

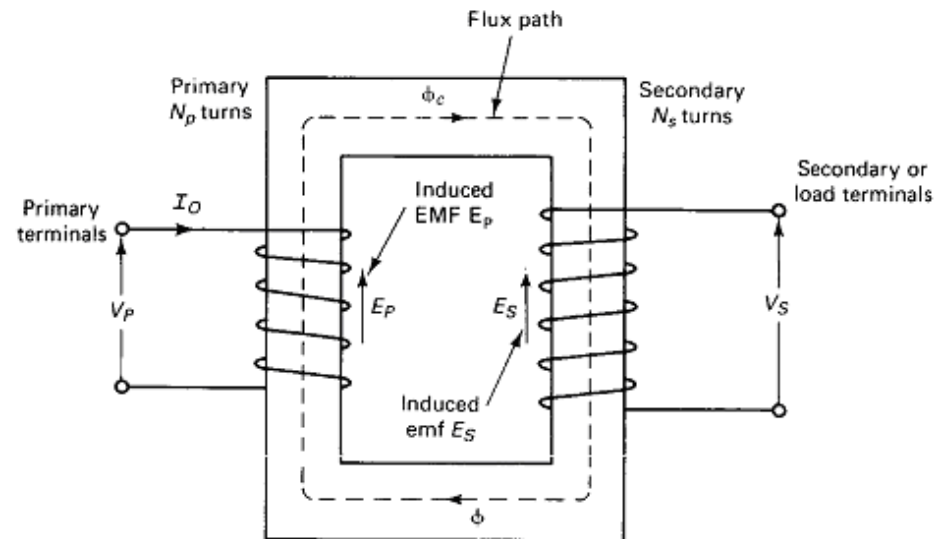
Lecture Notes

ELEC-B7

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Transformers

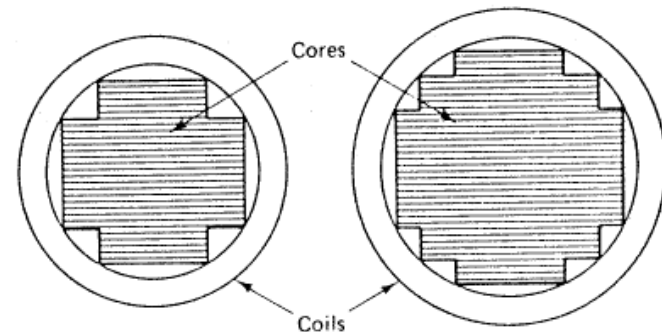
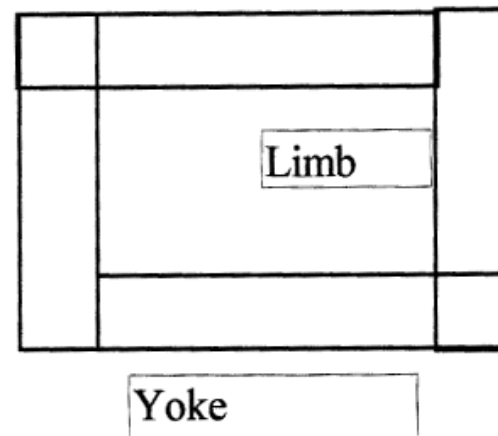
- **A transformer is a device which transfers electrical energy (power) from one voltage level to another voltage level.**
- **Unlike in rotating machines, there is no energy conversion.**
- **A transformer is a static device and all currents and voltages are AC.**
- **The transfer of energy takes place through the magnetic field.**



Transformer Construction

Iron Core

- **The iron core is made of thin laminated silicon steel (2-3 % silicon)**
- **Pre-cut insulated sheets are cut or pressed in form and placed on the top of each other .**
- **The sheets are overlap each others to avoid (reduce) air gaps.**



Transformer Construction Winding

- The winding is made of copper or aluminum conductor, insulated with paper or synthetic insulating material
- The windings are manufactured in several layers, and insulation is placed between windings.
- The primary and secondary windings are placed on top of each others but insulated by several layers of insulating sheets.
- The windings are dried in vacuum and impregnated to eliminate moisture.

Small transformer winding

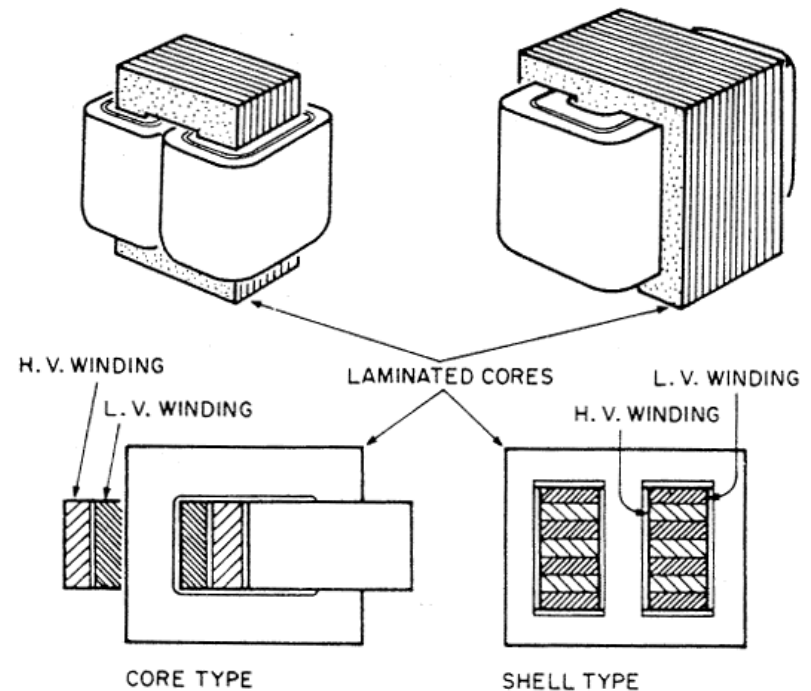


Figure 2-4 Transformer core types.

