Coverage Control for Multiple Event Types with Heterogeneous Robots

Armin Sadeghi Stephen L. Smith

Abstract— **This paper focuses on the problem of deploying a set of autonomous robots to efficiently monitor multiple types of events in an environment. There is a density function over the environment for each event type representing the weighted likelihood of the event at each location. The robots are heterogeneous in that each robot is equipped with a set of sensors and it is capable of sensing a subset of event types. The objective is to deploy the robots in the environment to minimize a linear combination of the total sensing quality of the events. We propose a new formulation for the problem which is a natural extension of the homogeneous problem. We propose distributed algorithms that drive the robots to locally optimal positions in both continuous environments that are obstacle-free, and in discrete environments that may contain obstacles. In both cases we prove convergence to locally optimal positions. We provide extension to the case where the density functions are unknown prior to the deployment in continuous environments. Finally, we present benchmarking results and physical experiments to characterize the solution quality.**

I. Introduction

In this paper, we focus on the problem of deploying multiple heterogeneous robots to cover an environment with different event types. Each event type has a different spatial distribution in the environment and can be sensed only with a specific type of sensor. The robots are heterogeneous in that each of the robots is equipped with a subset of sensors. Thus, each robot is capable of measuring a subset of event types. The quality of sensing for an event at a location is a decreasing function of the distance of the sensor from the location. The objective is to deploy the robots to maximize the total coverage quality of the events of different types. This appears in several applications including reconnaissance, surveillance [1] and monitoring [2], as well as in the deployment of vehicles with different capacities and/or capabilities in urban transportation systems [3], [4].

In convex environments, Cortes *et al.* [5] propose a distributed algorithm for homogeneous robots that utilizes Voronoi partitioning and the Lloyd descent algorithm. The proposed control law allows the robots to converge to a local maximum sensing quality over the events in the environment and requires communication only between neighboring robots in the Voronoi partition.

Several studies address different types of heterogeneity in the robots. In [6], the authors consider the problem of sensing an event where the sensors have different functions governing their sensing quality. The approach defines a generalized Voronoi partition based on the sensing functions and provides a Lloyd descent type algorithm for controlling each robot. Other types of heterogeneities for mobile sensors addressed in the literature include the sensors with different sensing ranges [7], [8], [9] and different additive weights on the sensing quality of robots [10]. In [7], the authors address the coverage problem for circular sensors with different radii. In [9], authors provide a gradient descent algorithm for the distributed control of circular sensors with different radii in a non-convex environment.

All the aforementioned studies consider the coverage control for a single event type. In [11], the authors introduced the coverage control problem for multiple event types, each with a different density function. An event of a certain type can be sensed by a robot if the robot is equipped with the required sensor. The proposed approach considers a Voronoi partition generated by all the robots, and the objective function is a convex combination of the sensing quality of each robot for the events inside its Voronoi cell and the sensing quality over the whole environment. In this paper, we consider the same heterogeneous problem, but we pose a different objective function, and thus distributed control law. As such, we compare our approach to [11] in detail, and demonstrate its advantages in simulation. In our approach, we define a Voronoi partition of the environment for each event type, each generated by the robots with the suitable sensors. With these partitions, we are able to ensure that each event is sensed by the closest robot with the required sensor. Moreover, our formulation captures the case where sensors for different events have different sensing functions.

Several studies considered the coverage control in an environment where the event density is unknown prior to deployment [12], [13]. In [12], the authors assume a basis function approximation for the event density with adaptive weights for each robot and propose a decentralized algorithm that updates the weights estimating the true density. The authors in [13] provide a simple stochastic gradient descent algorithm without estimating the real event density and prove convergence to a locally optimal solution. Built on the results in [13], we extend our analysis to environments with multiple events types each with an unknown density.

In a discrete environment represented by a graph, the objective of the coverage problem is to sense events occurring on the vertices of the graph. A closely related problem is the p-median problem [14], [15] where a set of service units are located on the vertices of a graph servicing demands arriving on the vertices with the objective of minimizing the total service time. In [16], a distributed algorithm is proposed for partitioning and coverage of a non-convex environment. Yun and Rus [17] presented a distributed vertex swap algorithm for the robots that converges to locally optimal solutions with two-hop communication. To the best of our knowledge, the existing literature on the distributed coverage control on

This research is partially supported by the Natural Sciences and Engineering Research Council of Canada (NSERC).

The authors are with the Department of Electrical and Computer Engineering, University of Waterloo, Waterloo ON, N2L 3G1 Canada (a6sadegh@uwaterloo.ca; stephen.smith@uwaterloo.ca)

graphs is limited to homogeneous robots sensing a single event type. In contrary, we consider multiple event types with heterogeneous robots both in the sensing capability and quality.

Contributions: The contributions of this paper are three-fold. First, we propose a new formulation for the problem of coverage control for heterogeneous mobile robots, which is a natural extension of the homogeneous formulation. Second, we provide a distributed algorithm for maximizing the sensing quality with the new formulation. Third and finally, we extend the results to discrete environments with different event types.

II. Problem Formulation

In this section, we formulate the coverage problem for continuous and discrete environments. We consider a set of m robots in an environment with k different event types. Each event type is measured with a different sensor in $\mathcal{S}$, and a robot i is equipped with a subset of these sensors, i.e., $S_i \subseteq \mathcal{S}$. The objective is to position the robots in the environment maximizing the total sensing quality of k event types.

