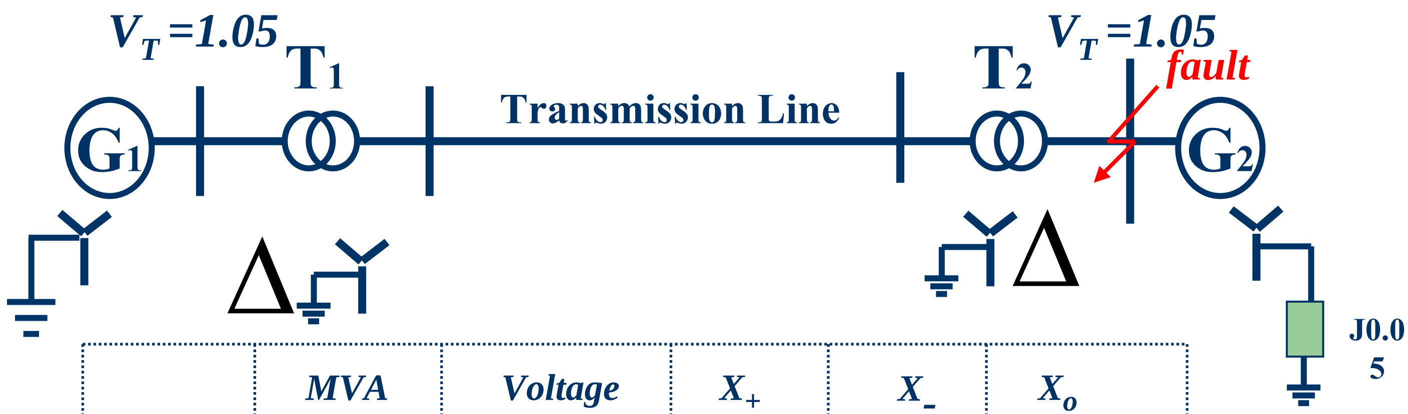
ELE B7 Power Systems Engineering

Unbalanced Fault Analysis

Unbalanced Fault Analysis

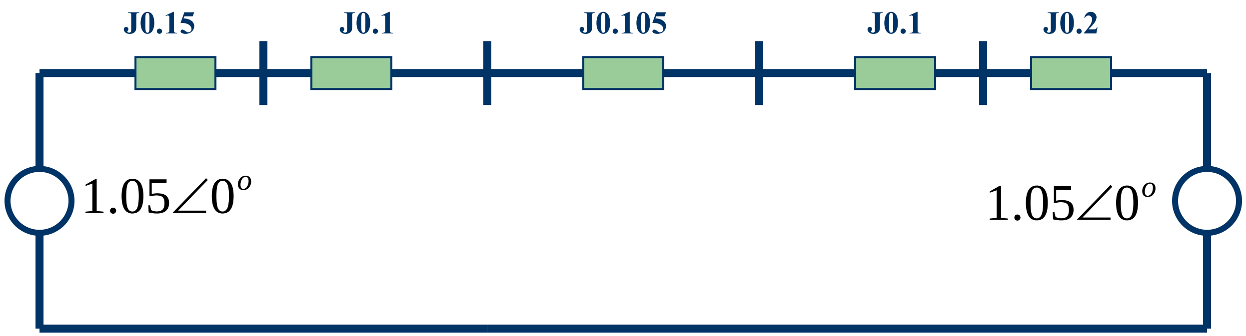
- The first step in the analysis of unbalanced faults is to assemble the three sequence networks.
- Consider the following example



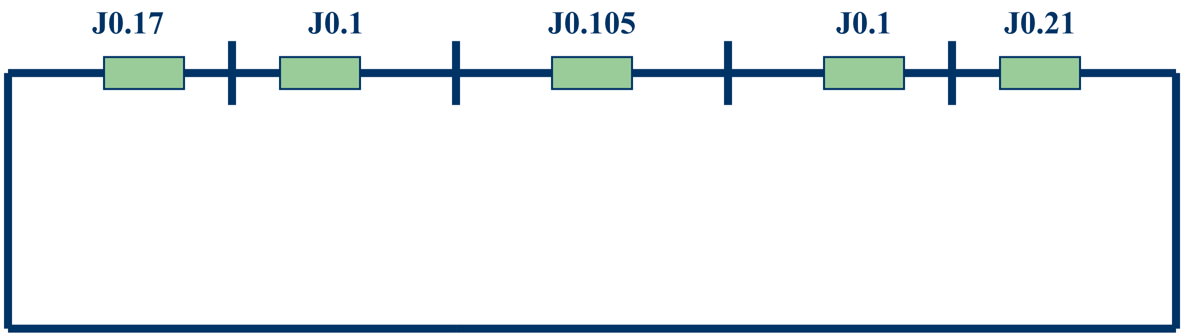
	MVA	Voltage	X_+	X_-	X_o
G1	100	11 kV	0.15	0.17	0.05
G2	100	11 kV	0.20	0.21	0.1
T1	100	11/220kV	0.1	0.1	0.1
T2	100	11/220kV	0.1	0.1	0.1
Line	100	220kV	0.105	0.105	0.315