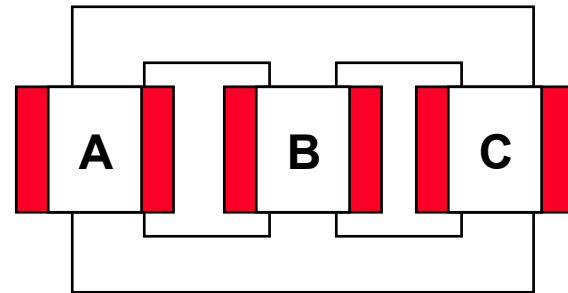
Transformer Construction

Iron Cores

The three phase transformer iron core has three legs.

- A phase winding is placed in each leg.
- The high voltage and low voltage windings are placed on top of each other and insulated by layers or tubes.
- Larger transformer use layered construction shown in the previous slides.

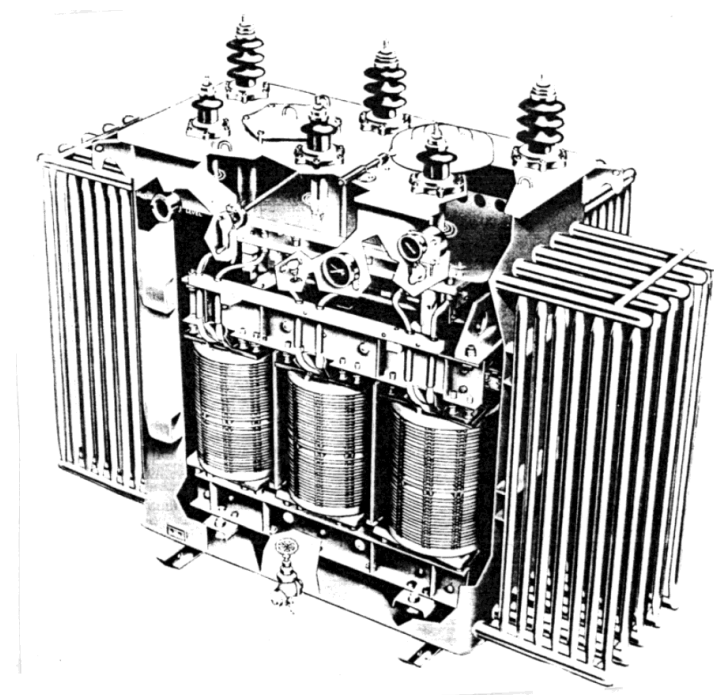
Three phase transformer iron core



Transformer Construction

- The dried and treated transformer is placed in a steel tank.
- The tank is filled, under vacuum, with heated transformer oil.
- The end of the windings are connected to bushings.

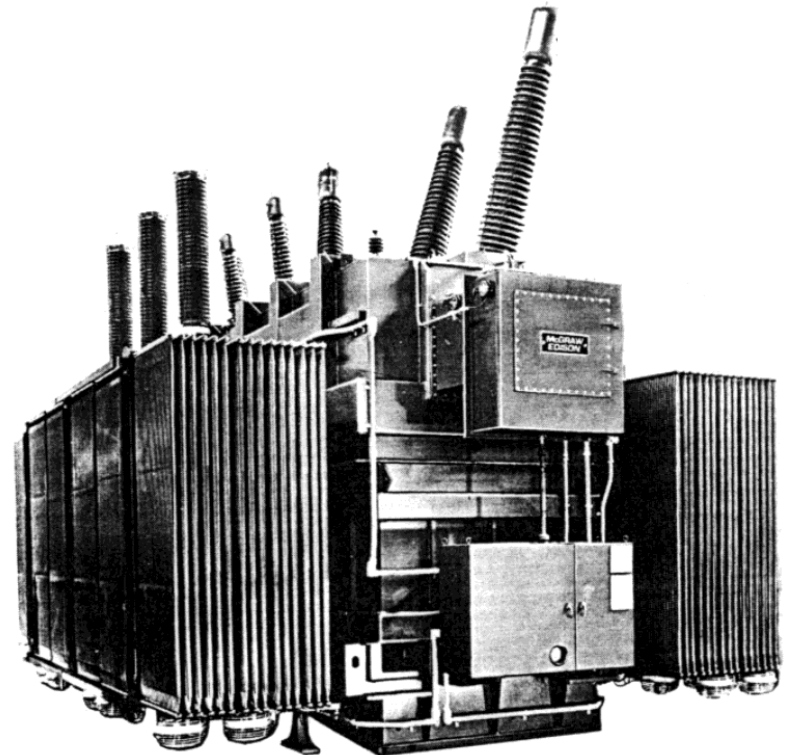
Three phase oil transformer



Transformer Construction

- **The transformer is equipped with cooling radiators which are cooled by forced ventilation.**
- **Cooling fans are installed under the radiators.**
- **Large bushings connect the windings to the electrical system.**
- **The oil is circulated by pumps and forced through the radiators.**
- **The oil temperature, pressure are monitored to predict transformer performance.**

