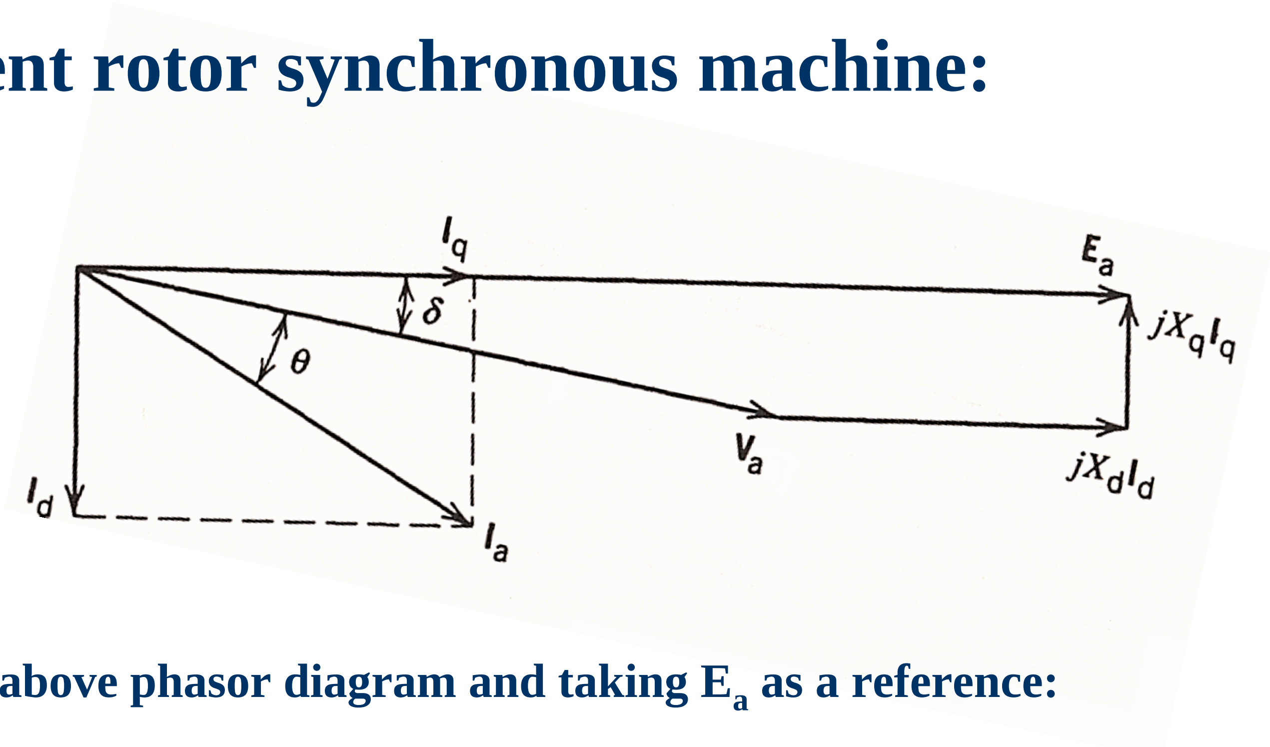


Neglecting R and taking E_a as a reference phasor,

$$\vec{V}_a = V_a \angle -\delta^o \qquad \& \qquad \vec{E}_a = E_a \angle 0^o$$

Synchronous Generator- Power-Angle Cc's

- **Salient rotor synchronous machine:**



Using the above phasor diagram and taking E_a as a reference:

$$I_q = \frac{V_a \sin\delta}{X_q} \quad \& \quad I_d = \frac{E_a - V_a \cos\delta}{X_d}$$

- **Salient rotor synchronous machine:**
  - **The complex power is given by:**

$$\vec{S} = P + jQ = 3\,\vec{V}_a\,\bar{I}_a^*$$
$$= 3\,V_a\angle -\delta\,\left(I_q - jI_d\right)^*$$
$$= 3\,V_a\angle -\delta\,\left(I_q\angle 0 + I_d\angle -90\right)$$

