ELE-B7

TRANSMISSION LINES

Development of Line Models

- Goals of this section are

1) Develop a simple model for transmission lines
2) Gain an intuitive feel for how the geometry of the transmission line affects the model parameters

Primary Methods for Power Transfer

The most common methods for transfer of electric power are

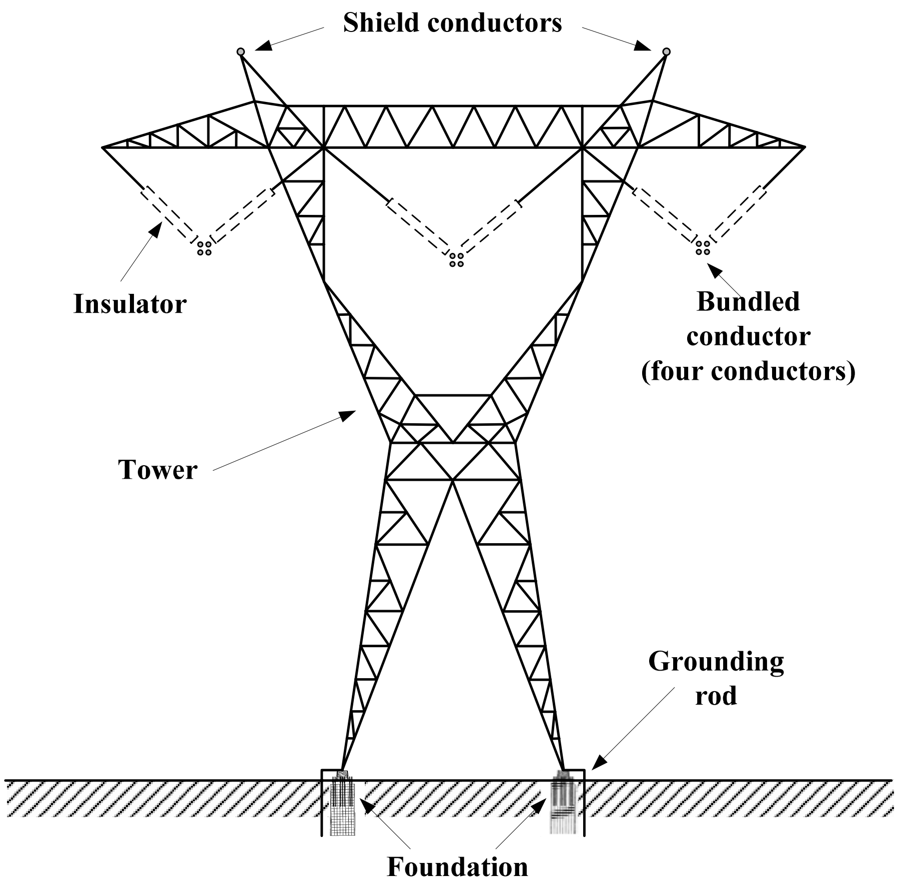
1) Overhead AC
2) Underground AC
3) Overhead DC
4) Underground DC

Transmission lines and cables

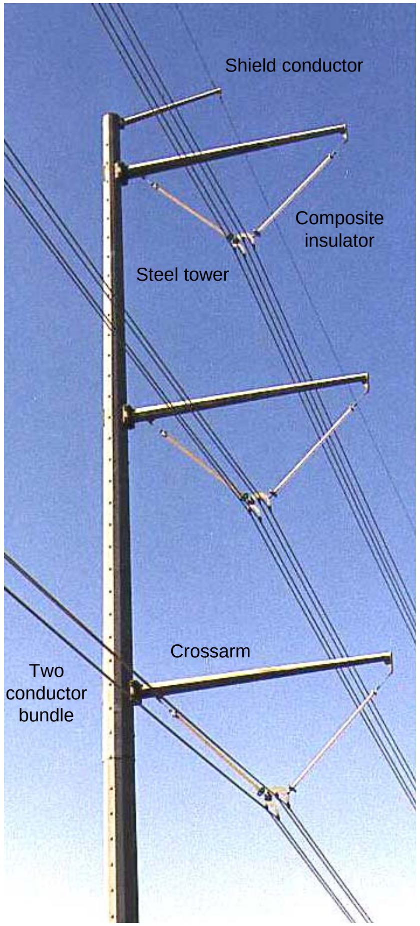
- **Extra-high-voltage lines**
  - Voltage: 345 kV, 500 kV, 765 kV
- **High-voltage lines**
  - Voltage: 115 kV, 230 kV
- **Sub-transmission lines**
  - Voltage: 46 kV, 69 kV
- **Distribution lines**
  - Voltage: 2.4 kV to 46 kV, with 15 kV being the most commonly used
- **High-voltage DC lines**
  - Voltage: ±120 kV to ±600 kV

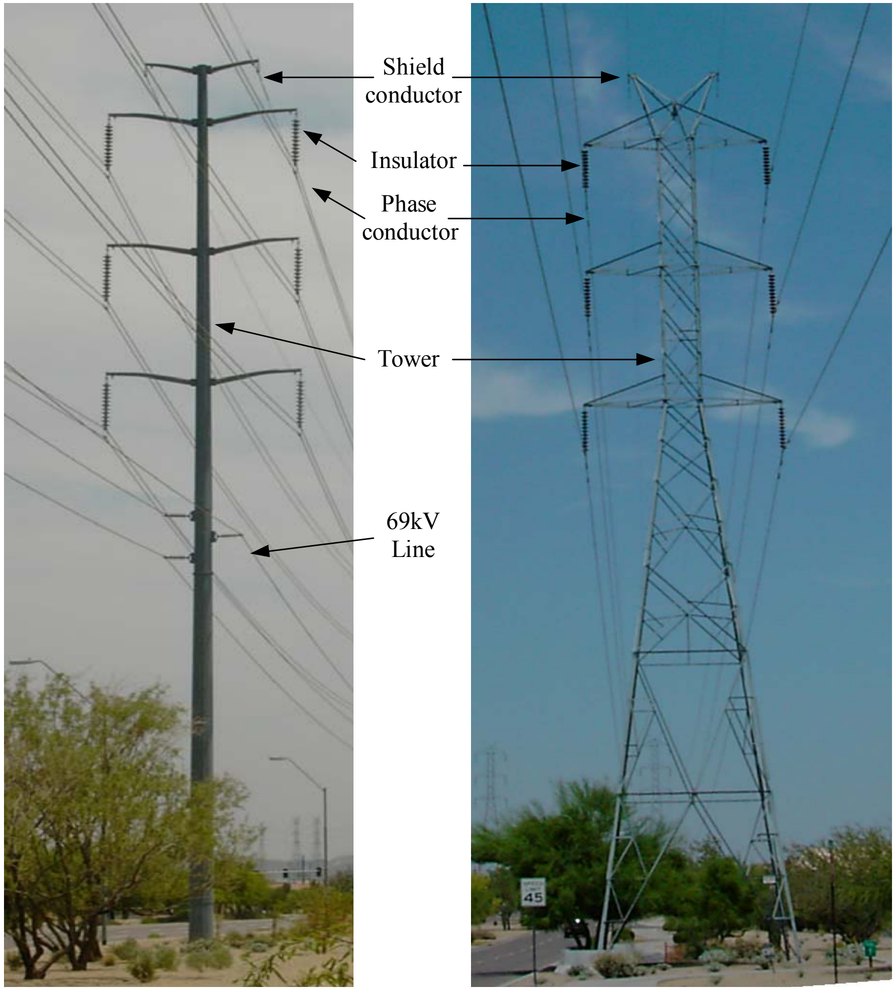
Transmission lines and cables

- Three-phase conductors, which carry the electric current;
- Insulators, which support and electrically isolate the conductors;
- Tower, which holds the insulators and conductors;
- Foundation and grounding; and
- Optional shield conductors, which protect against lightning