Sequence Diagrams for Example

Positive Sequence Network

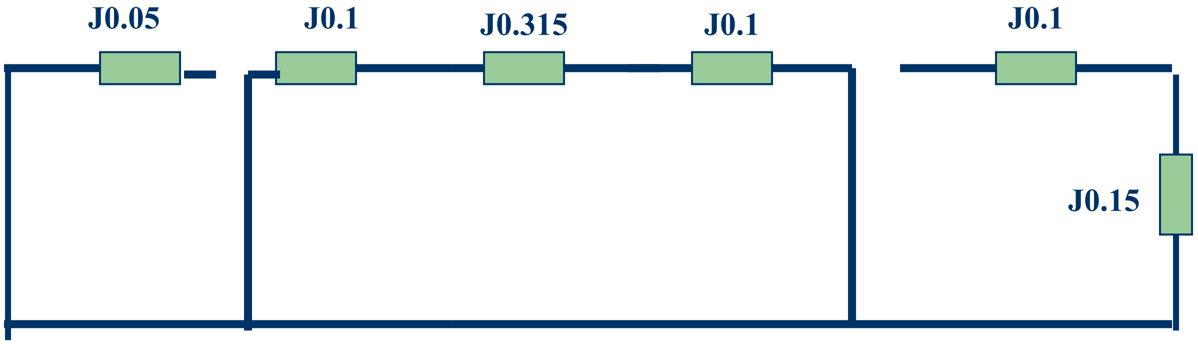


Negative Sequence Network



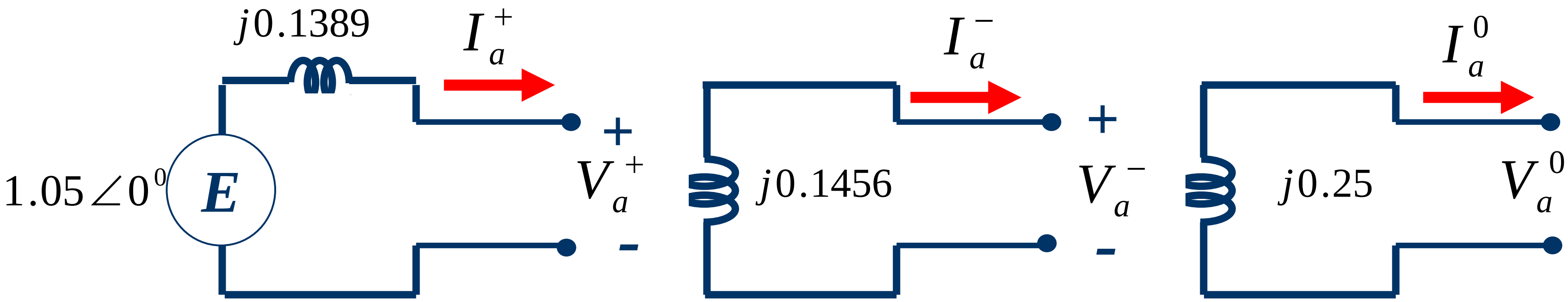
Sequence Diagrams for Example

Zero Sequence Network



Create Thevenin Equivalents

- Second is to calculate the Thevenin equivalents as seen from the fault location. In this example the fault is at the terminal of the right machine so the Thevenin equivalents are:



$$Z_{th}^{+} = j0.2 \text{ in parallel with } j0.455$$

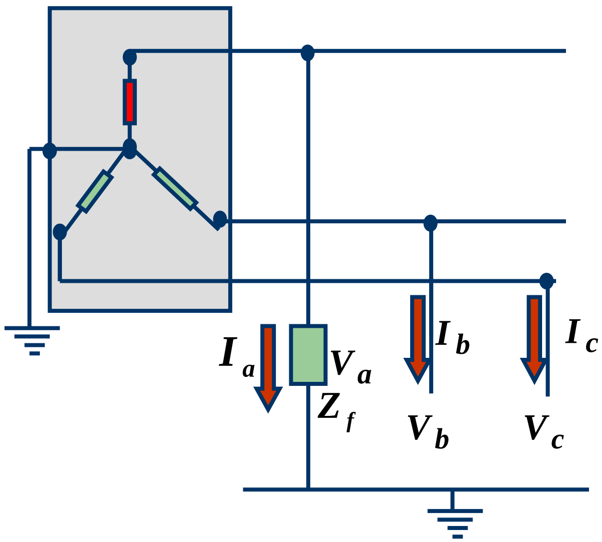
$$Z_{th}^{-} = j0.21 \text{ in parallel with } j0.475$$

Single Line-to-Ground (SLG) Faults

- Unbalanced faults unbalance the network, but only at the fault location. This causes a coupling of the sequence networks. How the sequence networks are coupled depends upon the fault type. We'll derive these relationships for several common faults.
- With a SLG fault only one phase has non-zero fault current -- we'll assume it is phase A.

SLG Faults, cont'd

Ignoring prefault currents, the SLG fault can be described by the following voltage and current relationships:



$$I_b = 0 \quad \& \quad I_c = 0$$

$$V_a = I_a Z_f$$

The terminal unbalance currents at the fault point can be transferred into their sequence components as follows:

$$\begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix} \longrightarrow I_a^0 = I_a^+ = I_a^- = \frac{I_a}{3}$$

SLG Faults, cont'd

During fault,

$$I_a = \frac{V_a}{Z_f} \quad \text{and} \quad I_a^0 = \frac{V_a}{3Z_f}$$

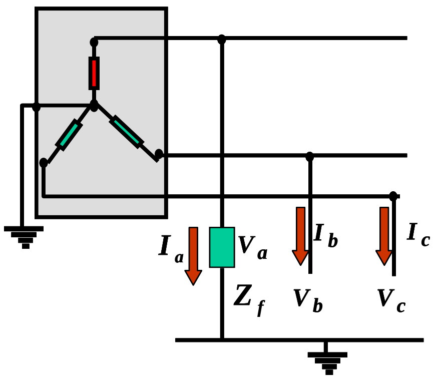
The terminal voltage at phase "a" can be transferred into its sequence components as:

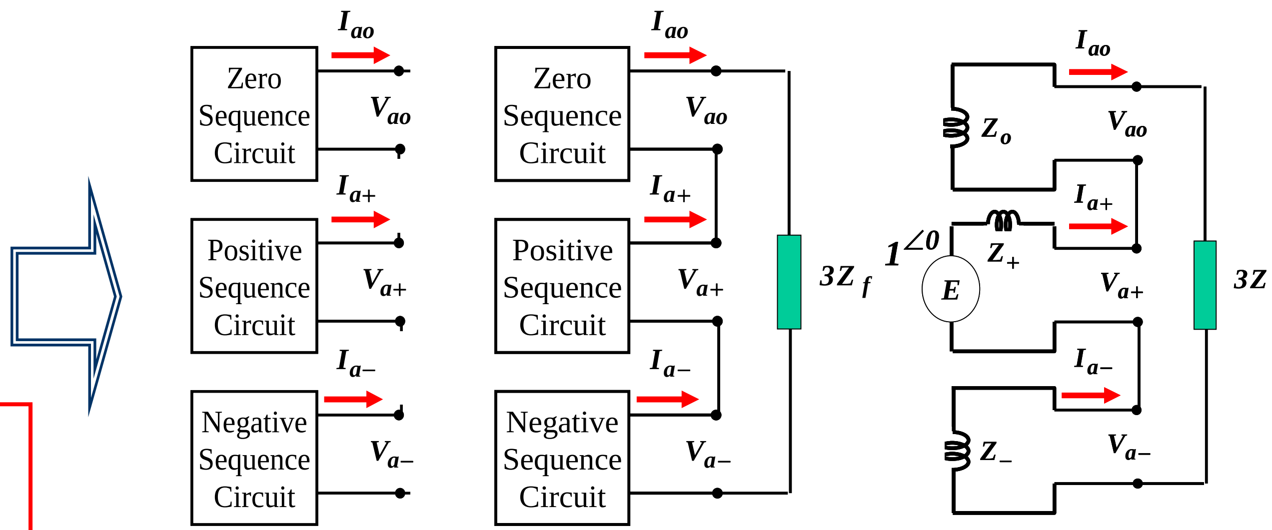
$$V_a = V_a^0 + V_a^+ + V_a^-$$

$$I_a^0 = \frac{V_a}{3Z_f} = \frac{V_a^0 + V_a^+ + V_a^-}{3Z_f}$$

SLG Faults, cont'd

The only way that these two constraint can be satisfied is by coupling the sequence networks in series