substitute for I_q *&* I_d *and simplify*

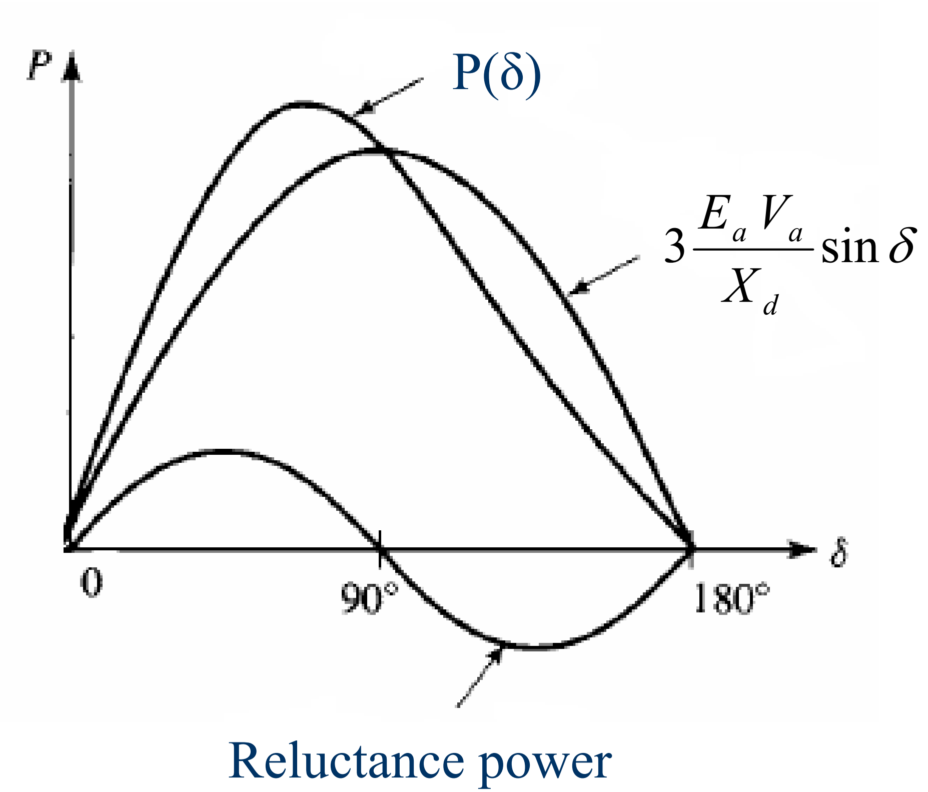
$$\therefore\ P = 3\left[\frac{E_a V_a}{X_d}\sin\delta + \frac{V_a^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\sin 2\delta\right]$$

$$Q = 3\left[\frac{E_a V_a}{X_d}\cos\delta - V_a^2\left(\frac{\sin^2\delta}{X_q} + \frac{\cos^2\delta}{X_d}\right)\right]$$

Synchronous Generator- Power-Angle Cc's

ELE B7

- Power angle curve of a salient-pole synchronous generator



Synchronous Generator- Power-Angle Cc's

ELE B7

- Salient rotor synchronous machine:

- Observations:
  - The first term of the active power is identical to the power delivered by a round rotor synchronous generator;
  - The second term represents the effect of generator saliency, and it is called reluctance power;
  - In salient pole machine, $X_d > X_q$. When a salient machine approaches a round rotor, the values of X_d & X_q will both approach X_s and P &Q of salient pole machine are reduced to that of round rotor machine.

Synchronous Generator- EXAMPLE 5

A 25 kVA, 230 V three phase, four pole, 60 Hz, Y-connected synchronous generator has a synchronous reactance of 1.5 Ω/phase and a negligible armature resistance. The generator is connected to an infinite bus of constant voltage (230 V) and frequency (60 Hz), find:

a. The generated EMF (E_a) when the machine is delivering rated kVA at 0.8 power factor lagging.
b. If the field current I_f is increased by 20 % without changing the power input find the stator current I_a.
c. With the field excitation current I_f as in part (a), the input power from prime mover is increased very slowly. What is the steady state limit? Determine the stator current I_a, power factor, and reactive power

Synchronous Generator- EXAMPLE 5-Sol.

a. The terminal voltage is taken as reference phasor. Thus,

$$\mathbf{V}_t = (230/\sqrt{3})\angle 0^\circ = 132.8\angle 0^\circ \text{ V} \quad \text{(line-to-neutral)}$$

The stator current is obtained as follows:

$$\mathbf{I}_a = \frac{25,000}{230\sqrt{3}}\angle -\cos^{-1} 0.8 = 62.8\angle -36.9^\circ \text{ A}$$

The excitation voltage is calculated as follows:

$$\begin{aligned}\mathbf{E}_a &= \mathbf{V}_t + \mathbf{I}_a(R_a + jX_s) \\ &= 132.8\angle 0^\circ + (62.8\angle -36.9^\circ)(0 + j1.5) \\ &= 203.8\angle 21.7^\circ \text{ V} \quad \text{(line-to-neutral)}\end{aligned}$$

Therefore, the line-to-line excitation voltage magnitude is

$$E_a = 203.8\sqrt{3} = 353 \text{ V} \quad \text{(line-to-line)}$$

Synchronous Generator- EXAMPLE 5-Sol.