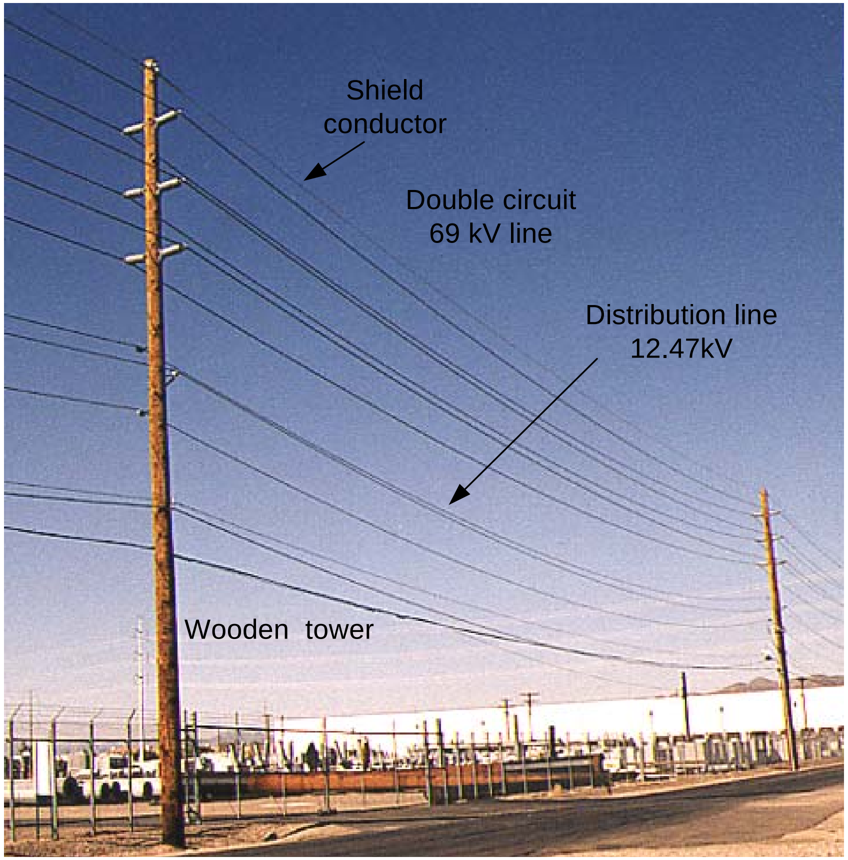
Transmission lines and cables



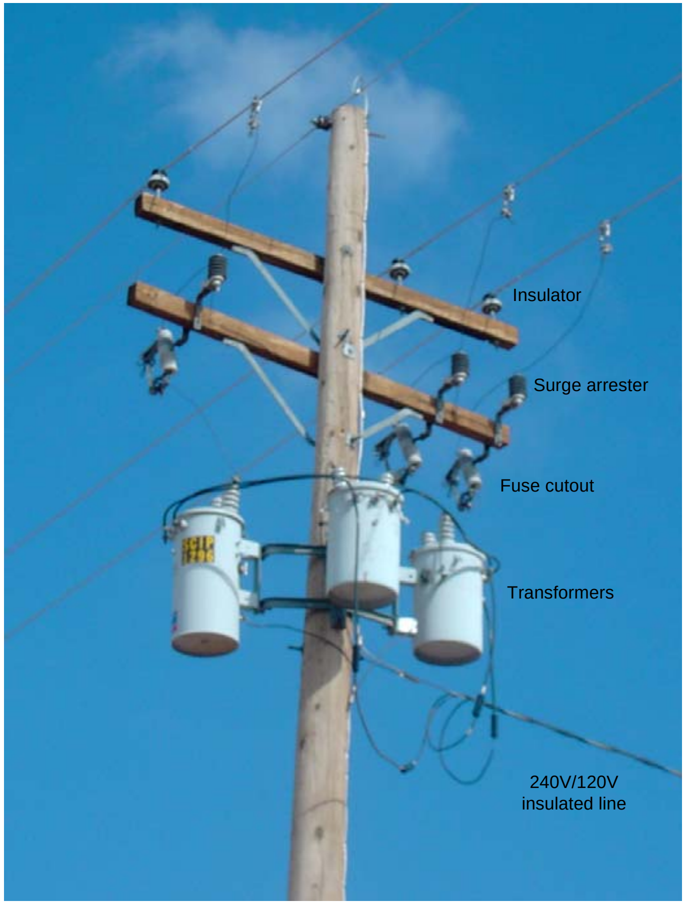


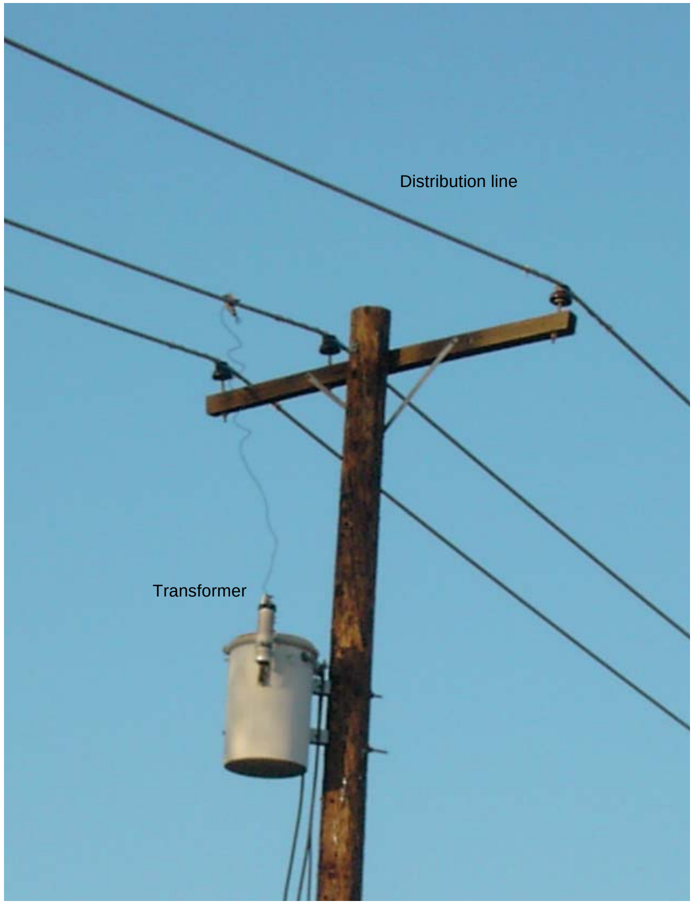
Transmission lines and cables

Distribution Line

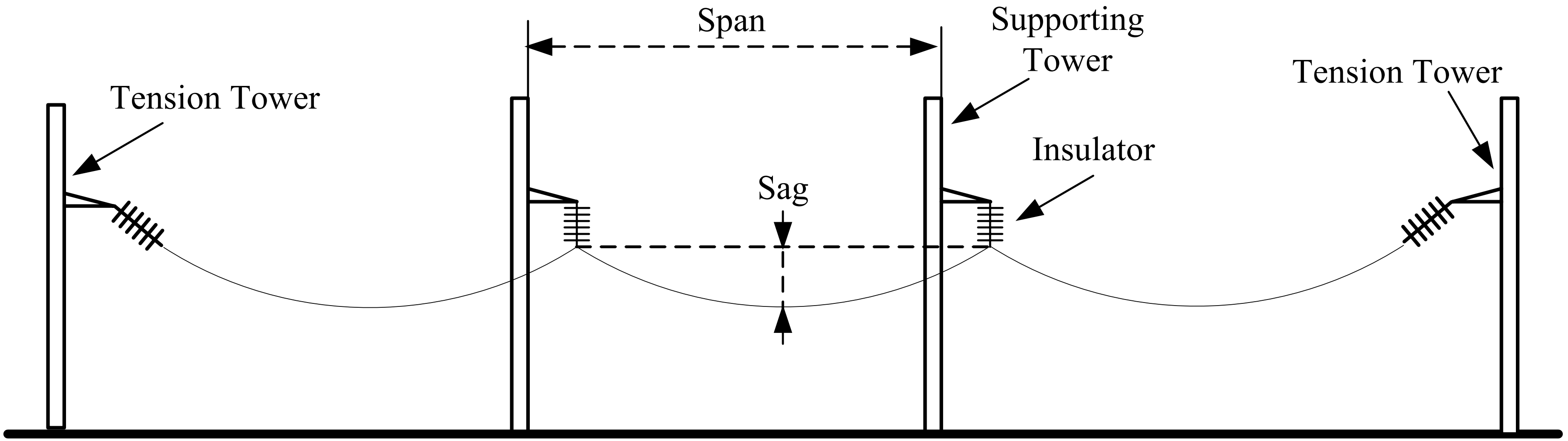


Transmission lines and cables





Transmission lines and cables

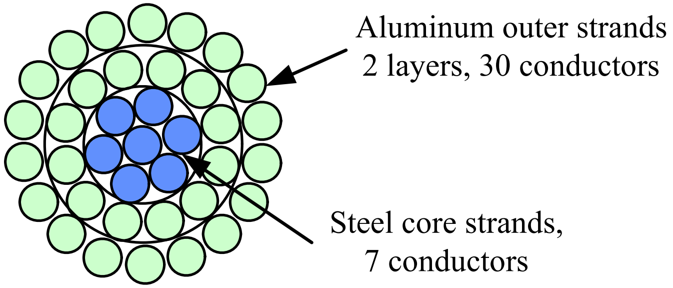


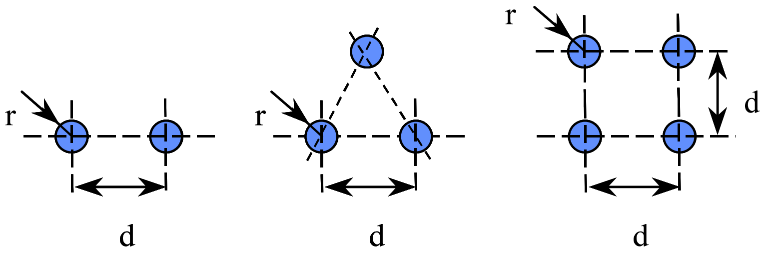
Definition of Parameters

Transmission lines and cables

- Aluminum Conductor Steel Reinforced (ACSR);
- All Aluminum Conductor (AAC); and
- All Aluminum Alloy Conductor (AAAC).