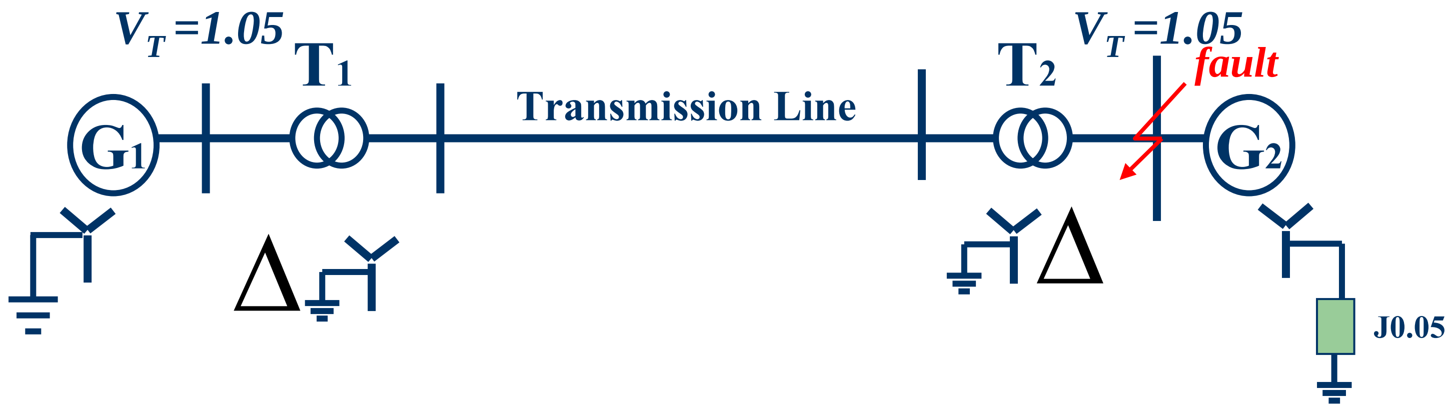


$$I_a^0 = I_a^+ = I_a^-$$

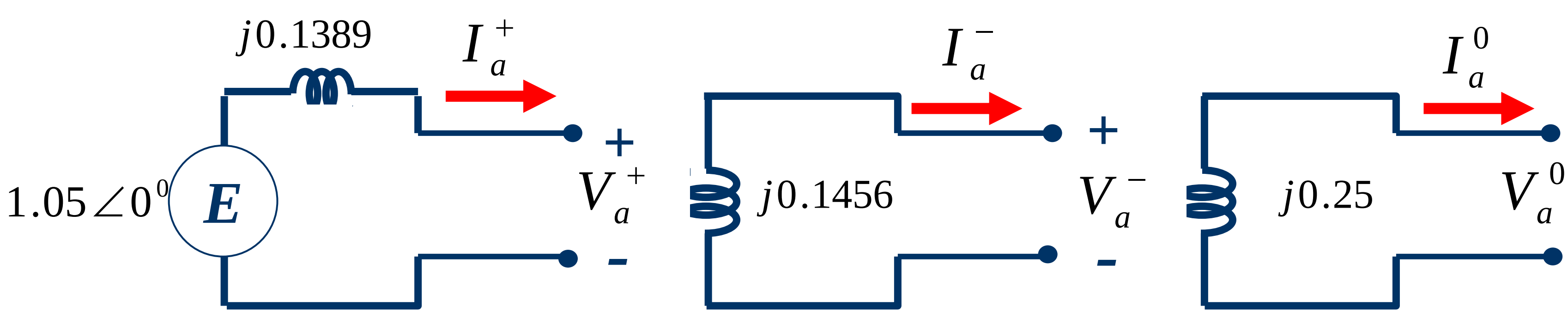
$$I_a^0 = \frac{V_a}{3Z_f} = \frac{V_a^0 + V_a^+ + V_a^-}{3Z_f}$$

Example:

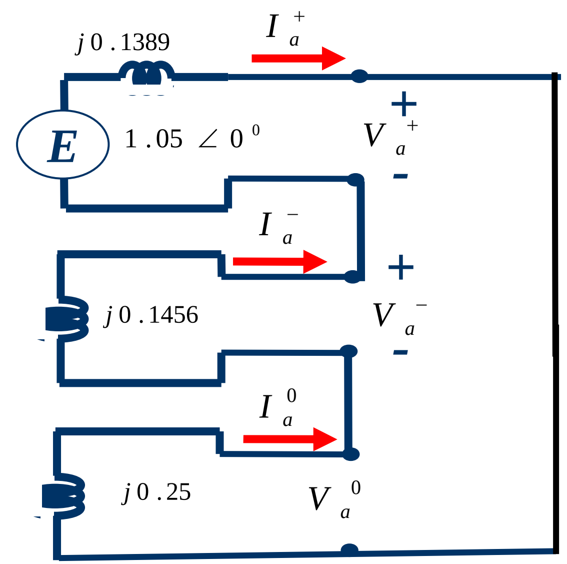
- Consider the following system



- Its Thevenin equivalents as seen from the fault location are:



Example, cont'd



With the sequence networks in series, we can solve for the fault currents

$$I_a^+ = I_a^- = I_a^0 = \frac{1.05\angle 0^0}{j(0.1389 + 0.1456 + 0.25)} = -j1.964$$

$$\mathbf{I} = \mathbf{A}\mathbf{I}_\mathbf{s} \rightarrow I_a = -j5.8 \text{ (of course, } I_b = I_c = 0)$$

NOTE 1: These are the currents at the SLG fault point. The currents in the system during the SLG fault should be computed by analyzing the sequence circuits.

Example, cont'd

From the sequence currents we can find the sequence voltages as follows:

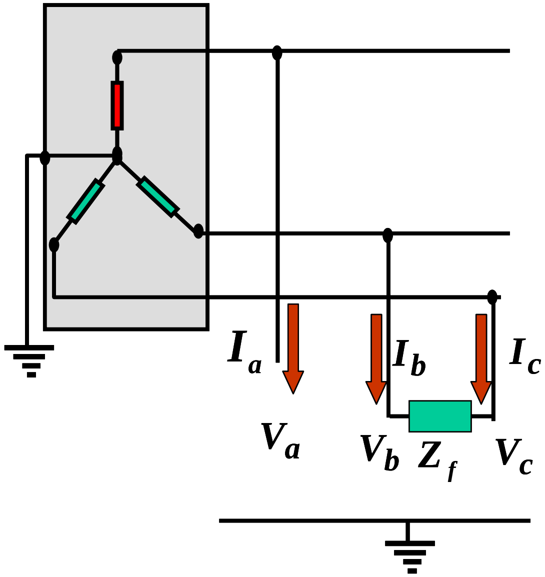
$$V_a^+ = 1.05\angle 0^0 - I_a^+ Z^+, V_a^- = -I_a^- Z^-, V_a^0 = -I_a^o Z^o$$

$$\mathbf{V} = \mathbf{A}\mathbf{V_s} \rightarrow V_a = 0, V_b = 1.166 - j0.178, V_c = 1.166 + j0.178$$

NOTE 2: These are the voltages at the SLG fault point. The voltages at other locations in the system (during the SLG fault) should be computed by analyzing the sequence circuits.

Line-to-Line (LL) Faults

- The second most common fault is line-to-line, which occurs when two of the conductors come in contact with each other. With out loss of generality we'll assume phases b and c.



