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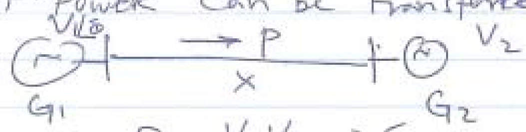
Power System Transient Stability

Introduction:

- Power system stability refers to the ability of the various synchronous machines in the system to remain in synchronism, or stay in step, with each other following a disturbance (sudden changes in the system variables). $i \uparrow$ or Load $\uparrow\downarrow$.
- – synchronism requires that (for 2 pole machines), the rotors turn at exactly the same speed.
- Loss of synchronism results in a condition in which NO NET POWER can be transferred between the machines



$$P = \frac{V_1 V_2}{X} \sin\delta$$

$f = \frac{P\,n}{120}$ speed

- Stability may be classified to:

→ Steady-state stability: Refers to the ability of the various machines to REGAIN & MAINTAIN synchronism after a small & slow disturbance, such as gradual change in load.

→ Transient stability: is stability after a sudden large disturbance such as a fault, loss of a generator, and a sudden load change.

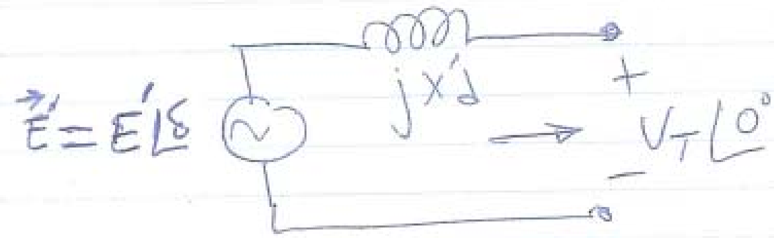
The stability problem:

Stability studies evaluates the impact of disturbances on the electro-mechanical behavior of the power system.

Therefor, we need to develop both electrical & mechanical models ~~of~~ for the synch. generators.

Generator Electric Model:-

The synch. generator is modeled as a voltage source behind the direct-axis transient reactances. The voltage magnitude is fixed, but its angle changes according to the mechanical dynamics.

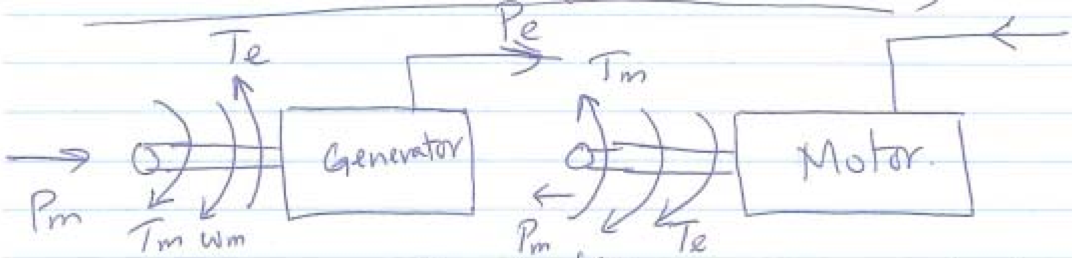


Assumptions:

1- The machine is operating under balanced 3 ϕ +ve sequence conditions.
2- Machine excitation is constant (DC flux is constant)
3- Machine losses, saturation, & saliency are neglected.

$$P_e(\delta) = \frac{|V_T||E'|}{X'_d}\sin\delta$$

Mechanical Model (Swing equation)



Schematic description of powers & torques in synchronous machines.

The rotor motion of synch. machine is governed by Newton's 2nd law

$$J\alpha_m = T_m - T_e = T_a$$

Where,

J = total moment of inertia of the rotating masses, $Kg\,m^2$

α_m = rotor angular acceleration, rad/s^2

T_m = mechanical input torque (N-m). [+ve T_m means generator]

T_e = equivalent electric torque accounts for 3ϕ electric output power of generator + electric losses (N-m).

T_a = net accelerating torque (N-m).