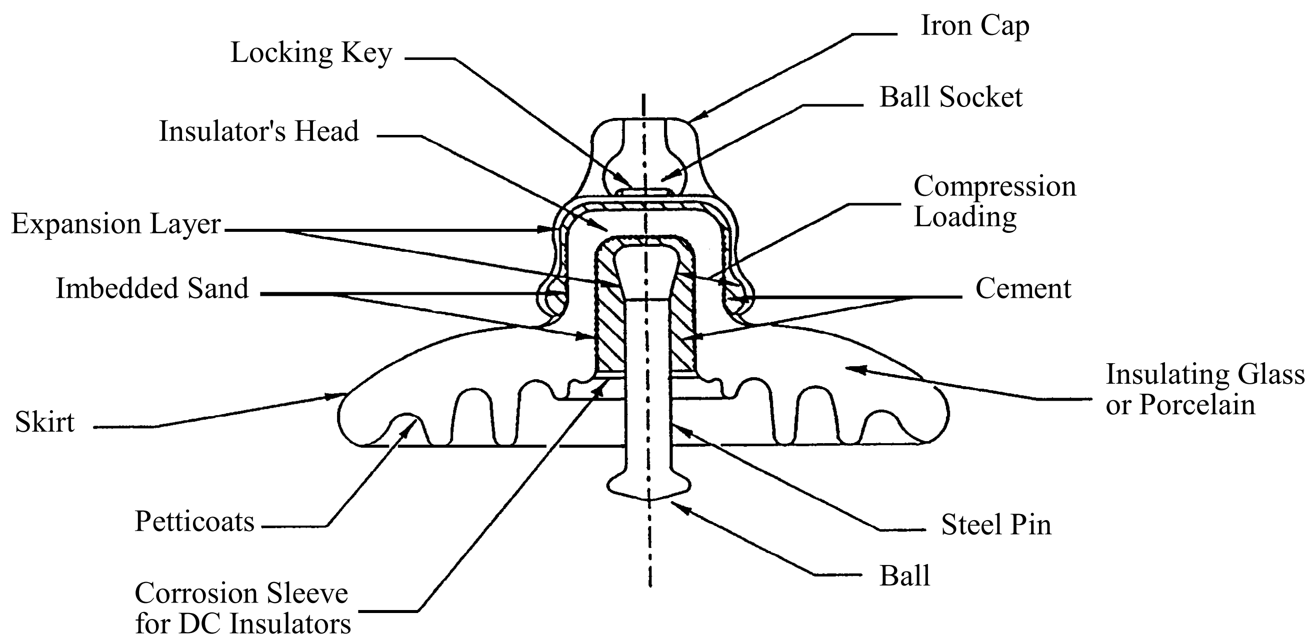
- ACSR Coductor





Transmission lines and cables

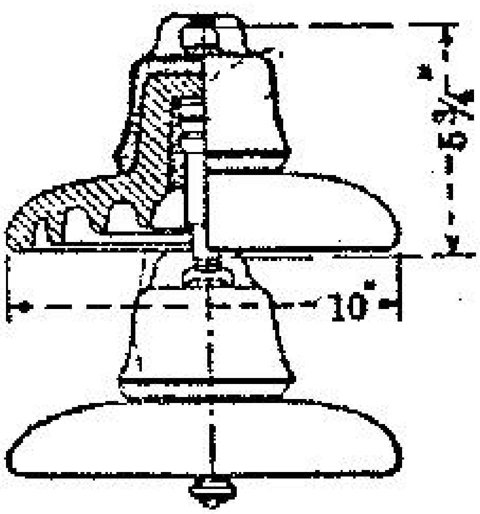
Insulators



Transmission lines and cables

Insulator Chain

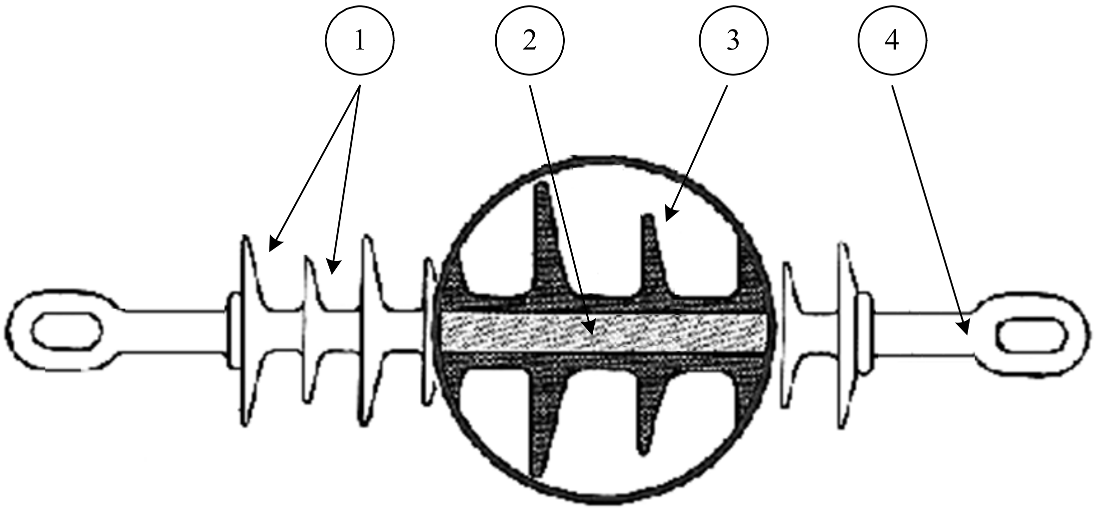
Line Voltage	Number of Insulators per String
69 kV	4–6
115 kV	7–9
138 kV	8–10
230 kV	12
345 kV	18
500 kV	24
765 kV	30–35



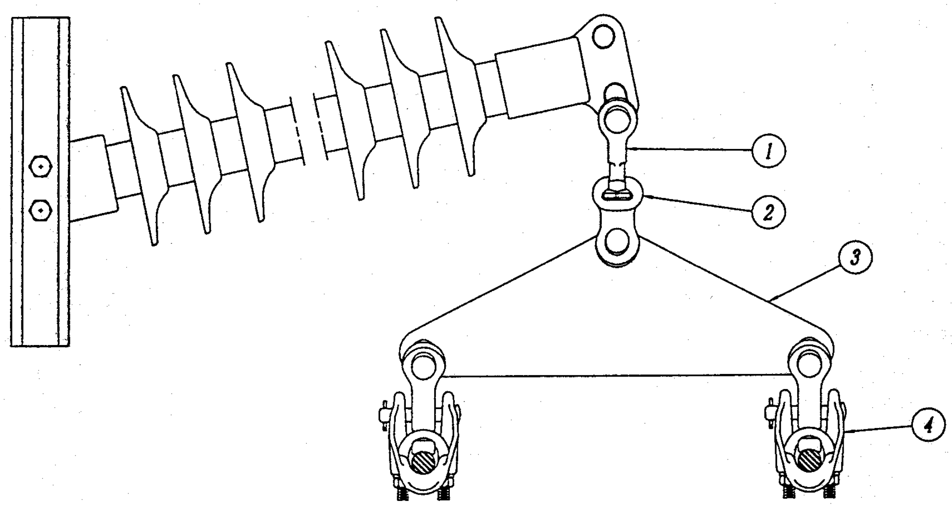
Transmission lines and cables

Composite insulator.

- (1) Sheds of alternating diameters prevent bridging by ice, snow and cascading rain.
- (2) Fiberglass reinforced resin rod.
- (3) Injection molded rubber (EPDM or Silicone) weather sheds and rod covering.
- (4) Forged steel end fitting, galvanized and joined to rod by swaging process.