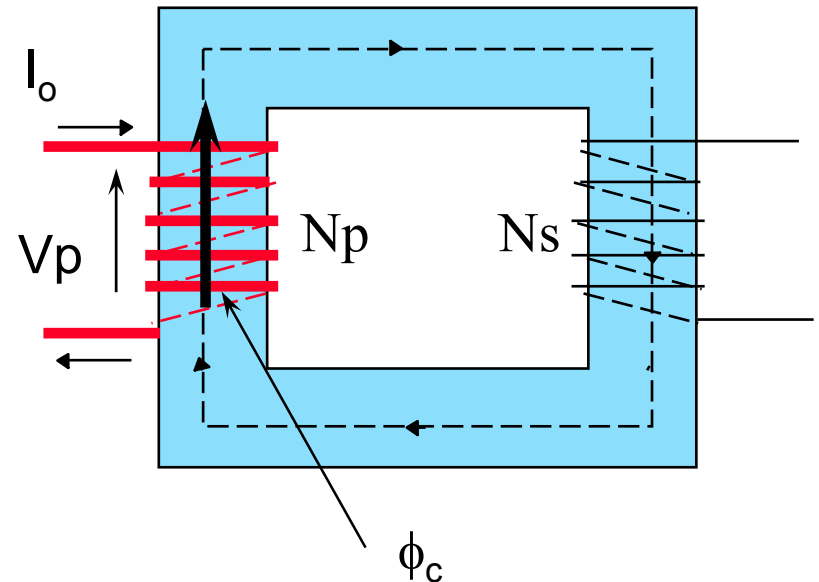
Large three phase oil transformer



Ideal Transformer

- A component of the current, called the magnetization current I_m , is required to set-up the magnetic field (or the flux in the iron core, Φ_c). This flux which is a time-varying flux links both the primary and secondary windings. Accordingly, voltages (emfs) are induced in both windings.
- Since the iron core is exposed to AC current, the source should also supply a component of current called the core loss component, I_c , to account for hysteresis and eddy current losses.
- Total No-load current, $I_o = I_m + I_c$.

Flux generation



Ideal Transformer

- **Induced Voltages:**

The induced emf in primary winding is:

$$E_p = 4.44 N_p \Phi_m f,$$

where N_p is the number of turns in primary winding, Φ_m , the maximum (peak) flux, and f the frequency of the supply voltage.

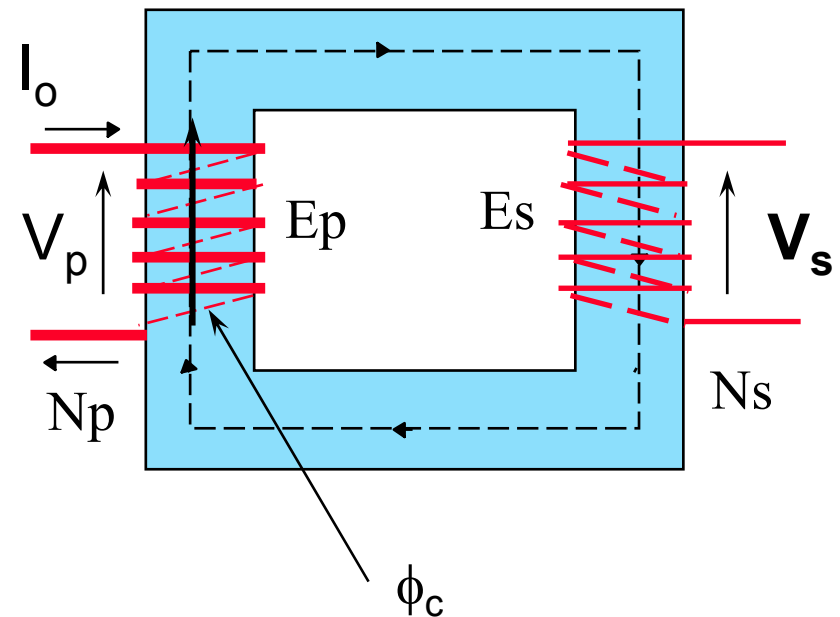
- Similarly, the induced emf in secondary winding,

$$E_s = 4.44 N_s \Phi_m f,$$

- where N_s is the number of turns in secondary winding.

- **Turns Ratio, $a = E_p/E_s = N_p/N_s$**

Voltage generation



Ideal Transformer

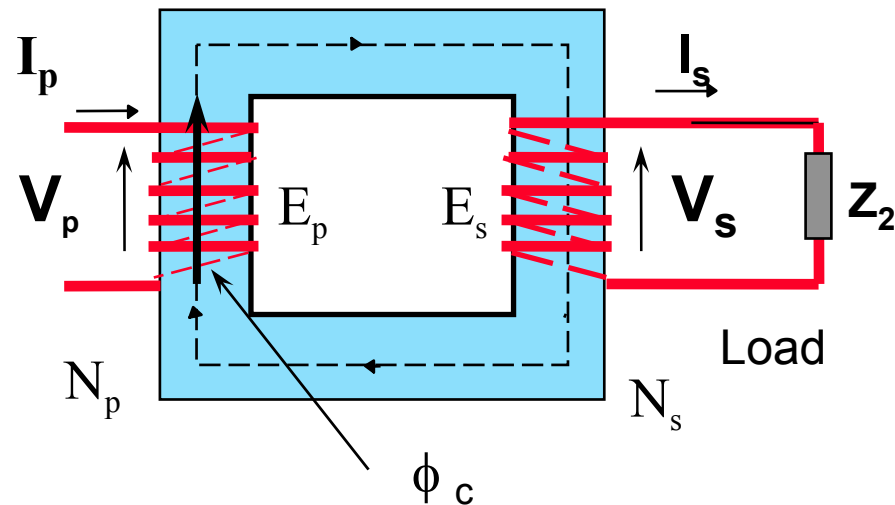
- If the transformer is ideal,
Input power = Output power.
- Assuming the power factor to be
same on both sides,