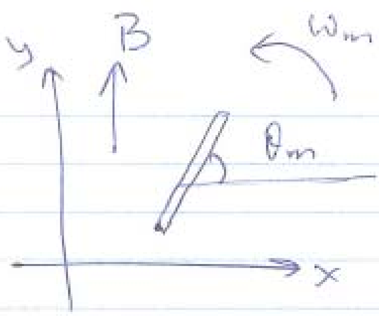
The rotor angular acceleration can be written as

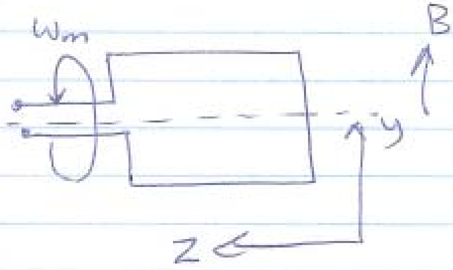
$$\alpha_m = \frac{d\omega_m}{dt} = \frac{d^2\theta_m}{dt}$$

$$J\frac{d^2\theta_m}{dt} = T_m - T_e = T_a$$

ω_m = rotor angular velocity, rad/s

θ_m = rotor angular position w.r.t. a stationary axis, rad.



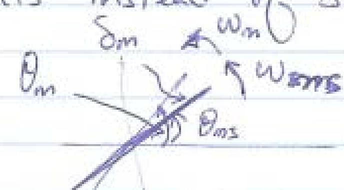


T_m & T_e are (+ve) for generator action.
In steady-state $T_m = T_e \Rightarrow T_a = 0. \Rightarrow \alpha_m = 0$
$\Rightarrow$ rotor ~~not~~ will rotat at a constant speed (synch.)

If $T_m > T_e \Rightarrow T_a$ is (+ve) $\Rightarrow \alpha_m$ is (+ve)
rotor speed $\uparrow$

If $T_m < T_e \Rightarrow T_a$ is (−ve) $\Rightarrow$ rotor speed $\downarrow$

It is convenient to measure θ_m w.r.t. synchronously rotating reference axis instead of stationary axis.



$\therefore \quad \theta_m = \omega_{ms} t + \delta_m.$

ω_{ms} = synch. angular velocity of the rotor, rad/s
δ_m = rotor angular position w.r.t. a synchronously rotating reference, rad.

$$\frac{d\theta_m}{dt} = \omega_{ms} + \frac{d\delta_m}{dt} \Rightarrow \frac{d^2\theta}{dt^2} = \frac{d^2\delta_m}{dt^2}$$

$$\therefore \quad J\frac{d^2\theta_m}{dt} = J\frac{d^2\delta_m}{dt^2} = T_m - T_e = T_a$$

Multiply by ω_m,

$$\boxed{J\omega_m \frac{d^2\delta_m}{dt^2} = P_m - P_e = P_a}$$ Swing eqn.

where,

$P_m = T_m\omega_m$ = mechanical power supplied by the prime mover minus mech. losses (W)

$P_e = T_e\omega_m$ = electrical power output + electric losses (W)

$P_a = P_m - P_e$ = net accelerating power (W).

The swing equation describes how the rotor moves, or swings, with respect to the synchronously rotating reference frame in the presence of a disturbance, that is, when the net accelerating power is NOT zero.

Divide the above equation by S_{rated} (3ϕ VA)

$$\frac{J\omega_m}{S_{rated}} \frac{d^2\delta_m}{dt^2} = \frac{P_m - P_e}{S_{rated}} = \frac{P_a}{S_{rated}} = P_{m,pu} - P_{e,pu} = P_{a_{pu}}$$

It is convenient to work with a normalized interia constant (H).

$$H = \frac{\text{Stored Kinetic energy at synch. speed}}{\text{generator VA ratings}}$$

$$= \frac{\frac{1}{2} J\omega_{ms}^2}{S_{rated}} \quad \text{Joule / VA. (pu-second)}$$

$$\therefore \left(\frac{2H\omega_m}{\omega_{ms}^2}\right) \frac{d^2\delta_m}{dt^2} = P_{m,pu} - P_{e,pu} = P_{a,pu}$$

Define, $\omega_{pu} = \frac{\omega_m}{\omega_{ms}}$ (Most of the time $\omega_{pu} = 1.0$)

$$\boxed{\therefore \frac{2H}{\omega_{ms}} \omega_{,pu} \frac{d^2\delta_m}{dt^2} = P_{m,pu} - P_{e,pu} = P_{a,pu}}$$

For a synchronous generator with P poles, the electrical angular acceleration α, electrical radian frequency ω, and power angle δ are,

$$\alpha = \frac{P}{2}\alpha_m \quad , \quad \omega = \frac{P}{2}\omega_m \quad , \quad \delta = \frac{P}{2}\delta_m .$$

$$\omega_s = \frac{P}{2}\omega_{ms}$$

$$\therefore \frac{2H}{\omega_s} \omega_{pu} \frac{d^2\delta}{dt^2} = P_{m,pu} - P_{e,pu} = P_{a,pu}$$

per-unit swing equation determines the rotor dynamics in transient stability.