Current relationships: $I_a = 0$ & $I_b = -I_c$

Voltage relationships: $V_b = V_c + I_b Z_f$

LL Faults, cont'd

Using the current relationships, we get

$$\begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} 0 \\ I_b \\ -I_b \end{bmatrix}$$

Therefore,

$$I_a^0 = 0$$

$$I_a^+ = \frac{1}{3}(\alpha - \alpha^2)I_b$$

$$I_a^- = \frac{1}{3}(\alpha^2 - \alpha)I_b$$

Hence

$$I_a^- = -I_a^+$$

NOTE

$$\alpha = 1\angle 120$$

$$\alpha = -0.5 + j0.866$$

$$\alpha^2 = 1\angle 240$$

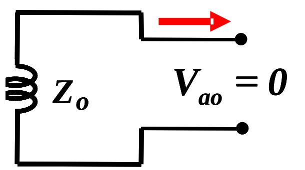
$$\alpha^2 = -0.5 - j0.866$$

$$\alpha^2 - \alpha = -j\sqrt{3}$$

$$\alpha - \alpha^2 = j\sqrt{3}$$

LL Faults, con'td

Therefore, it is obvious that, during a LL Faults *there is no zero sequence components* in the sequence circuit that represents this fault.



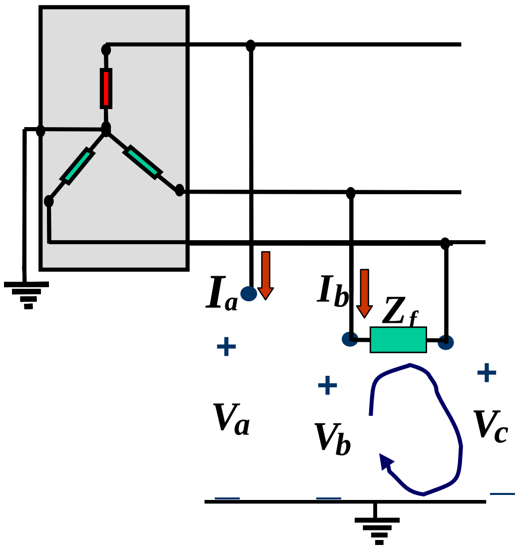
$$I_a^0 = 0$$

$$V_a^0 = -I_a^0 Z^0 = 0$$

During *LL fault*, we have:

$$I_a^0 = 0$$

$$V_b = V_c + I_b Z_f$$



Using the symmetrical components, then:

$$V_b = V_a^0 + \alpha^2 V_a^+ + \alpha V_a^-$$

$$V_c = V_a^0 + \alpha V_a^+ + \alpha^2 V_a^-$$

$$I_b Z_f = Z_f (I_a^0 + \alpha^2 I_a^+ + \alpha I_a^-)$$

LL Faults, con'td

Therefore,

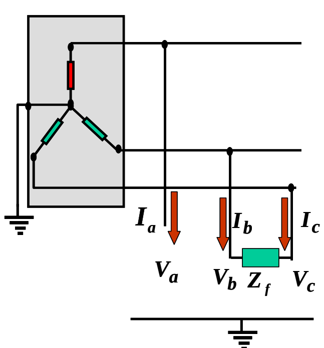
$$V_a^0 + \alpha^2 V_a^+ + \alpha\, V_a^- = V_a^0 + \alpha V_a^+ + \alpha^2 V_a^- + Z_f (I_a^0 + \alpha^2 I_a^+ + \alpha\, I_a^-)$$

Substitute for $I_a^0 = 0$ $\quad V_a^0 = -I_a^0 Z^0 = 0$ $\quad I_{a+} = -I_{a-}$

Then, $\quad (\alpha^2 - \alpha) V_a^+ = (\alpha^2 - \alpha) V_a^- + (\alpha^2 - \alpha) I_a^+ Z_f$

$$\boxed{V_a^+ = V_a^- + I_a^+ Z_f}$$

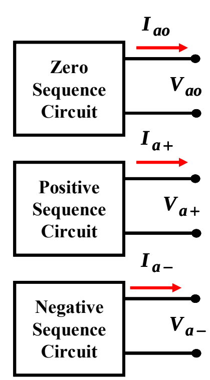
To satisfy $I_a^- = -I_a^+$, $\quad V_a^+ = V_a^- + I_a^+ Z_f$ and $I_a^0 = 0$, the positive and negative sequence networks must be connected in parallel

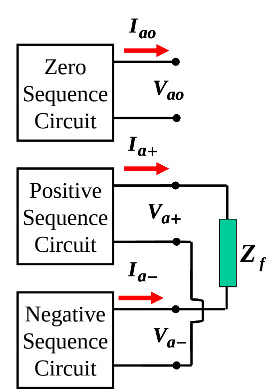


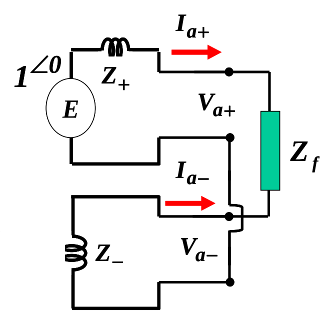
$$I_a^0 = 0$$

$$I_a^+ = -I_a^-$$

$$V_a^+ = V_a^- + I_a^+ Z_f$$

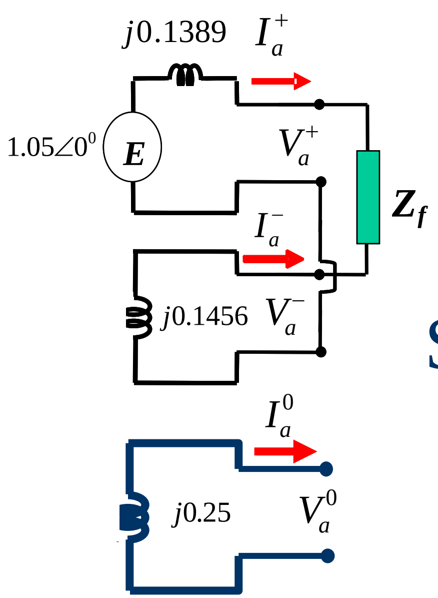


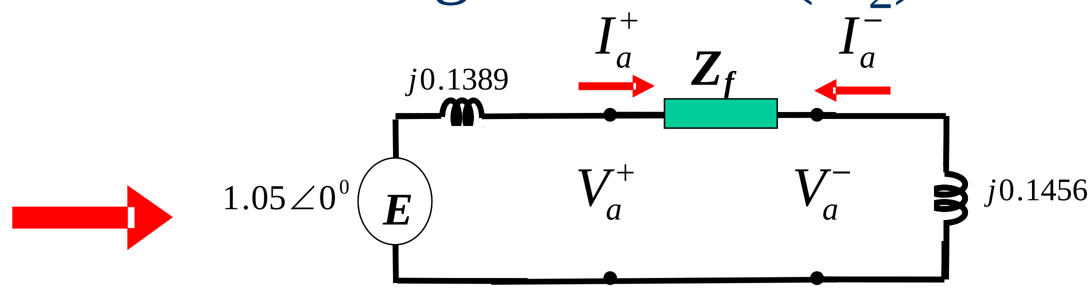




LL Faults-Example

In the previous example, assume a phase-b-to-phase-c fault occurs at the busbar of generator 2 (G_2)