$$V_p I_p = V_s I_s$$

Hence,

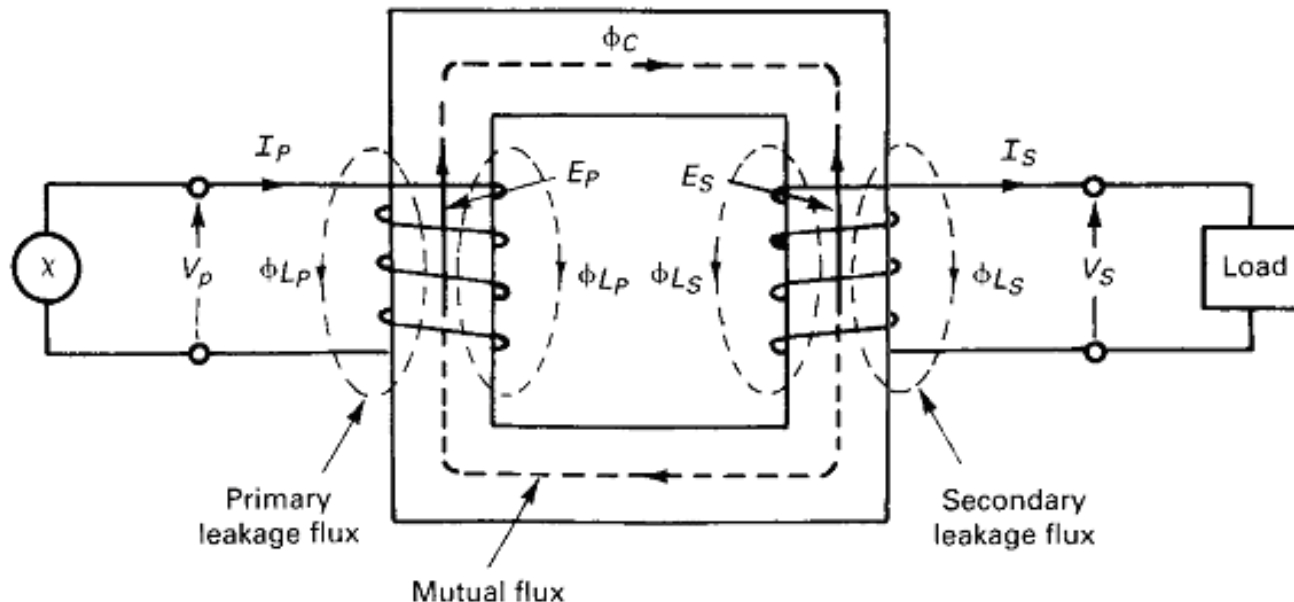
$$N_p/N_s = V_p/V_s = I_s/I_p$$

Transformer loaded



Real Transformer

- **Leakage Flux**: Not all of the flux produced by the primary current links the winding, but there is leakage of some flux into air surrounding the primary. Similarly, not all of the flux produced by the secondary current (load current) links the secondary, rather there is loss of flux due to leakage. These effects are modelled as leakage reactance in the equivalent circuit representation.



Real Transformer

- Equivalent circuit:

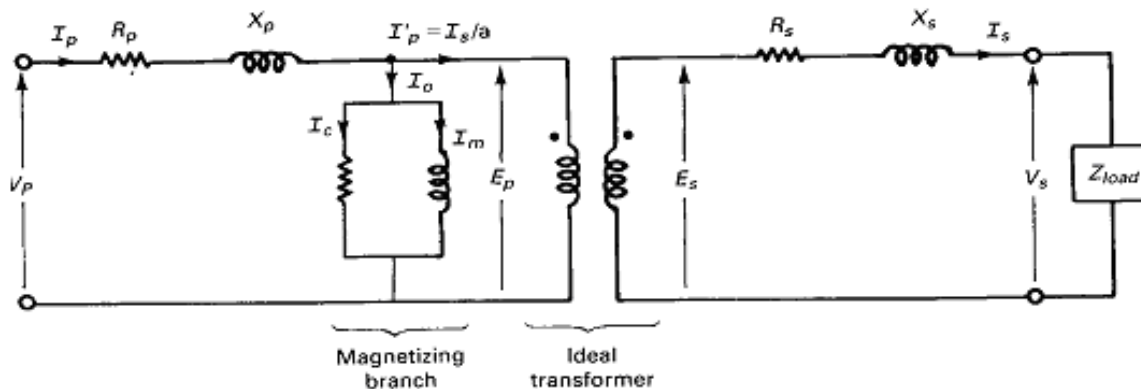
Equivalent Circuit of a Two-winding, 1-phase, Transformer

R_c => core loss component

X_m => magnetization component

R_p and X_p are resistance and reactance of the primary winding

R_s and X_s are resistance and reactance of the secondary winding



Real Transformer

- Impedance Transfer:

To model a transformer, it is important to understand how impedance are transferred from one side to another, that is primary to secondary or secondary to primary.

- Looking into the circuit from source side, let us define the impedance as $Z_{in} = V_p/I_p$
- Looking into the circuit from load side, let us define the impedance as $Z_L = V_s/I_s$
- Relating V_p/I_p in terms of V_s/I_s using the turns ration, a,