The above equation is a 2nd order differential equation which can be written as 2 1st order equations.

$$\boxed{\begin{aligned} \frac{d\delta}{dt} &= \omega - \omega_s \\ \frac{2H}{\omega_s}\,\omega\,\frac{d\omega}{dt} &= P_m - P_e = P_a \end{aligned}}$$

$$\omega_m = \frac{d\theta_m}{dt} = \omega_{ms} + \frac{d\delta_m}{dt}$$

or

$$\omega = \omega_s + \frac{d\delta}{dt}$$

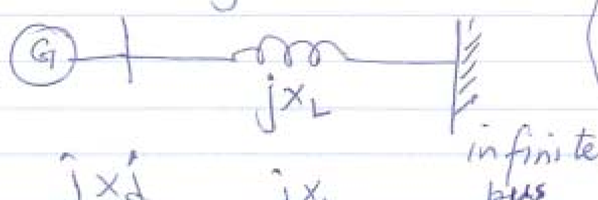
The above equations are non linear since P_e is function of δ

$$P_e = \left(\frac{E_a E_T}{X} \sin\delta\right)$$

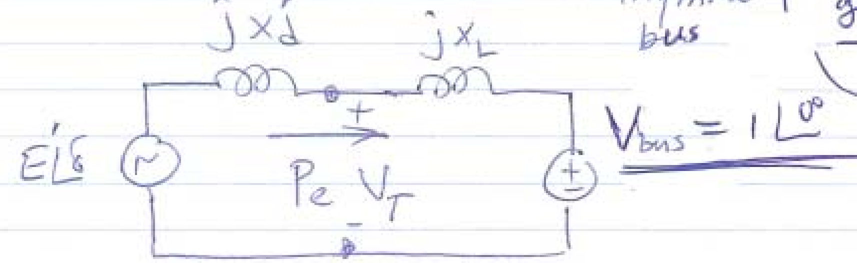
Single Machine Infinite Bus:

The models & the swing equation will be applied to a simple system of a synchronous generator connected to an infinite bus (a bus with fixed Voltage, angle and constant frequency).

- A complete stability analysis of a power system is an extensive & complicated task.
- To understand the transient stability problem, we'll consider the following system



(Infinite bus represent the rest of P.S seen by the generator terminals)



The real power delivered by the synchronous generator to the infinite bus

$$P_e = \frac{E' V_{bus}}{X_{eq}} \sin\delta \qquad (X_{eq} = X'_d + X_L)$$

at $\delta = 90° \Rightarrow P_e^{max} = \frac{E' V_{bus}}{X_{eq}} \Rightarrow \boxed{P_e = P_e^{max} \sin\delta}$

Note: during transient disturbance, both E' & V_{bus} are considered constant.

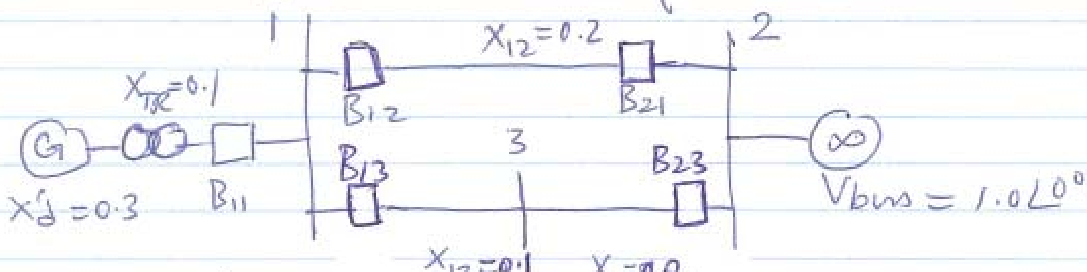
$$\Rightarrow P_e = f(\delta).$$

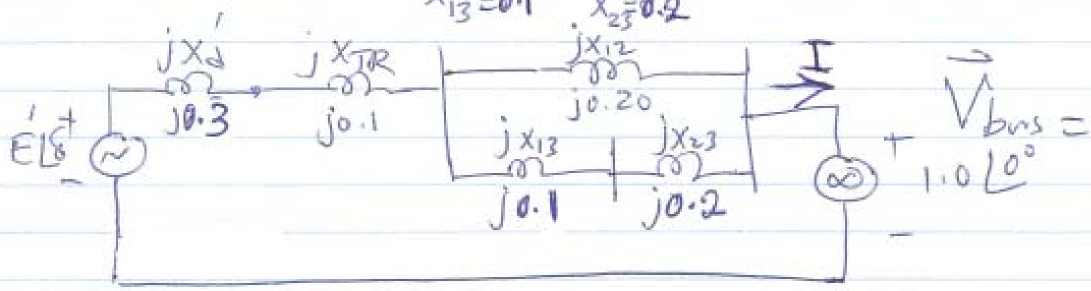
<u>Ex1:</u> In the following figure, all the reactances are in p.u. on a common bas.