Solving the network for the currents, we get

$$I_a^+ = \frac{1.05\angle 0^0}{j(0.1389+0.1456)} = 3.691\angle -90^0$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} 0 \\ 3.691\angle -90^0 \\ 3.691\angle 90^0 \end{bmatrix} = \begin{bmatrix} 0 \\ -6.39 \\ 6.39 \end{bmatrix}$$

Note: $Z_f = 0$

LL Faults-Example, cont'd

Solving the network for the voltages we get

$$V_a^+ = 1.05\angle 0° - j0.1389 \times 3.691\angle -90° = 0.537\angle 0°$$

$$V_a^- = -j0.1452 \times 3.691\angle 90° = 0.537\angle 0°$$

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.537 \\ 0.537 \end{bmatrix} = \begin{bmatrix} 1.074 \\ -0.537 \\ -0.537 \end{bmatrix}$$

Double Line-to-Ground Faults

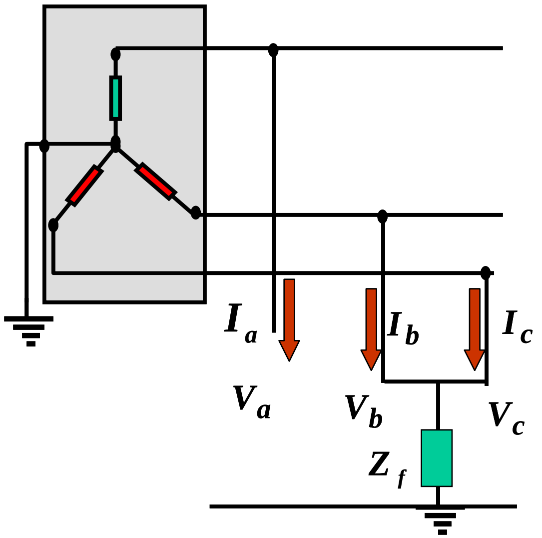
- With a double line-to-ground (DLG) fault two line conductors come in contact both with each other and ground. We'll assume these are phases b and c. The voltage and the current relationships are:

$$V_b = V_c$$

$$V_b = V_c = (I_b + I_c)Z_f$$

$$I_a = 0$$

$$I_a = I_a^0 + I_a^+ + I_a^- = 0$$



Note, because of the path to ground the zero sequence current is no longer zero.

DLG Faults, cont'd

Using the symmetrical components, the terminal voltages are:

$$V_b = V_b^0 + V_b^+ + V_b^-$$

$$V_b = V_a^0 + \alpha^2 V_a^+ + \alpha\, V_a^-$$

$$V_c = V_a^0 + \alpha\, V_a^+ + \alpha^2 V_a^-$$

$$V_b = V_c$$

$$V_a^0 + \alpha^2 V_a^+ + \alpha\, V_a^- = V_a^0 + \alpha\, V_a^+ + \alpha^2 V_a^-$$

$$(\alpha^2 - \alpha) V_a^+ = (\alpha^2 - \alpha) V_a^-$$

$$\boxed{V_a^+ = V_a^-}$$

DLG Faults, cont'd

Using the symmetrical components, the terminal currents are:

$$I_b = I_a^0 + \alpha^2 I_a^+ + \alpha\ I_a^-$$

$$I_c = I_a^0 + \alpha\ I_a^+ + \alpha^2 I_a^-$$

The voltage between fault terminal and ground is:

$$V_b = V_c = (I_b + I_c) Z_f$$

Express the above equation in terms of its symmetrical components:

$$V_a^0 + \alpha^2 V_a^+ + \alpha\ V_a^- = (I_a^0 + \alpha^2 I_a^+ + \alpha\ I_a^- + I_a^0 + \alpha\ I_a^+ + \alpha^2 I_a^-) Z_f$$

Using $V_a^+ = V_a^-$, $1 + \alpha + \alpha^2 = 0$ & $I_a = I_a^0 + I_a^+ + I_a^- = 0$

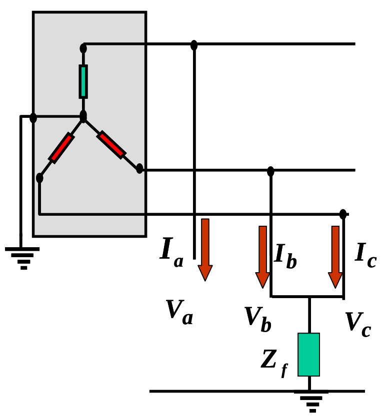
Then

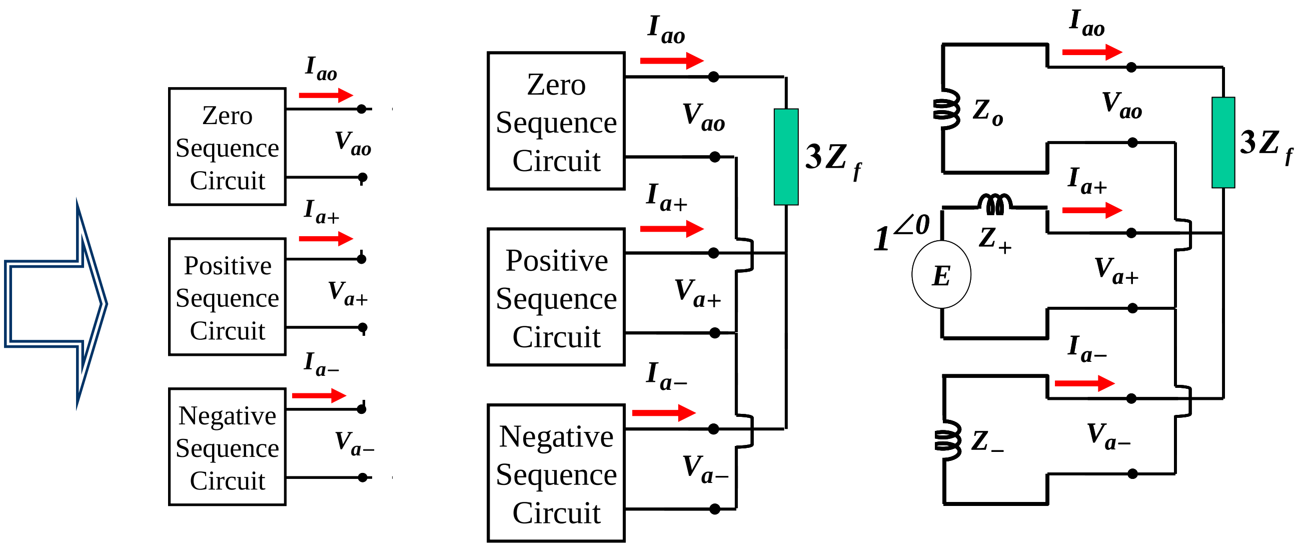
$$V_a^0 - V_a^+ = 3 I_a^0 Z_f$$

DLG Faults, cont'd

To satisfy $I_a = I_a^0 + I_a^+ + I_a^- = 0$, $V_a^+ = V_a^-$ & $V_a^0 - V_a^+ = 3I_a^0 Z_f$