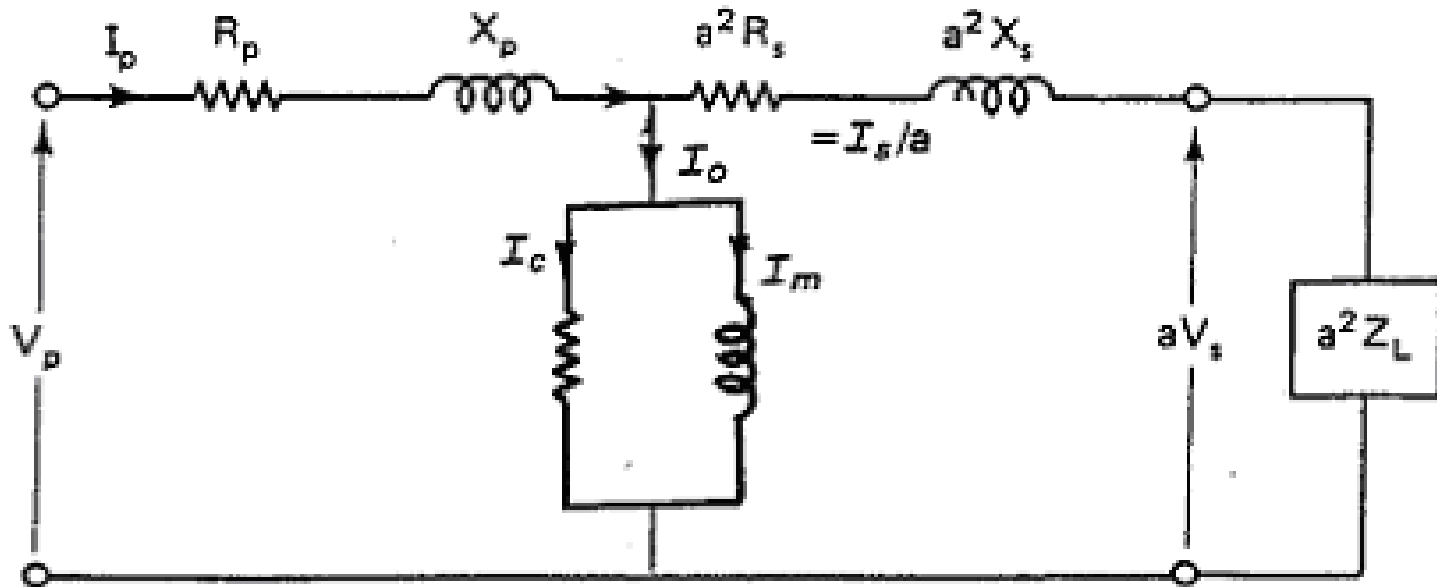
$$[V_p/I_p] = a^2 [V_s/I_s]$$

$$Z_{in} = a^2 Z_L$$

Hence, in general, any impedance transferred from secondary side to primary side must be multiplied by the square of the turns-ratio, a^2 .

Real Transformer

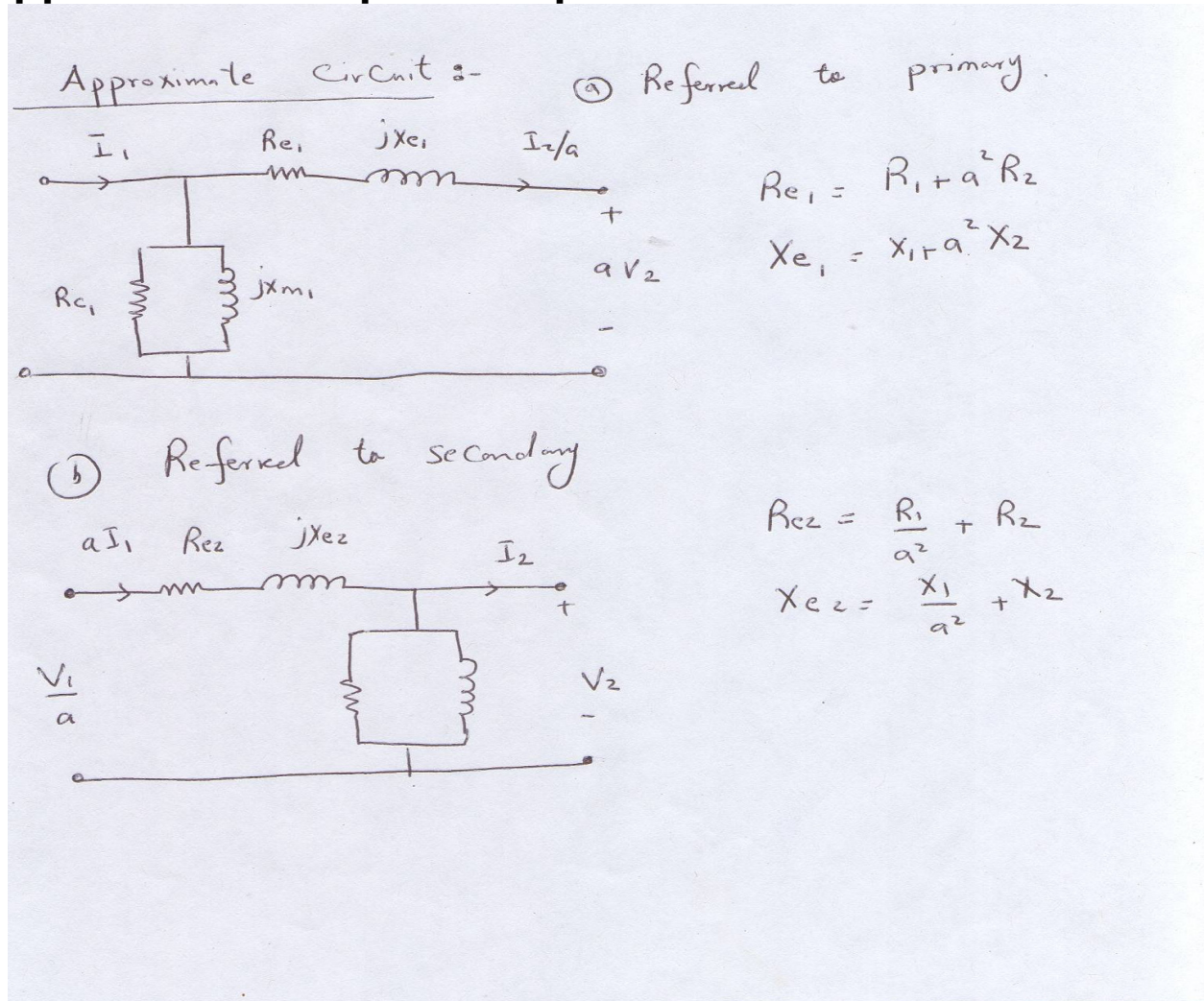
Equivalent Circuit - seen from primary side



Secondary parameters transferred to primary

Real Transformer

Approximate Simplified Equivalent Circuit



Example no. 1

A 100 kVA, 2300/230V, single phase transformer has the following parameters:

$$\mathbf{R_1 = 0.30 \ \Omega \quad R_2 = 0.0030 \ \Omega \quad R_c = 4.5 \text{ k} \ \Omega}$$

$$\mathbf{X_1 = 0.65 \ \Omega \quad X_2 = .0065 \ \Omega \quad X_m = 1.0 \text{ k} \ \Omega}$$