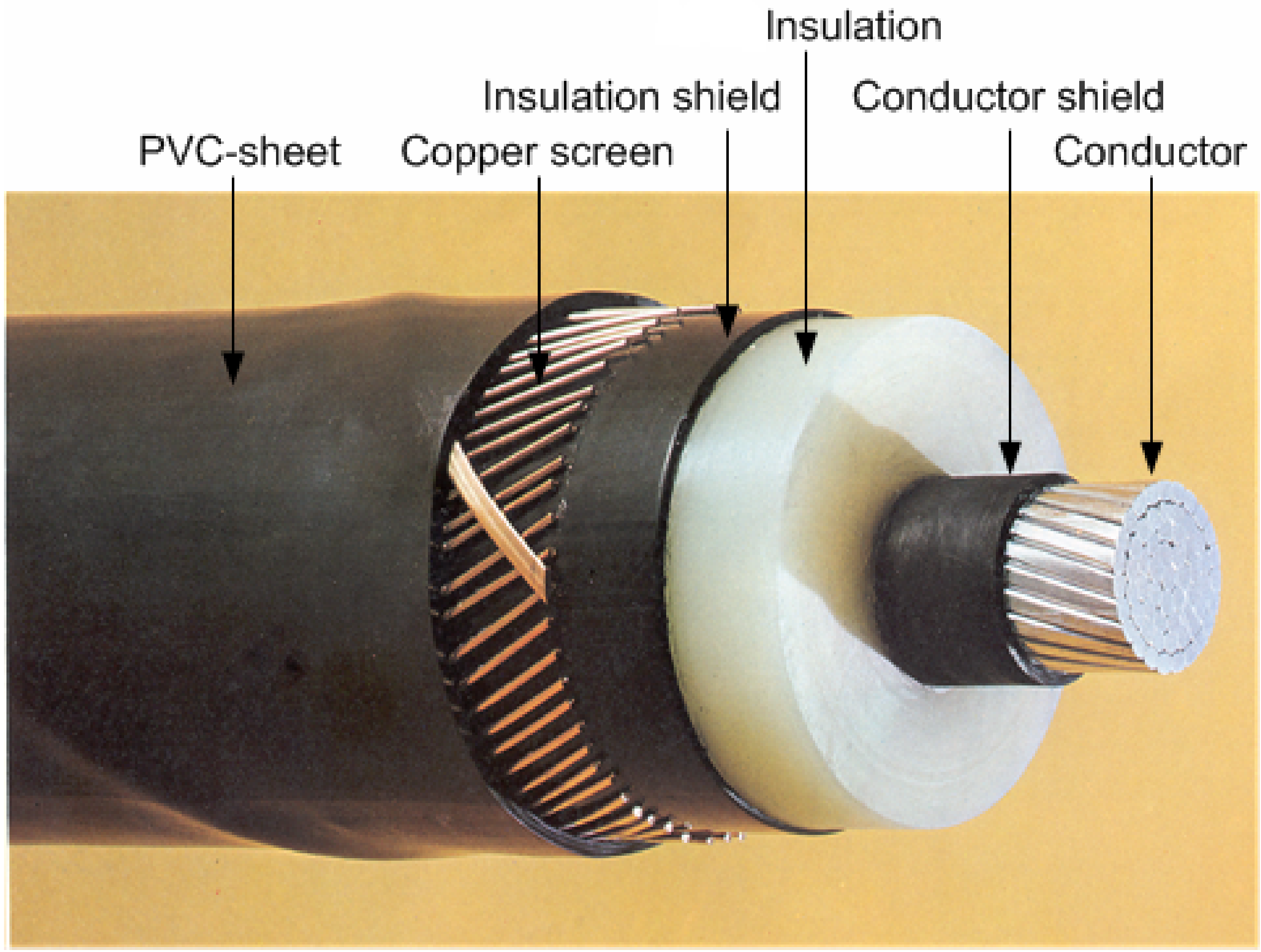
Transmission lines and cables



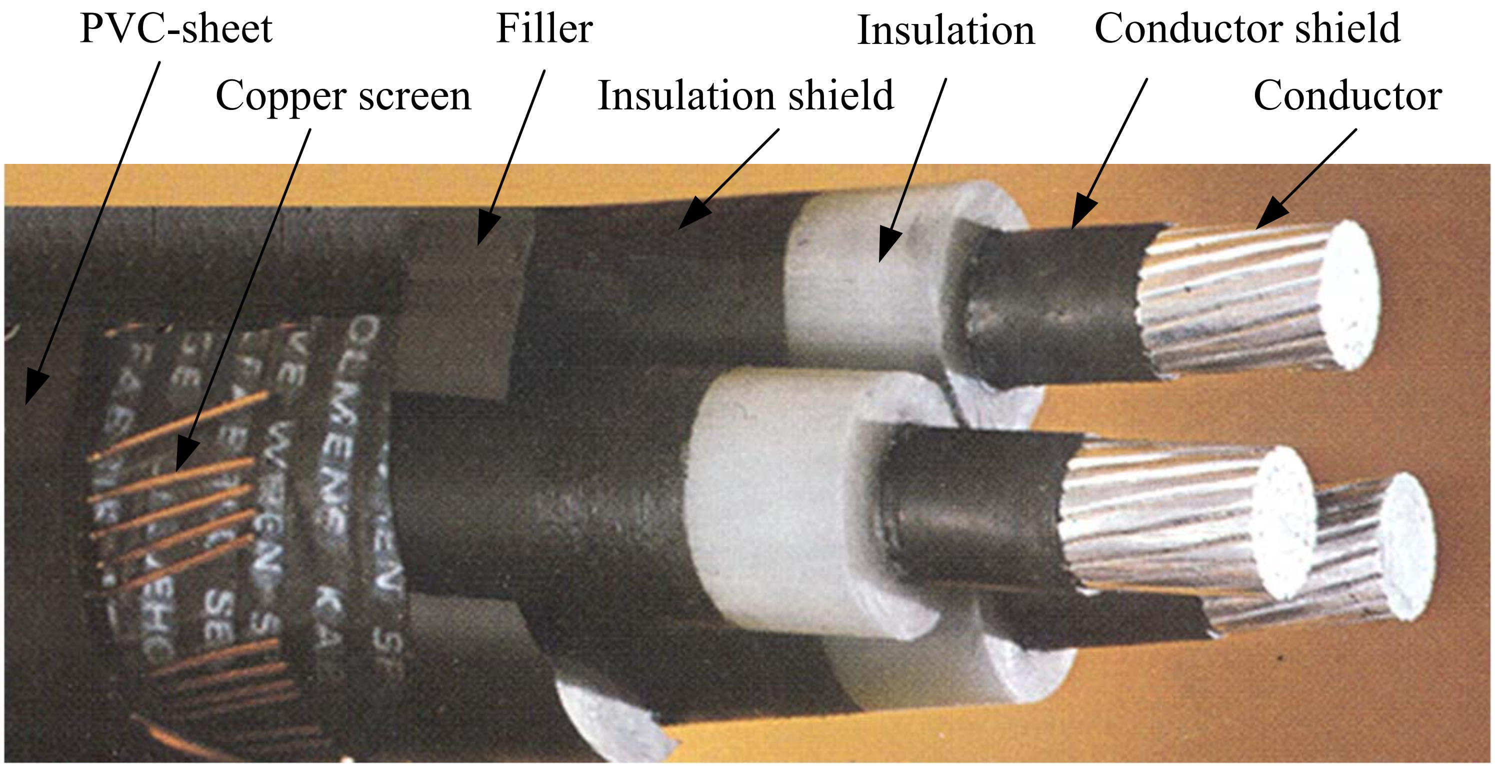
•Figure 4.15 Line post-composite insulator with yoke holding two conductors.

- (1) is the clevis ball,
- (2) is the socket for the clevis,
- (3) is the yoke plate, and
- (4) is the suspension clamp. (*Source*: Sediver)

Transmission lines and cables



Transmission lines and cables



Inductance of a Single Wire

The inductance of a magnetic circuit has a constant permeability can be obtained by determining the following:

a) Magnetic field intensity H, from Ampere's law.
b) Magnetic flux density B ($B = \mu H$)
c) Flux linkage λ
d) Inductance from flux linkage per ampere ($L = \lambda / I$)

Flux linkages within the wire :

$$\oint H \cdot dl = I_{enclosed}$$

$$H_x(2\pi x) = I_x \Rightarrow H_x = \frac{I_x}{2\pi x}$$

Assume uniform current distribution :

$$I_x = \left(\frac{x}{r}\right)^2 I \Rightarrow H_x = \frac{xI}{2\pi r^2}$$

For non - magnetic conductor :

$$B_x = \mu_0 H_x = \frac{\mu_0 x I}{2\pi r^2}$$

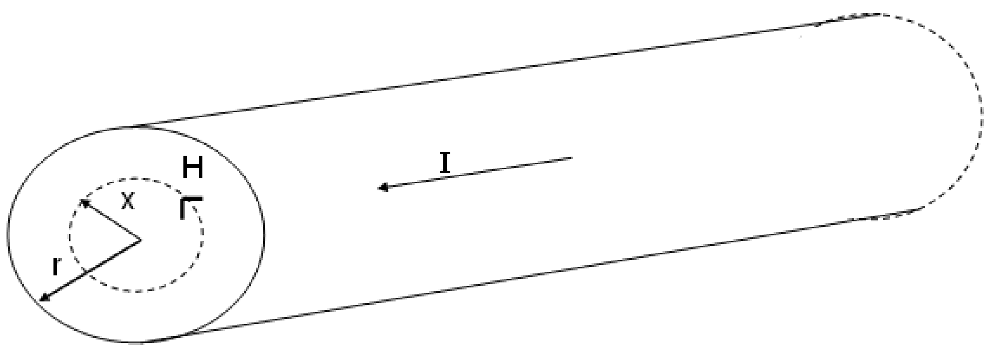
The differential flux $d\phi$ per unit length is :

$$d\phi = B_x dx$$

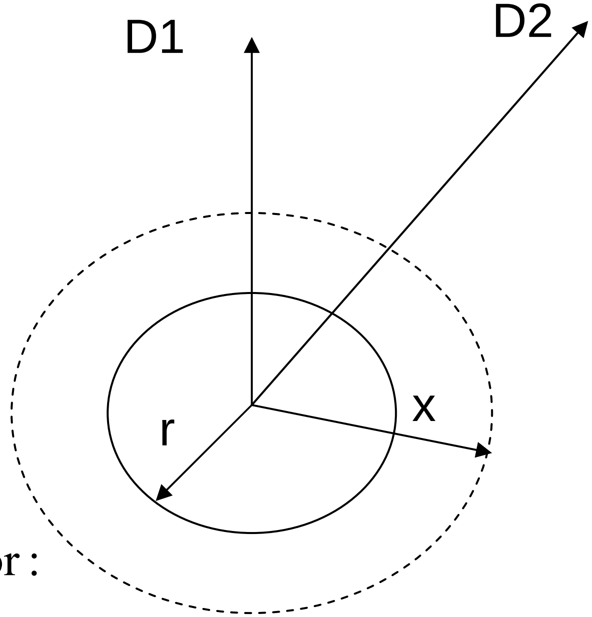
Since only the fraction $(x/r)^2$ of the total current is linked by the flux :

$$d\lambda = \left(\frac{x}{r}\right)^2 d\phi = \frac{\mu_0 x^3 I}{2\pi r^4} dx$$

$$\lambda_{int} = \int_0^r d\lambda = \frac{1}{2} \times 10^{-7} I \Rightarrow L_{int} = \frac{\lambda_{int}}{I} = \frac{1}{2} \times 10^{-7} H / m$$



Flux linkages outside of the wire :



$$\oint H \cdot dl = I_{enclosed}$$

$$H_x(2\pi x) = I \Rightarrow H_x = \frac{I}{2\pi x}$$

$$B_x = \mu_0 H_x = 4\times\pi\times 10^{-7}\frac{I}{2\pi x} = 2\times 10^{-7}\frac{I}{x}$$

The differential flux dϕ per unit length is :

$$d\phi = B_x dx$$

Since the entire current is linked by the flux outside the conductor :

$$d\lambda = d\phi = 2\times 10^{-7}\frac{I}{x}dx$$

$$\lambda_{12} = \int_{D1}^{D2} d\lambda = 2\times 10^{-7} I \times \ln\left(\frac{D1}{D2}\right) \Rightarrow L_{12} = \frac{\lambda_{12}}{I} = 2\times 10^{-7}\ln\left(\frac{D1}{D2}\right) H/m$$

Total flux linkage:

Total flux linkage λ_p linking the conductor out to external point P at distance D is the sum of the interanl and external flux linkages. Since $D_1 = r$ and $D_2 = D$, then

$$\lambda_p = \frac{1}{2}\times 10^{-7}\,\mathrm{I} + 2\times 10^{-7}\,\mathrm{I}\times \ln\left(\frac{D}{r}\right) = 2\times 10^{-7}\,\mathrm{I}\times\left(\ln e^{1/4} + \ln\frac{D}{r}\right) = 2\times 10^{-7}\,\mathrm{I}\times \ln\frac{D}{r'}$$

where : $r' = e^{-1/4} r$

$$L_p = \frac{\lambda_p}{I} = 2\times 10^{-7}\ln\left(\frac{D}{r'}\right) H/m$$

Total flux linkage, cont.:

Finally, consider the array of M solid conductors. Assume each conductor m carries current I_m and the sum of the conductor currents is zero, then :

the flux linkage λ_{kPk} which links conductor k out to point P due to current I_k

$$\lambda_{kpk} = 2 \times 10^{-7} I_k \times \ln \frac{D_{Pk}}{r'_k}$$

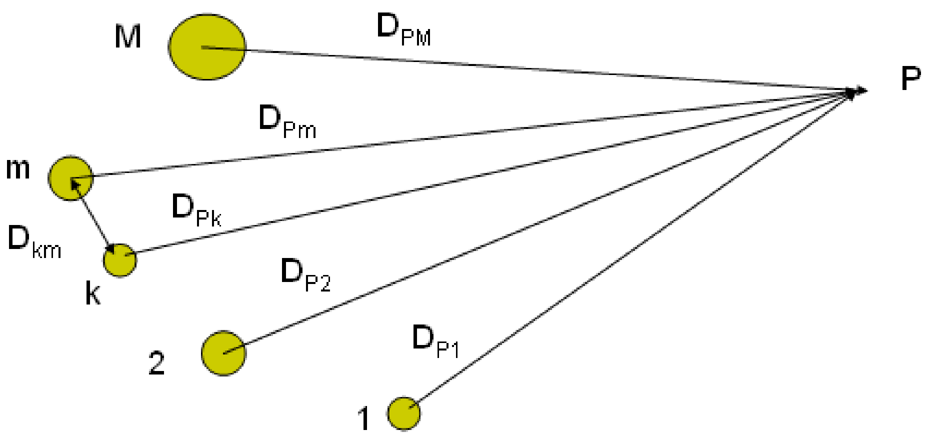
and λ_{kPm} which links conductor k out to point P due to current Im is :

$$\lambda_{kpm} = 2 \times 10^{-7} I_m \times \ln \frac{D_{Pm}}{D_{km}}$$

After some mathematical manipulation :

$$\lambda_k = 2 \times 10^{-7} \sum_{m=1}^{M} I_m \times \ln \frac{1}{D_{km}}$$

where : λ_k gives the total flux linking conductor k in an array of M conductors



Inductance of single-phase, two-wire

Since the sum of the two currents is zero the previous relation is valid and hence :

$$\lambda_x = 2\times10^{-7}\left(\mathrm{I_x}\times\ln\left(\frac{1}{D_{xx}}\right)+\mathrm{I_y}\times\ln\left(\frac{1}{D_{xy}}\right)\right)$$

$$\lambda_x = 2\times10^{-7}\left(\mathrm{I}\times\ln\left(\frac{1}{r'_x}\right)-\mathrm{I}\times\ln\left(\frac{1}{D}\right)\right)$$

$$\lambda_x = 2\times10^{-7}\, I\times\ln\left(\frac{D}{r'_x}\right)$$

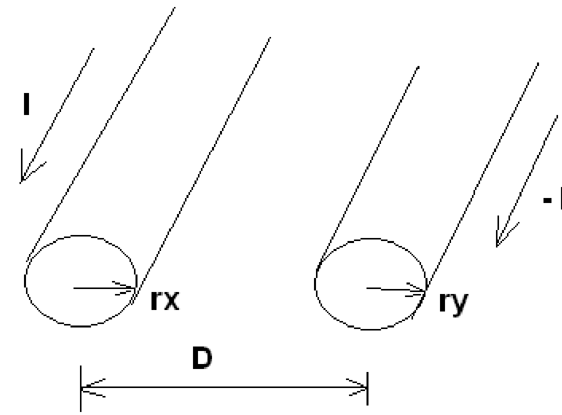
where : $\mathrm{r'_x} = e^{-1/4} r_x$ *similarly*

$$\lambda_y = -2\times10^{-7}\, I\times\ln\left(\frac{D}{r'_y}\right)$$

$$L_x = \frac{\lambda_x}{I_x} = 2\times10^{-7}\ln\left(\frac{D}{r'_x}\right)H/m \quad \text{per conductor, and } L_y = \frac{\lambda_y}{I_y} = 2\times10^{-7}\ln\left(\frac{D}{r'_y}\right)$$

$$L = L_x + L_y = 2\times10^{-7}\ln\left(\frac{D}{r'_x}+\frac{D}{r'_y}\right) = 2\times10^{-7}\ln\left(\frac{D^2}{r'_x r'_y}\right) = 4\times10^{-7}\ln\left(\frac{D}{\sqrt{r'_x r'_y}}\right)$$

If $\mathrm{r'_x = r'_y = r'} \Rightarrow L = 4\times10^{-7}\ln\left(\frac{D}{r'}\right)$



Inductance of three-phase, three-wire with equal phase spacing

Since the sum of the three currents is zero the previous relation is valid and hence :

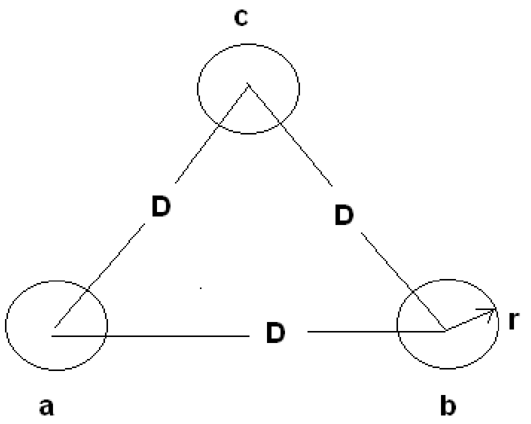
$$\lambda_a = 2\times10^{-7}\left(\mathrm{I_a}\times\ln\left(\frac{1}{r^{'}}\right) + \mathrm{I_b}\times\ln\left(\frac{1}{D}\right) + \mathrm{I_c}\times\ln\left(\frac{1}{D}\right)\right)$$

$$\lambda_a = 2\times10^{-7}\left(\mathrm{I_a}\times\ln\left(\frac{1}{r^{'}}\right) + (\mathrm{I_b} + I_c)\times\ln\left(\frac{1}{D}\right)\right)$$

$$\lambda_a = 2\times10^{-7}\left(\mathrm{I_a}\times\ln\left(\frac{1}{r^{'}}\right) - \mathrm{I_a}\times\ln\left(\frac{1}{D}\right)\right)$$

$$\lambda_a = 2\times10^{-7} I_a \times\ln\left(\frac{D}{r^{'}}\right)$$

$$L_a = 2\times10^{-7}\ln\left(\frac{D}{r^{'}}\right)$$



Inductance of composite conductors

The total flux ϕ_k linking subconductor k of conductor X is :

$$\phi_k = 2\times10^{-7}\left[\frac{I}{N}\sum_{m=1}^{N}\ln\frac{1}{D_{km}} - \frac{I}{M}\sum_{m=1'}^{M}\ln\frac{1}{D_{km}}\right]$$

Since only the fraction (1/N) of the total current I is linked by this flux, the flux linkage of λ_k subconductor k is :

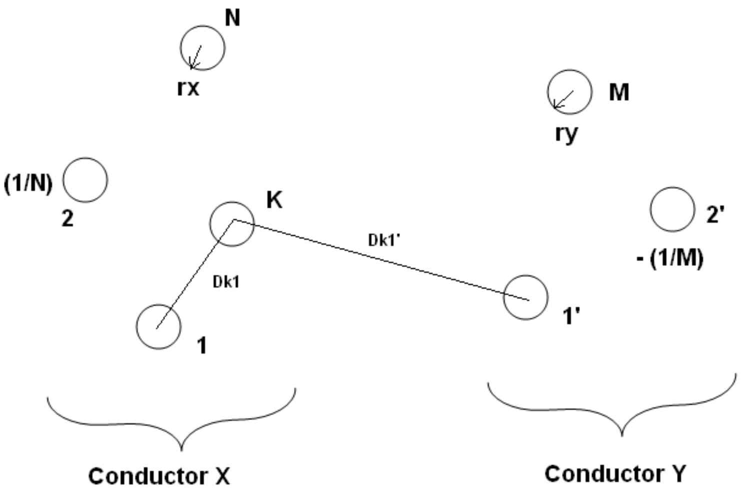
$$\lambda_k = \frac{\phi_k}{N} = 2\times10^{-7} I \times\left[\frac{1}{N^2}\sum_{m=1}^{N}\ln\frac{1}{D_{km}} - \frac{1}{NM}\sum_{m=1'}^{M}\ln\frac{1}{D_{km}}\right]$$

The total flux linkage of conductor x is :

$$\lambda_x = \sum_{k=1}^{N}\lambda_k = 2\times10^{-7} I \times \sum_{K=1}^{N}\left[\frac{1}{N^2}\sum_{m=1}^{N}\ln\frac{1}{D_{km}} - \frac{1}{NM}\sum_{m=1'}^{M}\ln\frac{1}{D_{km}}\right]$$

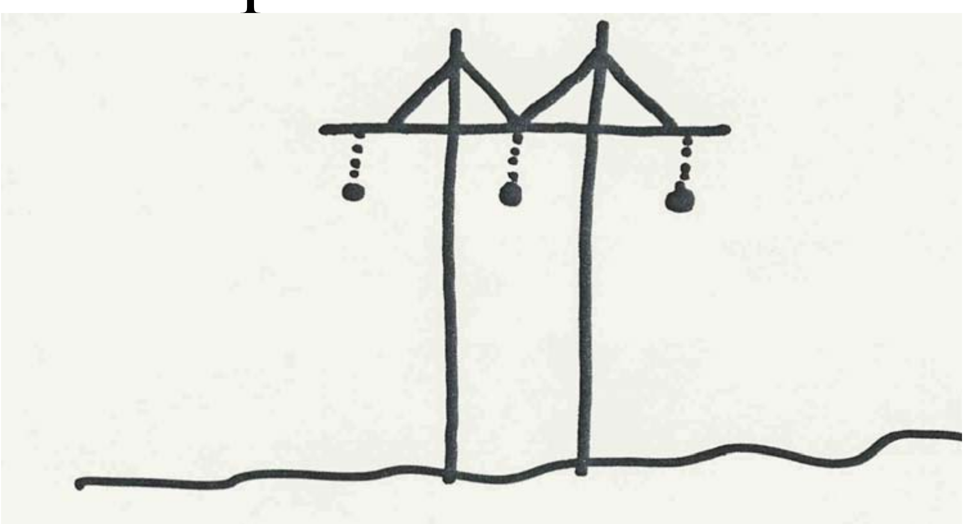
$$\lambda_x = 2\times10^{-7} I \times \ln\prod_{k=1}^{N}\frac{\left(\prod_{m=1'}^{M}D_{km}\right)^{1/NM}}{\left(\prod_{m=1}^{N}D_{km}\right)^{1/N^2}}$$

$$L_x = \frac{\lambda_x}{I} = 2\times10^{-7}\ln\frac{D_{xy}}{D_{xx}} \quad \text{where}: D_{xy} = \sqrt[MN]{\prod_{k=1}^{N}\prod_{m=1'}^{M}D_{km}} = GMD \ \text{ and } \ D_{xx} = \sqrt[N^2]{\prod_{k=1}^{N}\prod_{m=1}^{N}D_{km}} = GMR$$



Inductance of unequal phase spacing

- The problem with the line analysis we've done so far is we have assumed a symmetrical tower configuration. Such a tower figuration is seldom practical.