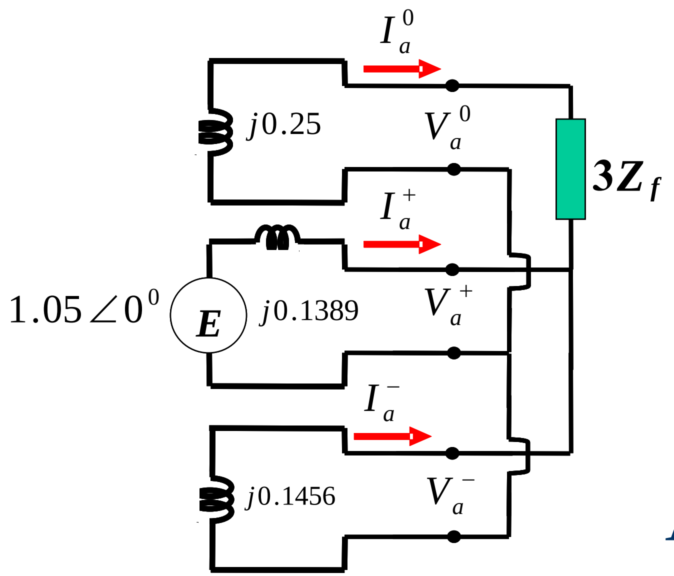
the three symmetrical circuits, during a double line to ground fault, are connected as follows:

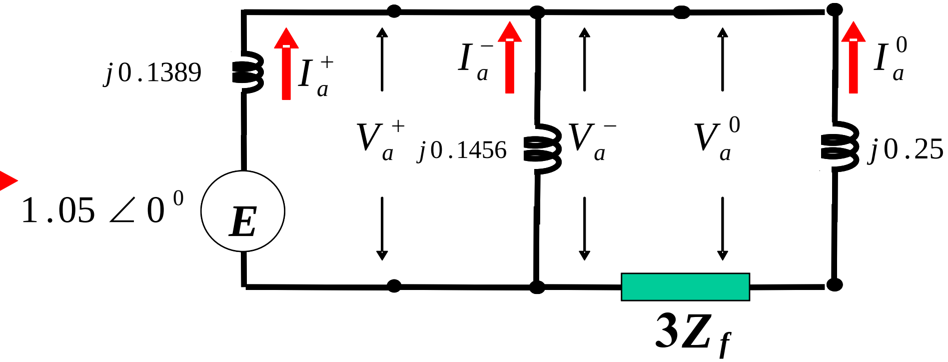




DLG Faults-Example

In previous example, assume DLG fault occurred at G2 bus.



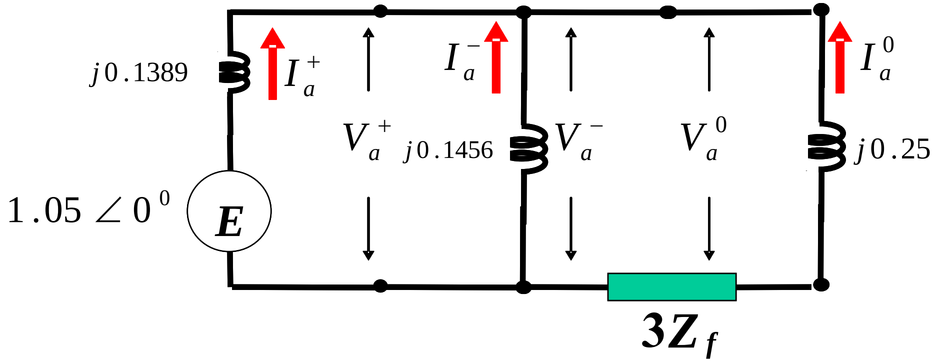


Assuming $Z_f=0$, then

$$I_a^+ = \frac{V_a^+}{Z^+ + Z^- //(Z^0 + 3Z_f)} = \frac{1.05\angle 0^0}{j(0.1389 + j0.092)}$$

$$= 4.547\angle -90^0$$

DLG Faults, cont'd



$$V_a^+ = 1.05 - 4.547\angle -90° \times j0.1389 = 0.4184$$

$$I_a^- = -0.4184 / j0.1456 = j2.874$$

$$I_a^0 = -I_a^+ - I_a^- = j4.547 - j2.874 = j1.673$$

Converting to phase: $I_b = -1.04 + j6.82$

$$I_c = 1.04 + j6.82$$

Unbalanced Fault Summary

- SLG: Sequence networks are connected in series, parallel to three times the fault impedance
- LL: Positive and negative sequence networks are connected in parallel; zero sequence network is not included since there is no path to ground
- DLG: Positive, negative and zero sequence networks are connected in parallel, with the zero sequence network including three times the fault impedance

Generalized System Solution

- Assume we know the pre-fault voltages
- The general procedure is then

1. Calculate Z_{bus} for each sequence
2. For a fault at bus i, the Z_{ii} values are the Thevenin equivalent impedances; the pre-fault voltage is the positive sequence Thevenin voltage
3. Connect and solve the Thevenin equivalent sequence networks to determine the fault current; how the sequence networks are interconnected depends upon the fault type

Generalized System Solution, cont'd

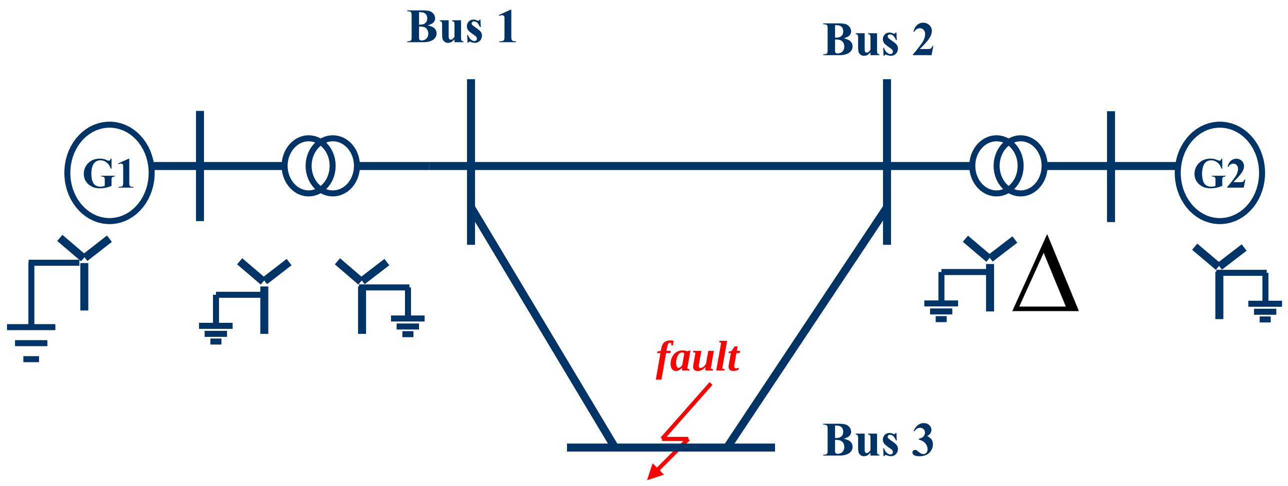
4. Sequence voltages throughout the system are given by

$$\mathbf{V} = \mathbf{V}^{prefault} + Z \begin{bmatrix} 0 \\ \vdots \\ 0 \\ -I_f \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

This is solved for each sequence network!