ELE B7

The power angle is the phase angle by which $\mathbf{E}_a$ leads $\mathbf{V}_t$, and it is given by

$$\delta = 21.7°$$

b. The excitation voltage magnitude is increased by 20%; that is,

$$E'_a = 1.20E_a = (1.2)(203.8) = 244.6\text{V} \quad \text{(line-to-neutral)}$$

Since the input power from the prime mover remains unchanged, $P' = P$. Therefore,

$$3(E'_a V_t / X_s)\sin\delta' = 3(E_a V_t / X_s)\sin\delta$$
$$244.6\sin\delta' = 203.8\sin 21.7°$$

Solving for the new power angle yields

$$\delta' = 17.9°$$

Synchronous Generator- EXAMPLE 5-Sol.

Hence, the stator current is calculated as follows:

$$\mathbf{I}_a = \frac{\mathbf{E}_a - \mathbf{V}_t}{jX_s}$$

$$= \frac{244.6 \angle 17.9^\circ - 132.8 \angle 0^\circ}{j1.5} = 83.4 \angle -53^\circ \text{ A}$$

The power factor is

$$PF = \cos 53^\circ = 0.60 \text{ lagging}$$

The reactive power is found as follows:

$$Q = 3V_t I_a \sin\theta = 3(132.8)(83.4)\sin 53^\circ = 26.5 \text{ kVAR}$$

Alternatively, the reactive power may be obtained by using

$$Q = 3\left[\frac{(244.6)(132.8)}{1.5}\cos 17.9^\circ - \frac{(132.8)^2}{1.5}\right] = 26.5 \text{ kVAR}$$

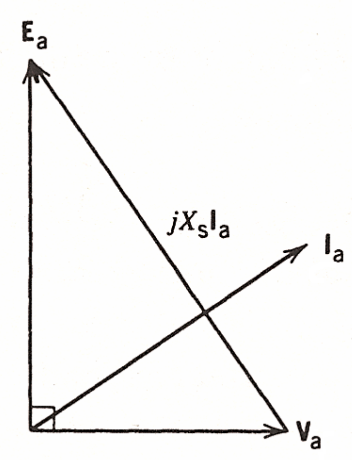
Synchronous Generator- **EXAMPLE 5-Sol.**

c. With the field excitation current as in part (a), the excitation voltage magnitude is

$$E_a = 203.8\text{V} \quad \text{(line-to-neutral)}$$

The steady-state limit is the maximum power P_{max} of the generator, and it occurs at $\delta = 90°$. Therefore,

$$P_{max} = 3E_a V_t / X_s = 3(203.8)(132.8)/1.5 = 54.13 \text{ kW}$$



Synchronous Generator- EXAMPLE 5-Sol.

ELE B7

From the previous phasor diagram,

$$\mathbf{I}_a = \frac{\mathbf{E}_a - \mathbf{V}_t}{jX_s}$$

$$= \frac{203.8\angle 90° - 132.8\angle 0°}{j1.5} = 162.2\angle 33.1° \text{ A}$$

The power factor is given by

$$PF = \cos(\angle \mathbf{V}_t - \angle \mathbf{I}_a) = \cos(0° - 33.1°) = 0.84 \text{ leading}$$

The reactive power is given by

$$Q = 3V_t I_a \sin\theta = 3(132.8)(162.2)\sin(-33.1°) = -35.3 \text{ kVAR}$$

Alternatively, the reactive power may be found by using Eq. 7.22.

$$Q = 3\left[\frac{(203.8)(132.8)}{1.5}\cos 90° - \frac{(132.8)^2}{1.5}\right] = -35.3 \text{ kVAR}$$

Synchronous Generator- EXAMPLE 6

ELE B7

A round rotor synchronous machine is connected to an infinite bus whose voltage is kept constant at 1.05 pu. The synchronous reactance of the machine is 0.4. The table given below relates to two operating conditions of the machine. Complete the table neglecting armature reaction.

	P	Q	E	δ
Condition A	?	?	1.3	40°
Condition B	2.0	0	?	?
Condition C	?	0	1.2	?

Synchronous Generator- EXAMPLE 7

ELE B7

A salient pole synchronous machine is connected to an infinite bus whose voltage is kept constant at 1.00 pu. The direct and quadrature axis reactances of the machine are 0.6 and 0.3 pu respectively. The table given below relates to three operating conditions of the machine. (Q_2 is the reactive power at machine terminals) Complete the table neglecting armature reaction.

	P	Q_2	E	δ
Condition A	?	0.0	1.12	?
Condition B	?	?	1.25	37.5°
Condition C	2.0	?	?	40°