If the infinite bus receives 1.0 pu real power at 0.95 pf lagging determine:

(a) the internal voltage of generator (E')

(b) P_e as a function of δ.





(a) $X_{eq} = j0.3 + j0.1 + [j0.2 \,||\, (j0.1 + j0.2)] = j\underline{0.52}$ pu

$$I = \frac{P}{V_{bus}(Pf)} \angle -\cos^{-1} Pf = \frac{(1.0)}{(1.0)(0.95)} \angle -\cos^{-1}(0.95)$$

$$= 1.05263 \angle -18.195^\circ$$

$$\vec{E}' = E'\angle\delta = \bar{V}_{bus} + jX_{eq}\bar{I} = 1.0\angle 0^\circ + (j0.52)(1.053\angle -18.^\circ)$$
$$= 1.171 + j0.52 = 1.2812 \angle 23.95^\circ$$

(b) $$P_e = \frac{E' V_{bus}}{X_{eq}} \sin\delta = \frac{(1.2812)(1.0)}{0.52} \sin\delta = 2.4638 \sin\delta \ \underline{pu}$$

Single Machine Infinite bus (SMIB):

SMIB - Qualitative Analysis:

In order to make a Qualitative analysis of one machine connected to an infinite bus, assume $\omega_{pu} = 1.0$. (neglect the damping factor).

$$\frac{2H}{\omega_s}\frac{d^2\delta}{dt^2} = P_m - P_e$$

or

$$\frac{2H}{\omega_s}\frac{d^2\delta}{dt^2} = P_m - P_e^{max}\sin\delta \quad \text{(non-linear system)}$$

SMIB - Equilibrium points:

Of fundamental importance to a non-linear system is its equilibrium points, i.e, the points in state space where all time derivatives vanish.

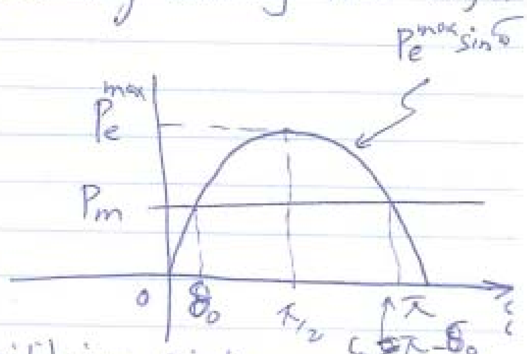
Equilibrium points are determined by setting the right-hand side to zero.

$$P_m - P_e^{max}\sin\delta = 0$$

or

$$P_m = P_e^{max}\sin\delta$$

$$\boxed{P_m^0 = P_e^{max}\sin\delta_0}$$

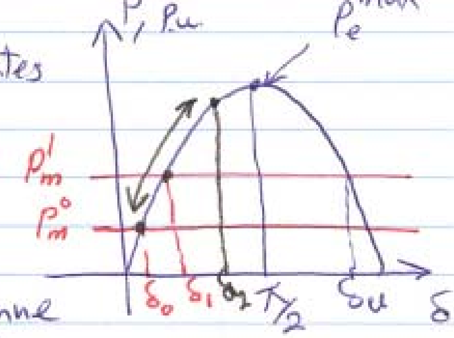


Notes:

1. If $P_m < P_e^{max}$ $\Rightarrow$ $\underline{2}$ equilibrium points $\Rightarrow$ δ_0 and $\pi - \delta_0$ for $0 \le \delta \le \pi$.
2. If $P_m = P_e^{max}$ $\Rightarrow$ $\underline{\text{one}}$ equilibrium point, i.e $\delta_0 = \pi/2$
3. If $P_m > P_e^{max}$, there are no equilibrium points

It is clear that if $P_m > P_e^{max}$, the systems is unstable. In this case, the rotor will accelerate until the protections trip the generator and the turbine.

Suppose the generator initially operated in steady state at $P_e = P_m = P_m^0$ and $\delta = \delta_0$, when a step change in P_m from P_m^0 to P_m^1 at $t=0$.