5. Phase values are determined from the sequence values

Unbalanced System Example



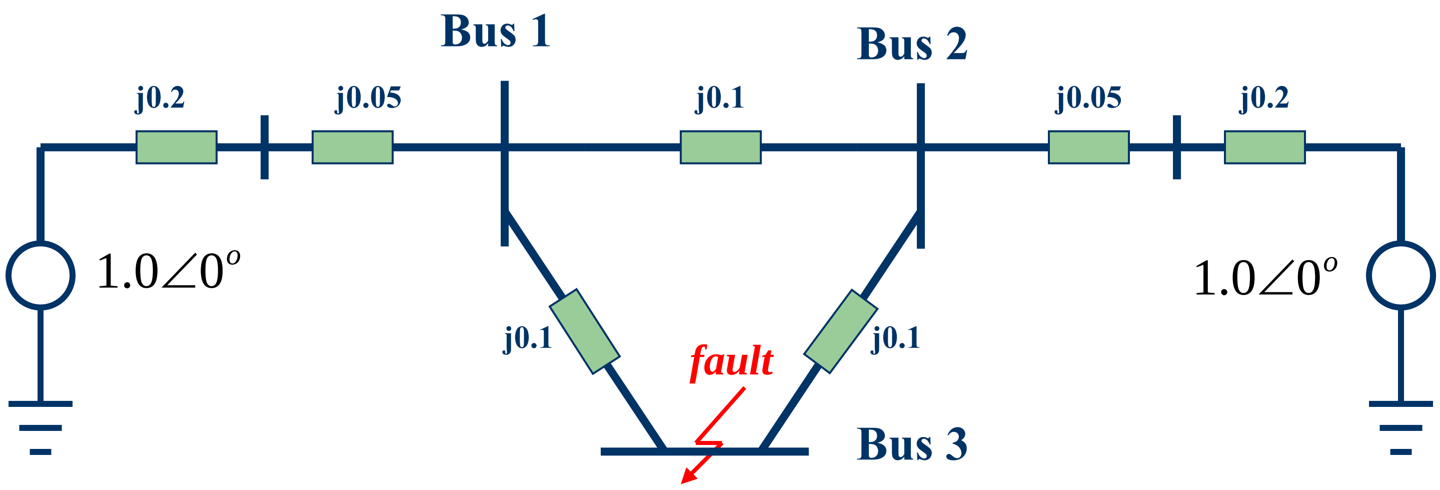
For the generators assume $Z^+ = Z^- = j0.2$; $Z^0 = j0.05$

For the transformers assume $Z^+ = Z^- = Z^0 = j0.05$

For the lines assume $Z^+ = Z^- = j0.1$; $Z^0 = j0.3$

Assume unloaded pre-fault, with voltages =1.0 p.u.

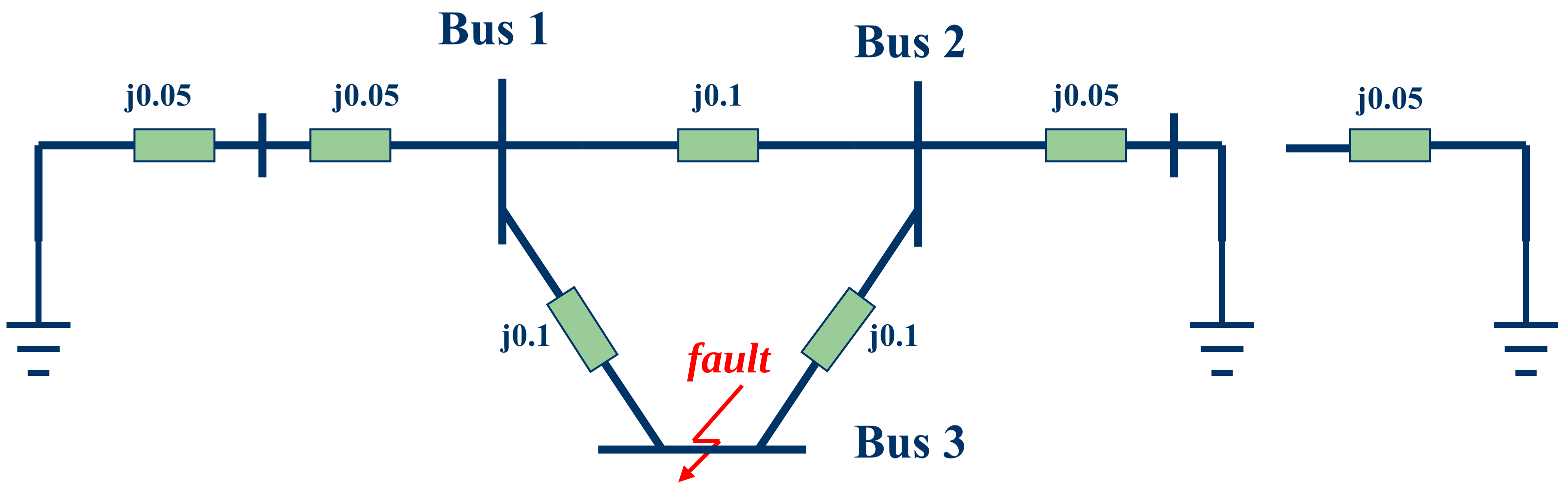
Positive/Negative Sequence Network



$$\mathbf{Y}_{bus}^{+} = j\begin{bmatrix} -24 & 10 & 10 \\ 10 & -24 & 10 \\ 10 & 10 & -20 \end{bmatrix} \qquad \mathbf{Z}_{bus}^{+} = j\begin{bmatrix} 0.1397 & 0.1103 & 0.125 \\ 0.1103 & 0.1397 & 0.125 \\ 0.1250 & 0.1250 & 0.175 \end{bmatrix}$$