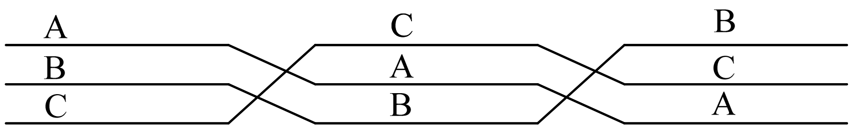
Typical Transmission Tower Configuration

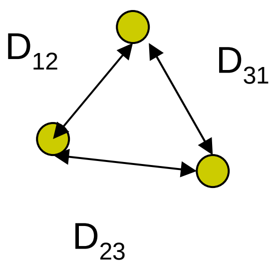
Therefore in general $D_{ab} \neq D_{ac} \neq D_{bc}$

Unless something was done this would result in unbalanced phases

Inductance of unequal phase spacing

- To keep system balanced, over the length of a transmission line the conductors are rotated so each phase occupies each position on tower for an equal distance. This is known as transposition.



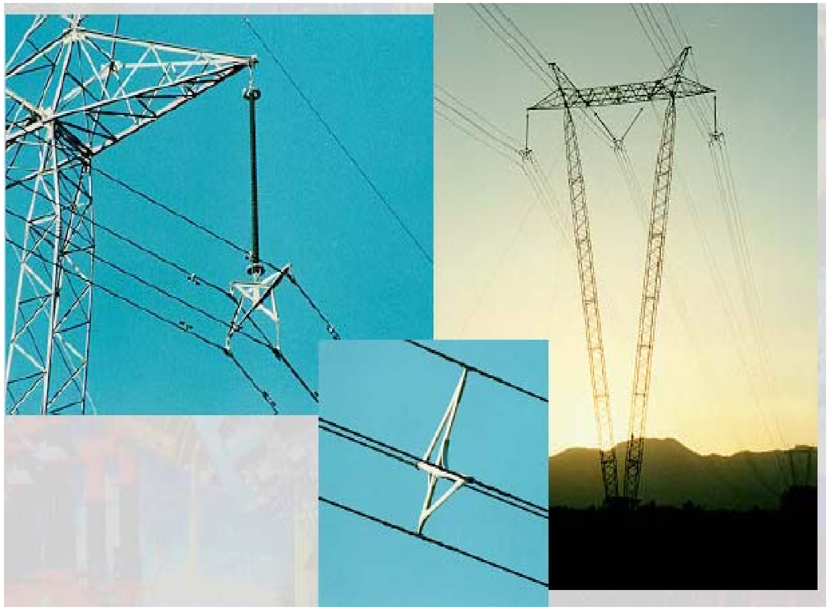


$$L = 2\times 10^{-7} \ln \frac{D_{eq}}{D_S} \quad \text{where:}$$

$$D_{eq} = \sqrt[3]{D_{12}D_{23}D_{31}}$$

Conductor Bundling

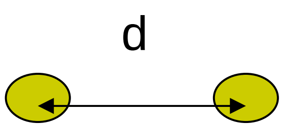
To increase the capacity of high voltage transmission lines it is very common to use a number of conductors per phase. This is known as conductor bundling. Typical values are two conductors for 345 kV lines, three for 500 kV and four for 765 kV.



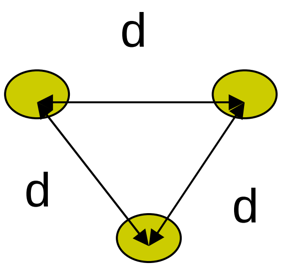
Inductance of bundled conductors

$$L = 2\times10^{-7}\ln\frac{D_{eq}}{D_{SL}} \quad \text{where:}$$

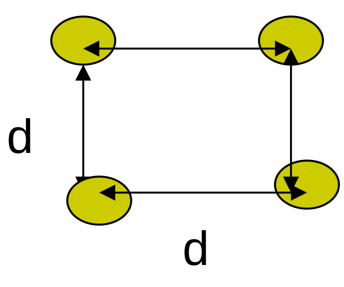
$$D_{eq} = \sqrt[3]{D_{12}D_{23}D_{31}} \quad \text{and } D_{SL} \text{ is calculated as follows:}$$



$$D_{SL} = \sqrt[4]{(D_S\times d)^2} = \sqrt{(D_S\times d)}$$



$$D_{SL} = \sqrt[9]{(D_S\times d\times d)^3} = \sqrt[3]{\left(D_S\times d^2\right)}$$



$$D_{SL} = \sqrt[16]{\left(D_S\times d\times d\times d\sqrt{2}\right)^4} = 1.091\sqrt[4]{\left(D_S\times d^3\right)}$$

Electric field and voltage: Solid cylindrical conductor

The capacitance between conductors in a medium with constant permittivity can be obtained by finding the following:

a) Electric field from Gauss's law.

b) Voltage between conductors.

c) Capacitance from charge per unit volt ($C = q/V$)

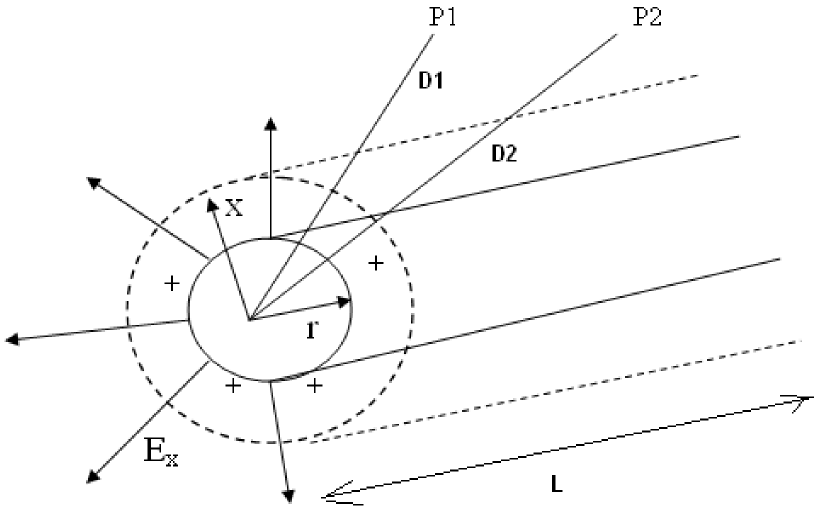
Gauss law: the total electric flux leaving a closed surface equals the total charge within the volume enclosed by the surface.

Electric field and voltage: Solid cylindrical conductor

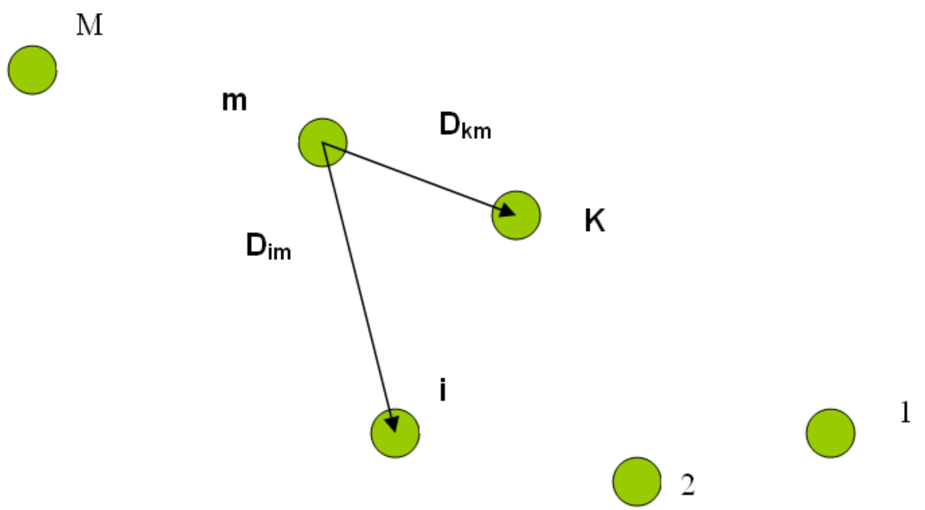
$$\oiint E ds = \frac{Q_{enclosed}}{\varepsilon}$$

$$E_x(2\pi x)(L) = \frac{qL}{\varepsilon} \Rightarrow E_x = \frac{q}{2\pi\varepsilon x}$$

$$V_{12} = \int_{D1}^{D2} E_x dx = \int_{D1}^{D2} \frac{q}{2\pi\varepsilon x} dx = \frac{q}{2\pi\varepsilon} \ln \frac{D_2}{D_1}$$