$\Rightarrow$ $P_m - P_e = (+ve) \Rightarrow$ rotor accelerates and δ increases.

when δ reaches δ_1, $P_e = P_m^1$ & $d^2\delta/dt^2$ becomes zero. But $\frac{d\delta}{dt}$ is still (+ve) and δ continue to increase overshooting its final steady state operating point and reaches δ_2

when $\delta > \delta_1 \Rightarrow P_m < P_e \Rightarrow P_m - P_e = (-ve)$
$\Rightarrow$ the rotor decelerates and δ swings back towards δ_1 and would continually oscilate around δ_1.

<u>Note:</u>

1- Damping (omitted from the swing eqn.) due to mechanical and electric losses causes δ to settle down to its final steady state operating point δ_1.
2- If the power angle exceeds δ_u, then $P_m > P_e$ $\Rightarrow$ rotor accelerates $\Rightarrow$ further increase in δ $\Rightarrow$ loss of stability.
3- For system to be stable $0 < \delta \le \frac{\pi}{2}$

Transient Stability Analysis:-

For transient stability analysis, we need to consider 3 systems

1- prefault - before the fault occurs, the system is assumed to be on equilibrium point.

2- Faulted - the fault changes the system eqns moving the systems away from its equilibrium point

3- postfault - after fault is cleared, the system hopefully return to a new stable operating point.

Transient stability ~~problems~~ solution Methods:

There are 2 methods for solving the transient stability problem

1- Direct or Energy method: This method is Known as EQUAL AREA CRITERIA.
- Can be only applicable for 2 bus system.
- it is used to provide an intuitive insight into the transient stability problem.

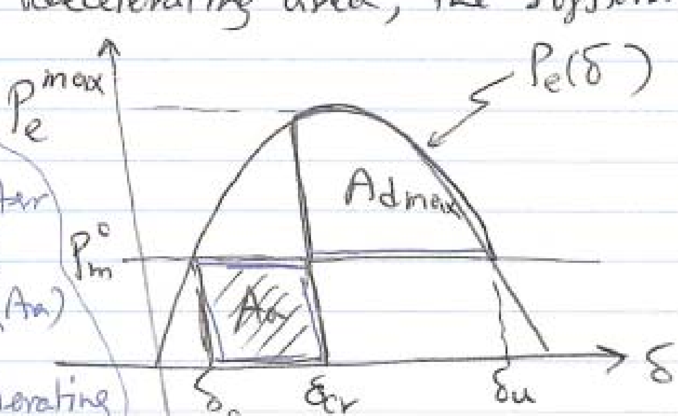
2- Numerical Integration: based on Euler's meth
- Most common for large systems.
- During the fault and after the fault, the power system differential equations are solved usin numerical methods.

2 Direct Method:

Equal-Area Stability Criterion:

If $A_a < A_{dmax}$, the system is transient-stable.
In words, if the accelerating area is less than the maximum decelerating area, the system is transient stable.

The system will be stable after the fault if the accelerating area (A_a) is less than the maximum decelerating area



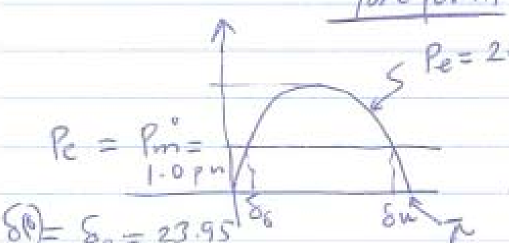
δ_{cr} = critical fault clearing angle = the angle at which the fault is cleared. $\Rightarrow$ ~~corresponds to t_{cr}~~.

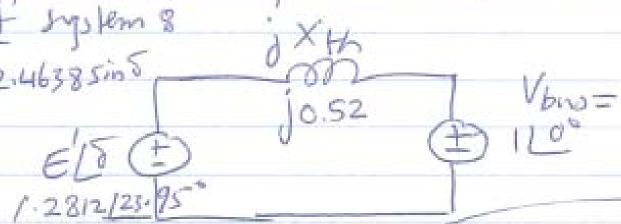
t_{cr} = critical clearing time = the longest ~~time~~ fault duration allowable for stability.

The goal of the equal area criteria is to try to determine whether a system is stable or not without having to completely integrate the system response.

EX2: In previous example, assume a temporary 3-ϕ fault occurred on line 1-3. 3 cycles later the fault is cleared by itself. Determine whether the system is stable or not & determine the max power angle. Assume $H = 3$ pu-sec, $\omega_{pu} = 1.0$ & P_m remains constant through the disturbance.