The transformer delivers 75 kW at 230 V and 0.85 power factor lagging, find:

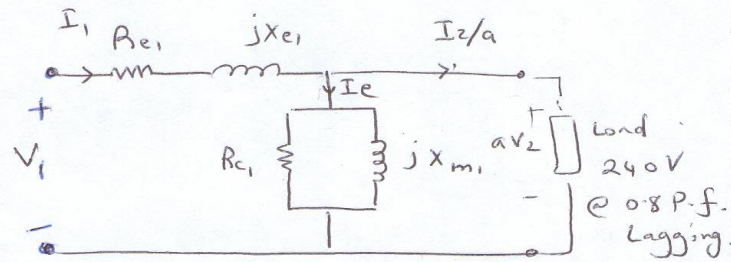
- a) The input current.**
- b) The input voltage.**

Prob # 4.16

$$Z_{e1} = 6 + j8.5 \Omega$$

$$R_{c1} = 50 \text{ k}\Omega$$

$$X_{m1} = 15 \text{ k}\Omega$$



$$a = \frac{2400}{240} = 10$$

$$V_2 = 240 \angle 0$$

$$\bar{I}_2 = \frac{15000}{240} \angle -\cos^{-1}(0.8) = 62.5 \angle -36.9$$

$$aV_2 = (10)(240) \angle 0 = 2400 \angle 0$$

$$\bar{I}_2/a = 6.25 \angle -36.9$$

$$\bar{I}_1 = \frac{\bar{I}_2}{a} + \bar{I}_e = \frac{\bar{I}_2}{a} + aV_2 \left(\frac{1}{R_{c1}} + \frac{1}{jX_{m1}} \right)$$

$$= 6.25 \angle -36.9 + 2400 \angle 0 \left(\frac{1}{50\text{k}} + \frac{1}{j15\text{k}} \right)$$

$$= (5 - j3.75) + (-0.48 - j0.16) = 5.048 - j3.912$$

$$= 6.386 \angle -37.8 \text{ A}$$

$$V_1 = aV_2 + \bar{I}_1(Z_{e1}) = 2400 \angle 0 + 6.386 \angle -37.8 (6 + j8.5)$$

$$= 2463.5 \angle 0.4$$

Voltage Regulation

$$\% \text{ V.R.} = (V_{NL} - V_{FL}) \times 100 / V_{FL}$$

$$= (V_p - aV_s) \times 100 / aV_s$$

$$= (V_p - V'_s) \times 100 / V'_s$$

Note: The primary side voltage is always adjusted to meet the load changes; hence, V'_s and V_s are kept constant. There is no source on the secondary side

Efficiency

As always, efficiency is defined as power output to power input ratio.

$$\eta = P_{\text{out}}/P_{\text{in}} \times 100 \%$$

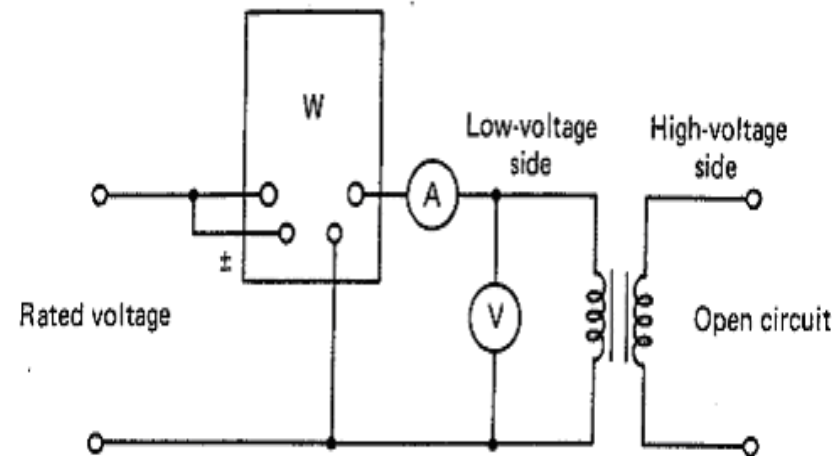
$$P_{\text{in}} = P_{\text{out}} + P_{\text{core}} + P_{\text{copper}}$$

P_{copper} represents the copper losses in primary and secondary windings. There are no rotational losses.

Equivalent circuit parameters

- **Open Circuit Test:** Secondary (normally the HV winding) is open, that means there is no load across secondary terminals; hence there is no current in the secondary.
- Winding losses are negligible, and the source mainly supplies the core losses, P_{core} .
- **Parameters obtained:** Test is done at rated voltage with secondary open. So, the ammeter reads the no-load current, I_o ; the wattmeter reads the core losses, and the voltmeter reads the applied primary voltage.

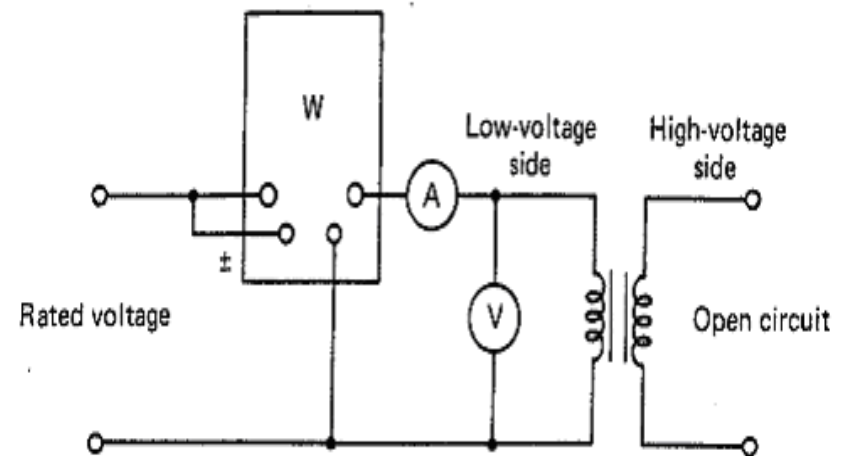
Open circuit test



Equivalent circuit parameters

- Wattmeter reading = $P_{oc} = P_{core}$
- Hence, $R_{c(LV)} = V_{(LV)}^2 / P_{oc}$
- Note: The open circuit test was done by energizing the LV (low voltage) side with secondary (HV) open.
- Once, $R_{c(LV)}$ is known, X_m can be found as follows.
- $I_{c(LV)} = V_{(LV)} / R_{c(LV)}$
- But, Ammeter reading = I_o .
- Therefore, $I_{m(LV)} = I_o - I_{c(LV)}$
- $X_m = V_{(LV)} / I_{m(LV)}$