Negative sequence is identical to positive sequence

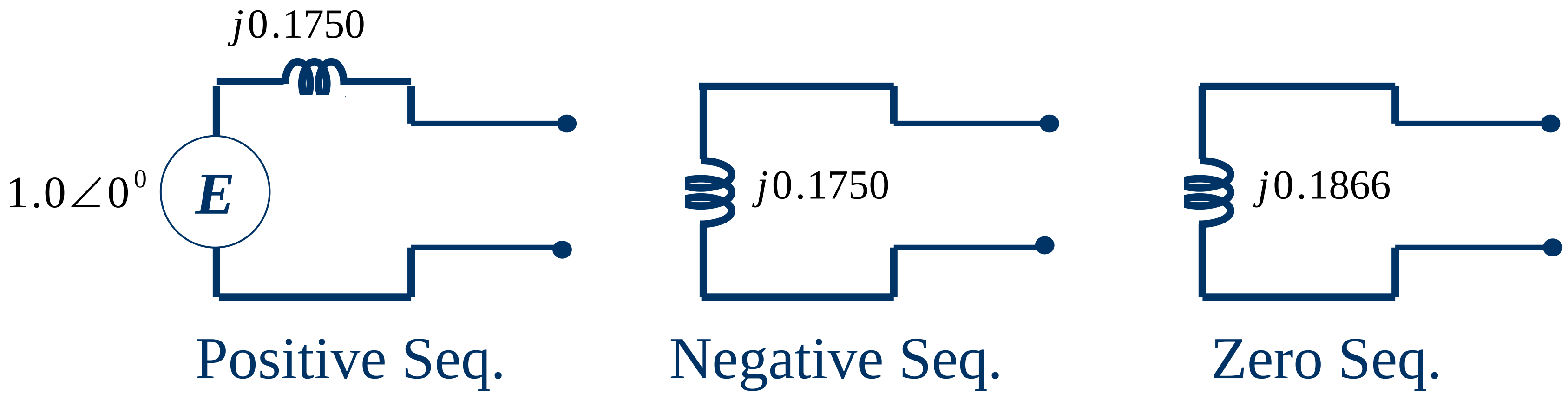
Zero Sequence Network



$$\mathbf{Y}_{bus}^{0} = j\begin{bmatrix} -16.66 & 3.33 & 3.33 \\ 3.33 & -26.66 & 3.33 \\ 3.33 & 3.33 & -6.66 \end{bmatrix} \qquad \mathbf{Z}_{bus}^{0} = j\begin{bmatrix} 0.0732 & 0.0148 & 0.0440 \\ 0.0148 & 0.0435 & 0.0.292 \\ 0.0440 & 0.0292 & 0.1866 \end{bmatrix}$$

For a SLG Fault at Bus 3

The sequence networks are created using the pre-fault voltage for the positive sequence thevenin voltage, and the Z_{bus} diagonals for the thevenin impedances



The fault type then determines how the networks are interconnected

Bus 3 SLG Fault, cont'd

$$I_f^+ = \frac{1.0\angle 0°}{j(0.1750 + 0.1750 + 0.1866)} = -j1.863$$

$$I_f^+ = I_f^- = I_f^0 = -j1.863$$

$$\mathbf{V}^+ = \begin{bmatrix} V_1^+ \\ V_2^+ \\ V_3^+ \end{bmatrix} = \begin{bmatrix} 1.0\angle 0° \\ 1.0\angle 0° \\ 1.0\angle 0° \end{bmatrix} + \mathbf{Z}_{bus}^+ \begin{bmatrix} 0 \\ 0 \\ j1.863 \end{bmatrix} = \begin{bmatrix} 0.7671 \\ 0.7671 \\ 0.6740 \end{bmatrix}$$

$$\mathbf{V}^- = \begin{bmatrix} V_1^- \\ V_2^- \\ V_3^- \end{bmatrix} = \mathbf{Z}_{bus}^- \begin{bmatrix} 0 \\ 0 \\ j1.863 \end{bmatrix} = \begin{bmatrix} -0.2329 \\ -0.2329 \\ -0.3260 \end{bmatrix}$$

Bus 3 SLG Fault, cont'd

$$\mathbf{V}^0 = \begin{bmatrix} V_1^0 \\ V_2^0 \\ V_3^0 \end{bmatrix} = \mathbf{Z}_{bus}^0 \begin{bmatrix} 0 \\ 0 \\ j1.863 \end{bmatrix} = \begin{bmatrix} -0.0820 \\ -0.0544 \\ -0.3479 \end{bmatrix}$$

We can then calculate the phase voltages at any bus

$$\mathbf{V}_3 = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \mathbf{A} \times \begin{bmatrix} -0.3479 \\ 0.6740 \\ -0.3260 \end{bmatrix} = \begin{bmatrix} 0 \\ -0.522 - j0.866 \\ -0.522 + j0.866 \end{bmatrix}$$

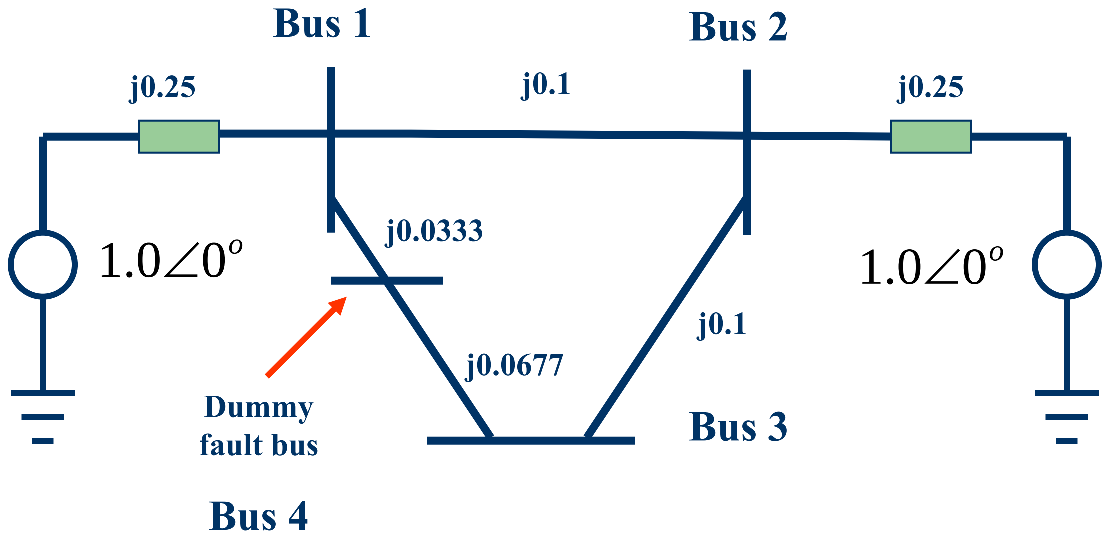
$$\mathbf{V}_1 = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \mathbf{A} \times \begin{bmatrix} -0.0820 \\ 0.7671 \\ -0.2329 \end{bmatrix} = \begin{bmatrix} 0.4522 \\ -0.3491 - j0.866 \\ -0.3491 + j0.866 \end{bmatrix}$$

Faults on Lines

- The previous analysis has assumed that the fault is at a bus. Most faults occur on transmission lines, not at the buses
- For analysis these faults are treated by including a dummy bus at the fault location. How the impedance of the transmission line is then split depends upon the fault location

Line Fault Example

Assume a SLG fault occurs on the previous system on the line from bus 1 to bus 3, one third of the way from bus 1 to bus 3. To solve the system we add a dummy bus, bus 4, at the fault location



Line Fault Example, cont'd

The Y_{bus} now has 4 buses

$$\mathbf{Y}_{bus}^{+} = j\begin{bmatrix} -44 & 10 & 0 & 30 \\ 10 & -24 & 10 & 0 \\ 0 & 10 & -25 & 15 \\ 30 & 0 & 15 & -45 \end{bmatrix}$$

Adding the dummy bus only changes the new row/column entries associated with the dummy bus

$$\mathbf{Z}_{bus}^{+} = j\begin{bmatrix} 0.1397 & 0.1103 & 0.1250 & 0.1348 \\ 0.1103 & 0.1397 & 0.1250 & 0.1152 \\ 0.1250 & 0.1250 & 0.1750 & 0.1417 \\ 0.1348 & 0.1152 & 0.1417 & 0.1593 \end{bmatrix}$$