Solution: From previous example (EX1), the simplified prefault system:





$\delta(0) = \delta_0 = 23.95$

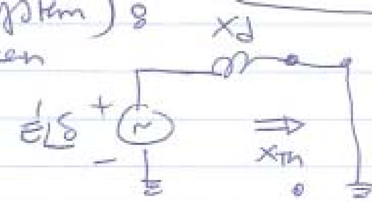
$\omega = \omega_s \Rightarrow \frac{d\delta(0)}{dt} = \omega_s - \omega_s = 0$

Assume fault happened at $t=0$

During the fault (Faulted system):

Find the Thevinen equivalent seen by the generator terminals

$X_{Th} = 0$ & $V_{Th} = 0$.



1 cycle = 1/60 sec

During the fault, no power can be transferred from the generator to the system.

$P_e = 0$

3 cycle = $3 \times (1/60) = 0.05$ sec

$0 \le t \le 0.05$ sec

$$\therefore \frac{2H}{\omega_s} \frac{d^2\delta(t)}{dt} = P_m$$

Integrate twice, with initial condition $\delta(0) = \delta_0$ & $d\delta(0)/dt = 0$.

$$d\delta(t)/dt = \frac{\omega_s P_m}{2H} t + 0. \leftarrow \frac{d\delta(0)}{dt}$$

or

$$\delta(t) = \frac{\omega_s P_m}{4H} t^2 + \delta_0$$

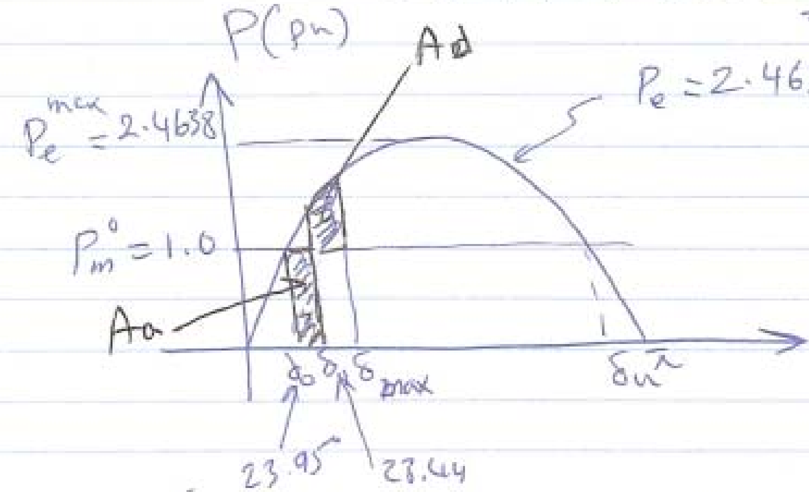
Post Fault system: In this particular example the system regain its structure.

~~at~~ $t = 3$ cycles $= 0.05$ sec.

$\delta_0 = 23.95^\circ = 0.4179$ rad

$$\delta_1 = \delta(0.05s) = \frac{2\pi 60}{4(3)}(0.05)^2 + 0.4179$$

$$= 0.4964 \text{ rad} = \underline{28.44^\circ}$$



$$A_a = \int_{\delta_0}^{\delta_1} P_m \, d\delta = \int_{\delta_0}^{\delta_1} 1 \, d\delta = (\delta_1 - \delta_0) = 0.4964 - 0.417$$
$$= \underline{0.0785}$$

At $t = 0.05$ sec, the fault & P_e instantaneously increased from zero to sinusoidal curve. δ continu
to increase until

$$A_d = A_a$$

or

$$A_d = \int_{\delta_1}^{\delta_{max}} (P_{max} \sin\delta - P_m)\, d\delta = A_a = 0.0785$$

$$= \int_{0.4964}^{\delta_{max}} (2.4638 \sin\delta - 1.0)\, d\delta = 0.0785$$

$$= 2.4638\left[\cos(0.4964) - \cos\delta_{max}\right] - (\delta_{max} - 0.4964) = 0.07$$

or $2.4638 \cos\delta_{max} + \delta_{max} = 2.5843$

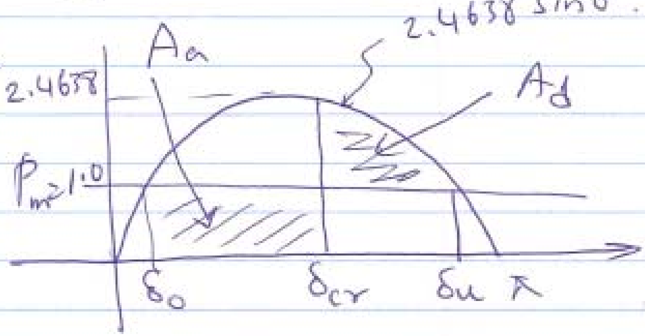
the above equation is non-linear and can be solved interatively

$\therefore$ max angle, $\delta_{max} = 0.7003$ rad $= \underline{40.12^\circ}$

since δ_{max} does not exceeds $\delta_u = 180 - \delta_o = 156.05^\circ$, stability is maintained.