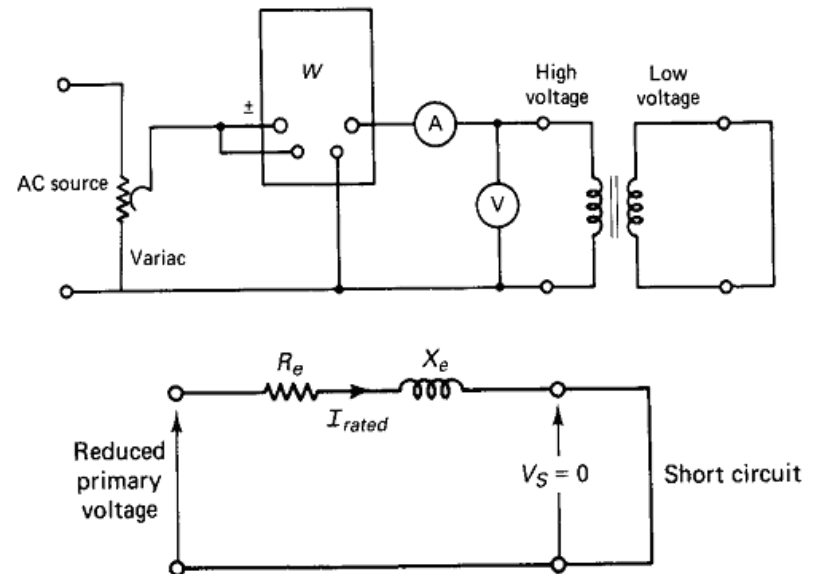
Open circuit test



Equivalent circuit parameters

- Secondary (normally the LV winding) is shorted, that means there is no voltage across secondary terminals; but a large current flows in the secondary.
- **Parameters obtained: Test is done at reduced voltage (about 5% of rated voltage) with full-load current in the secondary. So, the ammeter reads the full-load current, I_p ; the wattmeter reads the winding losses, and the voltmeter reads the applied primary voltage.**

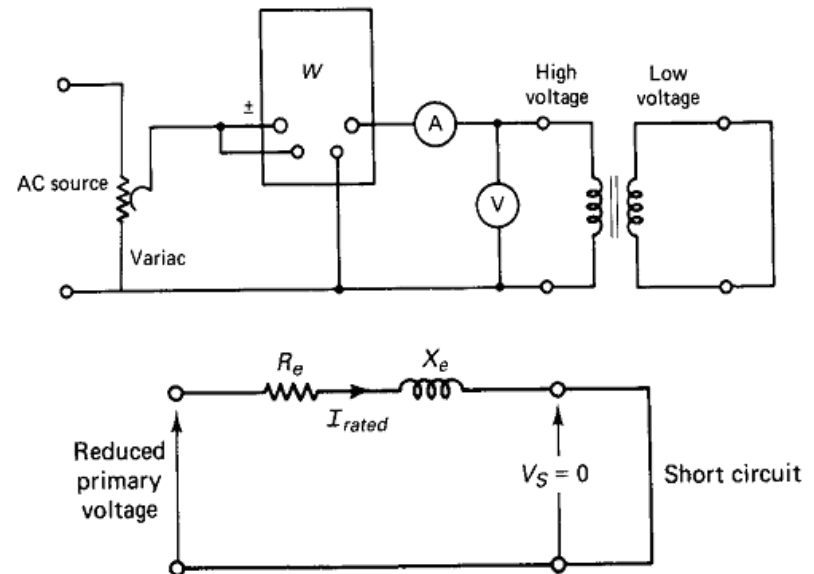
Short-circuit test



Equivalent circuit parameters

- Core losses are negligible as the applied voltage is \ll rated voltage.
- $R_{ep} = P_{sc} / I_{sc}^2$
- But, $Z_{ep}(HV) = V_{sc}(HV) / I_{sc}$ hence, $X_{ep}(HV)$ can be obtained

Short-circuit test



Example no. 2:

a 50 kVA, 2400/240 V transformer has the following test data:

| | Voltage (v) | Current (A) | Power (W) |
|---------------------------|--------------------|--------------------|------------------|
| Short circuit test | 55 | 20.8 | 600 |
| Open circuit test | 240 | 5.0 | 450 |

Calculate:

- 1. The voltage regulation and efficiency when the transformer is connected to a load that takes 156 A at 220 V and 0.8 power factor lagging.**

Find the transformer parameters:-

$$Y_2 = \frac{I_{oc}}{V_{oc}} = \frac{5}{240} = 0.02083$$

$$-\theta_2 = -\cos^{-1} \left(\frac{P_{oc}}{V_{oc} I_{oc}} \right) = -67.98$$

$$Y_2 = 0.02083 \angle -67.98 = 0.007081 - j0.0193$$

$$\Rightarrow R_{e2} = 128 \Omega \quad \& \quad X_{m2} = 51.8$$

(Since a o.c. test was conducted from the secondary side \Rightarrow all calculated values are referred to the sec. side.)