Electric field and voltage: Solid cylindrical conductor



Assume each conductor m has a charge q_m C/m, the voltage V_{kim} between conductors k and i due to the charge q_m acting alone is:

$$V_{kim} = \frac{q_m}{2\pi\varepsilon} \ln \frac{D_{im}}{D_{km}}$$

Using superposit iion, the voltage V_{ki} due to all charges is given by :

$$V_{ki} = \frac{1}{2\pi\varepsilon} \sum_{m=1}^{M} q_m \ln \frac{D_{im}}{D_{km}}$$

Capacitance for single phase two wire line

Assume conductor x has a uniform charge q C/m and conductor y has –q. Using the previous last equation with k = x, i = y and m = x, y

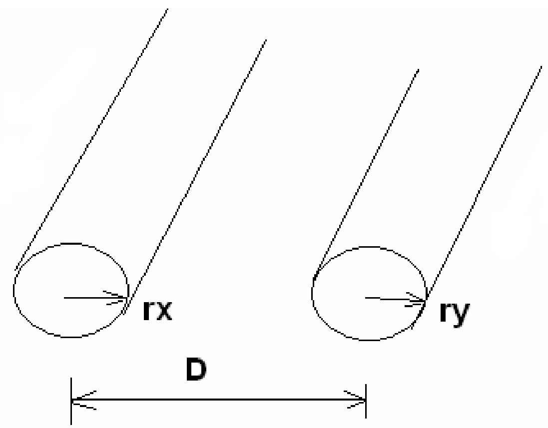
$$Vxy = \frac{1}{2\pi\varepsilon}\left[q\ln\frac{D_{yx}}{D_{xx}} - q\ln\frac{D_{yy}}{D_{xy}}\right] = \frac{q}{2\pi\varepsilon}\ln\frac{D_{yx}D_{xy}}{D_{xx}D_{yy}}$$

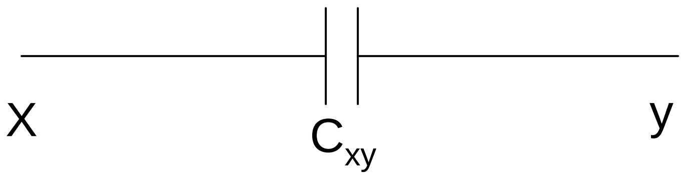
Using $D_{xy} = D_{yx} = D$, $D_{xx} = r_x$ and $D_{yy} = r_y$, then

$$Cxy = \frac{q}{Vxy} = \frac{\pi\varepsilon}{\ln\left(\frac{D}{\sqrt{r_x r_y}}\right)}$$

And if $r_x = r_y = r$, then

$$Cxy = \frac{\pi\varepsilon}{\ln\left(\frac{D}{r}\right)}$$





Capacitance for three phase with equal phase spacing

$$V_{ab} = \frac{1}{2\pi\varepsilon}\left[q_a \ln\frac{D_{ba}}{D_{aa}} + q_b \ln\frac{D_{bb}}{D_{ab}} + q_c \ln\frac{D_{bc}}{D_{ac}}\right]$$

Using $D_{ab} = D_{ca} = D_{cb} = D, D_{aa} = D_{yy} = r$, then

$$V_{ab} = \frac{1}{2\pi\varepsilon}\left[q_a \ln\frac{D}{r} + q_b \ln\frac{r}{D} + q_c \ln\frac{D}{D}\right] = \frac{1}{2\pi\varepsilon}\left[q_a \ln\frac{D}{r} + q_b \ln\frac{r}{D}\right]$$

similarly

$$V_{ac} = \frac{1}{2\pi\varepsilon}\left[q_a \ln\frac{D}{r} + q_c \ln\frac{r}{D}\right]$$

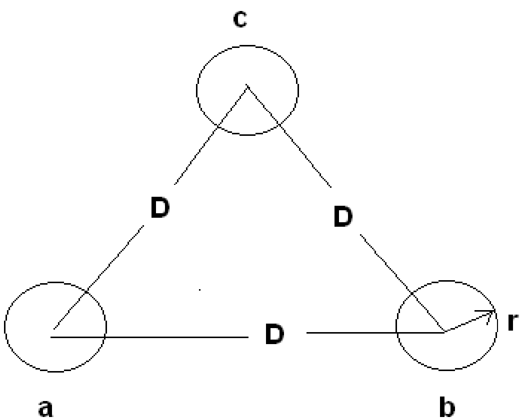
$$V_{ab} + V_{ac} = 3V_{an}$$

$$V_{an} = \frac{1}{3}\left(\frac{1}{2\pi\varepsilon}\right)\left[2q_a \ln\frac{D}{r} + (q_b + q_c)\ln\frac{r}{D}\right]$$

and with $q_b + q_c = -q_a$

$$V_{an} = \frac{1}{2\pi\varepsilon}\left[q_a \ln\frac{D}{r}\right]$$

$$C_{an} = \frac{2\pi\varepsilon}{\ln(D/r)}$$



Capacitance for stranded, unequal phase spacing and bundled conductors

$$C = \frac{2\pi\varepsilon}{\ln(D_{eq} / D_{sc})}$$

$$D_{eq} = \sqrt[3]{D_{ab} D_{bc} D_{ac}}$$

$$D_{sc} = \sqrt{rd} \quad \text{for two - conductor bundle}$$

$$D_{sc} = \sqrt[3]{rd^2} \quad \text{for three - conductor bundle}$$

$$D_{sc} = 1.091\sqrt[4]{rd^3} \quad \text{for four - conductor bundle}$$

Line Resistance

Line resistance per unit length is given by

$$R = \frac{\rho}{A} \quad \text{where } \rho \text{ is the resistivity}$$

Resistivity of Copper = 1.68×10^{-8} Ω-m

Resistivity of Aluminum = 2.65×10^{-8} Ω-m

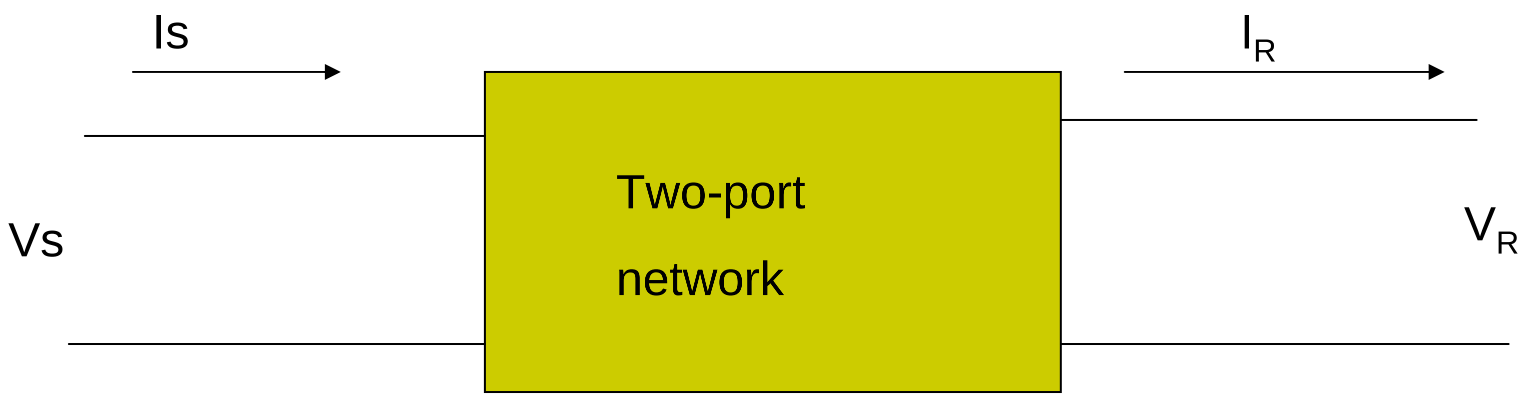
Example: What is the resistance in Ω / mile of a 1" diameter solid aluminum wire (at dc)?

$$R = \frac{2.65 \times 10^{-8}\ \Omega\text{-m}}{\pi \times 0.0127\text{m}^2} 1609 \frac{m}{mile} = 0.084 \frac{\Omega}{mile}$$

Line Resistance, cont'd

- Because ac current tends to flow towards the surface of a conductor, the resistance of a line at 60 Hz is slightly higher than at dc.
- Resistivity and hence line resistance increase as conductor temperature increases (changes is about 8% between 25°C and 50°C)
- Because ACSR conductors are stranded, actual resistance, inductance and capacitance needs to be determined from tables.