Ex 3

Assuming the temp. s.c. in previous example lasts longer than 3 cycles, calculate the critical clearing time. (t_{cr}).



Find δ_{cr} and hence t_{cr} using the solution of the swing equation.

$$A_a = A_d$$

$$\int_{\delta_o}^{\delta_{cr}} P_m \, d\delta = \int_{\delta_{cr}}^{\delta_u} (P_{max}\sin\delta - P_m)\, d\delta$$

$$\int_{0.4179}^{\delta_{cr}} 1.0 \, d\delta = \int_{\delta_{cr}}^{2.7236} (2.4638 \sin\delta - 1.0)\, d\delta$$

solving for δ_{cr}

$$(\delta_{cr} - 0.4179) = 2.4638\left[\cos\delta_{cr} - \cos(2.7236)\right] - (2.7236 - \delta_{cr})$$

$$2.4638 \cos\delta_{cr} = 0.05402$$

$$\Rightarrow \quad \delta_{cr} = 1.5489 \text{ rad} = \underline{\underline{88.74^\circ}}$$

From the solution to the swing equation given in previous example.

$$\delta(t) = \frac{\omega_s P_m}{4H} t^2 + \delta_0$$

solving, $\quad t = \sqrt{\frac{4H}{\omega_s P_m}\left(\delta(t) - \delta_0\right)}$

using $\delta_{cr} = 1.5489$ & $\delta_0 = 0.4179$ rad

$$t_{cr} = \sqrt{\frac{12}{(2\pi 60)(1.0)}(1.5489 - 0.4179)} = 0.1879 \text{ sec}$$

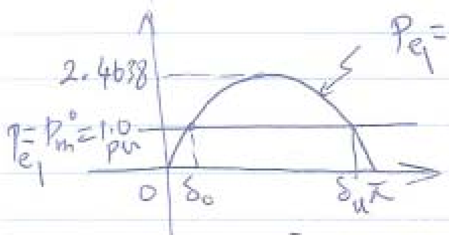
$$= \underline{\underline{11.38 \text{ cycl}}}$$

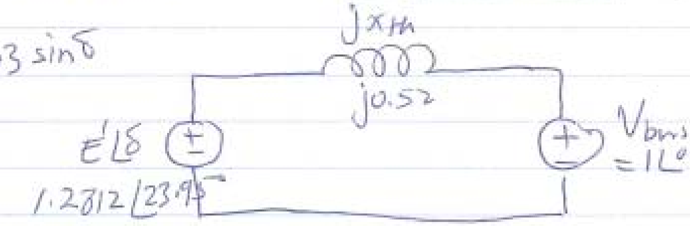
If the fault is cleared before $t = t_{cr} = 11.38$ cycle, stability is maintained. Otherwise, the generator goes out of synchronism with the infinite bus; that is stability is lost.

Ex 4: The synch. generator is previous example is initially operating in steady-state conditions given in Ex.3. when a permanent 3φ-to-ground bolted short circuit occurs on line 1-3 at bus 3. The fault is cleared by opening the circuit breakers at the end of line 1-3 & line 2-3. These CB, then remain open. Calculate the ~~clearing~~ critical clearing time using Equal area creterie ~~(...)~~.
Assume $H = 3.0$ pu-sec, $P_m = 1.0$ pu & $\omega_{pu} = 1.0$ φ

Solution :-

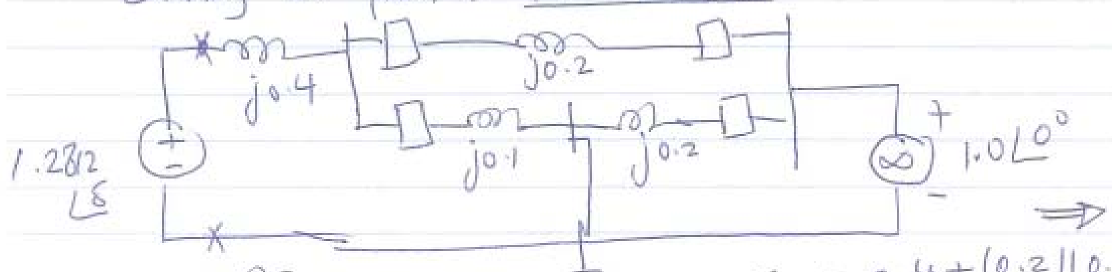
We first need to determine the pre-fault values.
from EX1 :





$\delta_0 = 23.95°$

During the fault : Faulted system.