$I_{sc} = 20.8 = I_{rated}$ in the primary side
 \Rightarrow s.c. test was conducted from the primary side.

$$R_{e1} = \frac{P_{sc}}{I_{sc}^2} = \frac{600}{(20.8)^2} = 1.387 \Omega$$

$$Z_{e1} = \frac{V_{sc}}{I_{sc}} = \frac{55}{20.8} = 2.644 \Omega$$

$$\Rightarrow X_{e1} = \sqrt{Z_{e1}^2 - R_{e1}^2} = 2.25 \Omega$$

⇒ Refer every thing to primary:

$$R_{c1} = a^2 R_{c2} = \left(\frac{2400}{240}\right)^2 12.8 = 12.8 \text{ k}\Omega$$

$$X_{m1} = \left(\frac{2400}{240}\right)^2 (51.8) = 5.18 \text{ k}\Omega$$

$$\textcircled{a} \quad \underline{I_2} = 156 \angle -\cos^{-1}(0.8)$$

$$= 156 \angle -36.9$$

$$\frac{I_2}{a} = 15.6 \angle -36.9$$

$$I_c = a V_2 \left[\frac{1}{R_{c1}} + \frac{1}{jX_{m1}} \right]$$

$$= (220)(10) \left[\frac{1}{12.8 \times 10^3} + \frac{1}{j5.18 \times 10^3} \right]$$

$$= 0.172 - j0.425 =$$

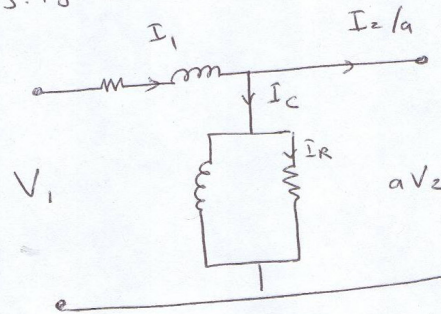
$$\underline{I_1} = \underline{I_2/a} + \underline{I_c} = 15.99 \angle -37.72$$

$$V_1 = I_1 Z_{e1} + a V_2 = 2239.6 \angle 0.38$$

$$V.R. = \frac{|V_1| - |aV_2|}{2200} = \frac{2239.6 - 2200}{2200} = 1.8\%$$

$$P_{core} = \underline{I_R}^2 R_{c1} = (0.172)^2 \times 12,800 = 378.69 \text{ W}$$

$$P_{cu} = \underline{I_1}^2 R_{e1} = (15.99)^2 \times 1.387 = 354.62 \text{ W}$$



$$\eta = \frac{P_{out}}{P_{in}}$$

$$= \frac{P_{out}}{P_{out} + \text{losses}}$$

$$P_{out} = (220)(156)(0.8) = 27,456$$

$$\Rightarrow \eta = \frac{27,456}{27,456 + 578.69 + 554.62} = 97.4\%$$

$$\text{or } P_{in} = V_1 I_1 \cos \theta$$

$$= (2239)(15.99) \cos(0.38 + 37.72)$$

$$= 28173.5$$

$$\Rightarrow \eta = \frac{27,456}{28173.5} = 97.4\%$$

Home work

Q1

A 4600/460 V, 200-kVA, 60-Hz, step-down, single-phase transformer has the following equivalent-circuit parameters referred to the high-voltage side.

$$R_{eq} = 1.35 \, \Omega$$

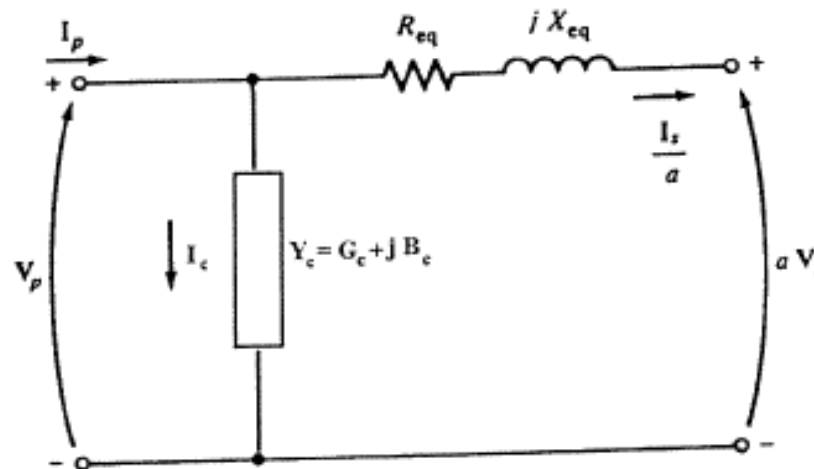
$$X_{eq} = 6.2 \, \Omega$$

The admittance of the shunt branch representing the core loss and magnetizing reactance referred to the primary side is given by:

$$Y_c = [0.2 - j1.1] \times 10^{-3} \, S$$

The transformer supplies a load of 180 kVA at a lagging power factor of 0.8 at 460 V and. Use the equivalent circuit of the transformer shown in Figure (2) to determine:

- b- The magnitude of the required primary voltage. [7.5 Marks]
- c- The power factor of the primary side, the input MVA, and the efficiency of the transformer. [7.5 Marks]



Home work

Q2

A 25-kVA, 2200/220 V, 60-Hz, single-phase transformer has the following equivalent-circuit parameters referred to the high-voltage side.

$$R_1 = 3.0 \, \Omega$$

$$R'_2 = 3.0 \, \Omega$$

$$X_{l1} = 12 \, \Omega$$

$$X'_{l2} = 12 \, \Omega$$

$$X_m = 20,000 \, \Omega$$

$$R_c = 50,000 \, \Omega$$

The transformer is supplying 17.5 kVA at 220-V and a lagging power factor of 0.8. Use the equivalent Cantilever model circuit of the transformer shown in Figure (1). Determine:

b- The magnitude of the required primary voltage. [7.5 points]

c- The power factor of the primary side, the input kVA, and the efficiency of the transformer. [7.5 points]