Tow-Port Network Model

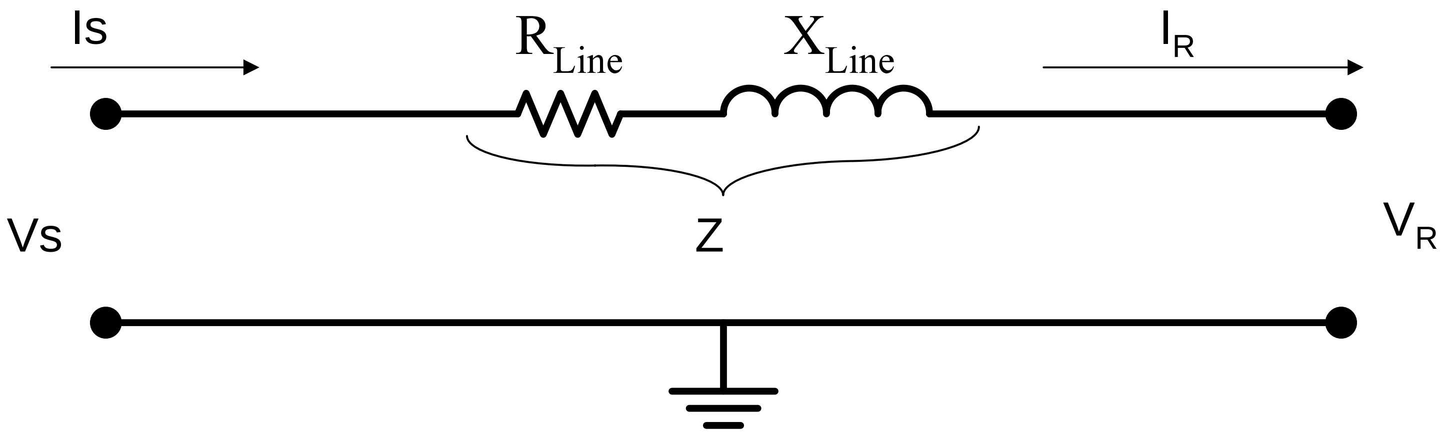


$$V_s = AV_R + BI_R$$

$$I_s = CV_R + DI_R$$

$$AD - BC = 1$$

Short transmission lines



$$V_S = V_R + ZI_R$$

$$I_S = I_R$$

$$A = D = 1$$

$$B = Z$$

$$C = o$$

Medium transmission lines

$$V_S = V_R + Z\left(I_R + \frac{V_R Y}{2}\right) = \left(1 + \frac{YZ}{2}\right)V_R + ZI_R$$

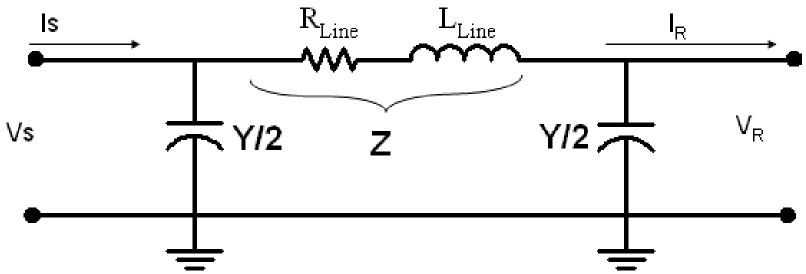
$$I_S = I_R + \frac{V_R Y}{2} + \frac{V_S Y}{2}, \text{subsitute the value of } \mathrm{V_s}$$

$$I_S = Y\left(1 + \frac{YZ}{4}\right)V_R + \left(1 + \frac{YZ}{2}\right)I_R$$

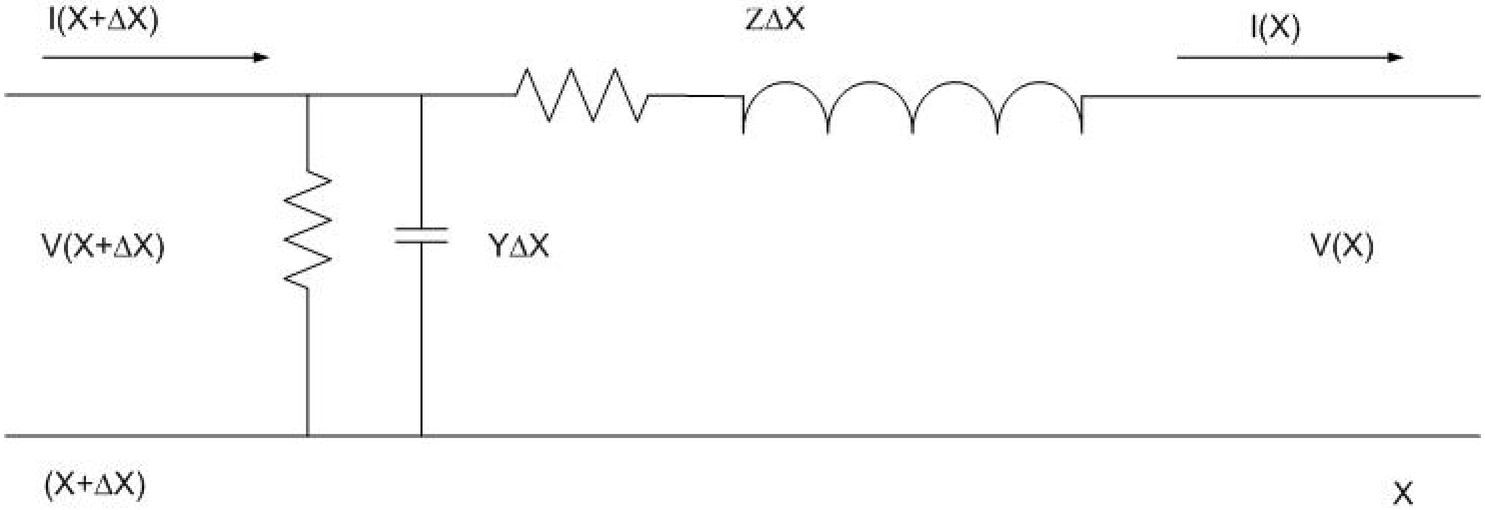
$$A = D = 1 + \frac{YZ}{2}$$

$$B = Z$$

$$C = Y\left(1 + \frac{YZ}{4}\right)$$



Long transmission lines



$$z = R + j\omega L \quad \Omega/\mathrm{m}$$

$$y = G + j\omega C \quad \mathrm{S/m}$$

Long transmission lines, cont.

$$V(x+\Delta x) = V(x) + (z\Delta x)I(x)$$

$$\frac{V(x+\Delta x) - V(x)}{\Delta x} = zI(x)$$

Taking the limit as Δx approaches zero :

$$\frac{dV(x)}{dx} = zI(x)$$

$$I(x+\Delta x) = I(x) + (y\Delta x)V(x+\Delta x)$$

$$\frac{I(x+\Delta x) - I(x)}{\Delta x} = yV(x)$$

Taking the limit as Δx approaches zero :

$$\frac{dI(x)}{dx} = yV(x)$$

$$\frac{d^2V(x)}{dx^2} = z\frac{dI(x)}{dx} = zyV(x)$$

$$\frac{d^2V(x)}{dx^2} - zyV(x) = 0$$

$$V(x) = A_1e^{\gamma x} + A_2e^{-\gamma x}$$

$\gamma = \sqrt{zy}$ is called the propagation constant

$$\frac{dV(x)}{dx} = \gamma A_1e^{\gamma x} - \gamma A_2e^{-\gamma x} = zI(x)$$

$$I(x) = \frac{A_1e^{\gamma x} - A_2e^{-\gamma x}}{Z_c}$$

$Z_c = \sqrt{\frac{z}{y}}$ is called the characteristic impedance.

Since $V_R = V(0) = A_1 + A_2$ and $I_R = I(0) = \frac{A_1 - A_2}{Z_c}$

$A_1 = \frac{V_R + Z_cI_R}{2}$ and $A_2 = \frac{V_R - Z_cI_R}{2}$

so :

$$V(x) = \cosh(\gamma x)V_R + Z_c\sinh(\gamma x)I_R$$

$$I(x) = \frac{1}{Z_c}\sinh(\gamma x)V_R + \cosh(\gamma x)I_R$$

Q1:

Home Work

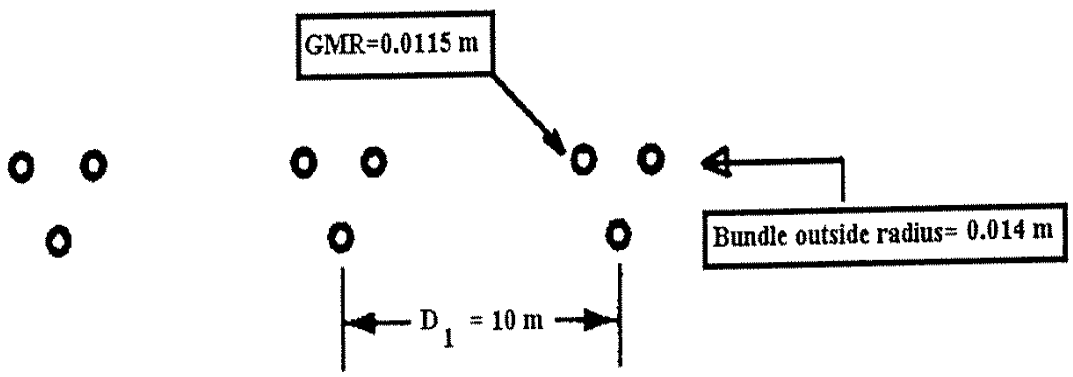
A 300 km, completely transposed 60 Hz, three phase line has flat horizontal phase spacing with 10 m between adjacent phases, as shown in Fig. (1). Each phase consists of a three-bundle conductor, with outside radius of 0.014 m, a GMR, D_s = 0.0115 m, and a bundle spacing of 0.4 m.

b- Calculate the positive-sequence inductive reactance of the line. [4 Marks]

c- Calculate the positive-sequence shunt capacitive susceptance of the line. [4 Marks]

d- Assume that the line has an X/R ratio of 5 and negligible shunt conductance. Find the exact value of the parameter A of the line. [4 Marks]

e- If the no load receiving end voltage of the line is 348 kV (line to line), find the value of the sending end voltage. [4 Marks]



Q2:

Home Work

Consider a 500-kV, 60 Hz three-phase transmission line modeled using the ABCD parameters as follows:

$$V_s = AV_r + BI_r$$
$$I_s = CV_r + AI_r$$
$$A^2 - BC = 1$$

Results of tests conducted at the receiving end the line involving open circuit ($I_r = 0$) and short circuit ($V_r = 0$) are given by:

$$Z_{oc} = \left.\frac{V_s}{I_s}\right|_{I_r=0} = 820\angle \underline{-88.8^o}$$

$$Z_{sc} = \left.\frac{V_s}{I_s}\right|_{V_r=0} = 200\angle \underline{78^o}$$

Find the line parameters A, B, and C. [10 Points]

Suppose that the load at the receiving end of the line of part b is 750 MVA at nominal voltage, and lagging power factor of 0.83 at rated voltage. Determine the sending end voltage, current, active and reactive power and power factor. [5 points]