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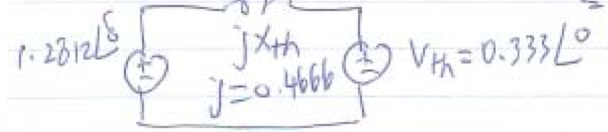


$X_{th} = 0.4 + (0.2 \| 0.1) = j0.$

$V_{th} = 1.0\angle 0° \left[\frac{X_{13}}{X_{13} + X_{12}}\right] = 1\angle 0° \left(\frac{0.}{0.}\right.$

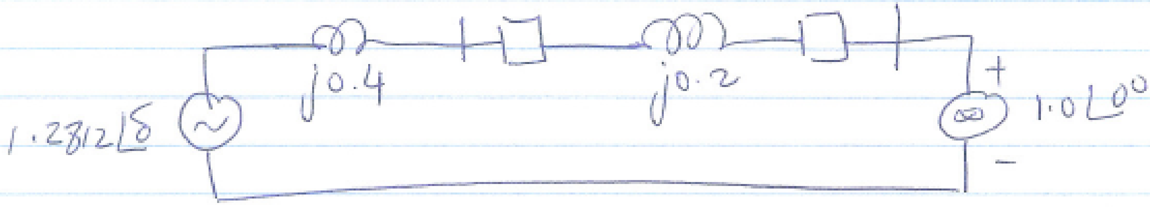
$= 0.333\angle 0°$

the power delivered by the generator during the fault

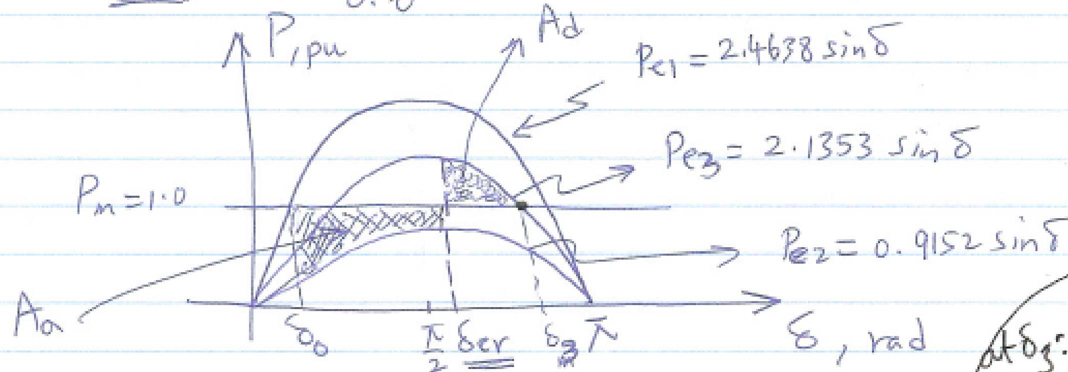
$$P_{e2} = \frac{E' V_{th}}{X_{th}} \sin\delta = \frac{(1.2812)(0.333)}{0.46666} \sin\delta$$

$$\underline{P_{e2} = 0.9152 \sin\delta} \quad pu$$

<u>post-fault~~ed~~ network</u>



$$\underline{P_{e3}} = \frac{(1.2812)(1.0)}{0.6} \sin\delta = \underline{2.1353 \sin\delta}$$



$$A_1 = \int_{\delta_0}^{\delta_{cr}} (P_m - P_{e2})\, d\delta = A_2 = \int_{\delta_{cr}}^{\delta_3} (P_{e3} - P_m)\, d\delta$$

$$\int_{0.4179}^{\delta_{cr}} (1.0 - 0.9152 \sin\delta)\, d\delta = \int_{\delta_{cr}}^{2.6542} (2.1353 \sin\delta - 1.0)\, d\delta$$

at δ_3: $P_m = P_{e3}(\delta_3)$ $\Rightarrow \delta_3 = 2.6542$

solve for $\delta_{cr} = 1.9812$ rad $= 113.5°$

If the fault cleared before $\delta = \overline{\delta_{cr}} = 113.5°$, stability is maintained.