A. Continuous Environments

Consider a convex environment $\mathcal{D} \subset \mathbb{R}^2$. Let $[k]$ denote the set $\{1, \dots, k\}$. The density function of an event type j is $\phi_j : \mathcal{D} \to \mathbb{R}_+, j \in [k]$, which represents the measure of information or the probability of an event type occurring over $\mathcal{D}$. Let $p_i \in \mathcal{D}$ be the position of robot i in the environment and $\mathcal{P} = \{p_1, \dots, p_m\}$. The sensing quality of a sensor j, denoted by f^j, is a decreasing function of the distance of the robot to the measured point. Thus, robot i is assigned to measure the events of type $j \in S_i$ at the points closest to p_i, i.e., for each event type $j \in S_i$ robot i measures the events in the Voronoi cell

$$V_i^j = \{q \in \mathcal{D} |\ ||q - p_i|| \leq ||q - p_r||, j \in S_i \cap S_r, \forall r \in [m]\}.$$

For simplicity we let V denote the set of all the Voronoi cells. Now we define the heterogeneous deployment problem in continuous environments as follows:

Problem II.1 (Coverage in Continuous Environments)**.** Given a set of m robots with dynamics $\dot{p_i} = u_i \ \forall i \in [m]$ where u_i is the control input, find a set of locations $\mathcal{P} = \{p_1, \dots, p_m\}$ that maximizes the total sensing quality function, i.e.,

$$\mathcal{H}(\mathcal{P}, V) = \sum_{i=1}^{m} \sum_{j \in S_i} \int_{V_i^j} f^j(||q - p_i||)\phi_j(q)dq. \qquad (1)$$

The sensing quality measure $\mathcal{H}(\mathcal{P}, V)$ is the total sensing quality of k event types over the environment by the robots located at $\mathcal{P}$. Contrary to the formulation in [11], by defining k Voronoi partitions for the environment, we ensure that an event of type j at location q is sensed by the closest robot with the required sensor. Moreover, the definition of $\mathcal{H}(\mathcal{P}, V)$ captures different sensing quality function f^j for each event type j. Observe that with $k = 1$, the quality measure $\mathcal{H}(\mathcal{P}, V)$ becomes the sensing quality for homogeneous robots as proposed in [5]. Figure 1 illustrates an instance of the coverage problem with two event types.

In [11], the authors considered the same heterogeneous coverage problem, but with the objective

$$\mathcal{H}_{het}(\mathcal{P}) = \sigma \sum_{i \in [m]} \int_{V_i} ||q - p_i||^2 \phi_{S_i} dq + (1 - \sigma) \sum_{i \in [m]} \int_{\mathcal{D}} ||q - p_i||^2 \phi_{S_i} dq$$

where $\sigma \in (0, 1]$, V_i is the Voronoi partition generated by the positions of all robots and $\phi_{S_i} = \sum_{j \in S_i} \phi_i^j$. To illustrate the differences with the proposed objective in Equation (1), Figure 2 shows a coverage problem on a line with three event types. Each robot senses the event type of the same color. The colored polygons represent the density function of each event type. As shown in Figure 2a, the objective in [11] has locally optimal configurations in which one event type is covered sub-optimally. This is in contrast to the configuration shown in Figure 2b, where a partition is generated for each event type.

B. Discrete Environments

In the discrete case, the environment is represented by an undirected graph $G = (\mathcal{V}, E, c)$. The graph could represent a roadmap of an environment with obstacles. Let $\mathcal{V}$ be the set of vertices, and let E be the edge set, which represents the paths between vertices. The cost function $c : E \to \mathbb{R}_+$ assigns a cost $c(e)$ for traversing each edge $e \in E$. The events occur on the vertices of the graph, and the function ϕ_u^j gives the mass of event type j at vertex $u \in \mathcal{V}$. Let $d(u, v)$ be the length of the shortest path from u to v on graph G. The goal is to deploy robots to a set of vertices of the graph. We do not consider deployments in which robots are located on the edges of the graph, and this is motivated by the result on the properties of locating robots on vertices given in Lemma IV.1.

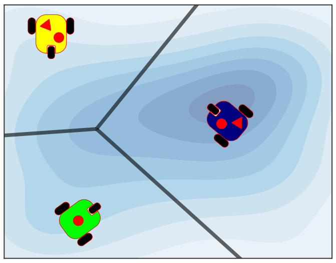
An event of type j is assigned to the closest robot among the robots that are equipped with the sensor type j. If there exists an even with two equally close robots $i, j \in [m]$, we assign the event to robot i if $i > j$ and robot j otherwise. The subset $W_i^j(\mathcal{P})$ is the set of vertices assigned to robot i to sense events of type $j \in S_i$ on those vertices. Observe that W_i^j is the discrete analogue of V_i^j.

For simplicity, we let W denote the set of all the subsets W_i^j. Now we define the heterogeneous deployment problem in discrete environments as follows:

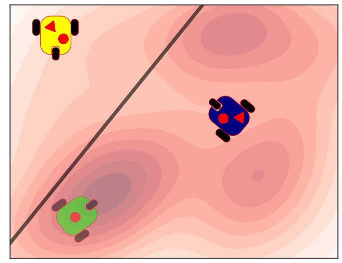
Problem II.2 (Coverage in Discrete Environments)**.** Given a set of m robots find a set of locations on the graph $\mathcal{P} = \{p_1, \dots, p_m\}$ such that maximizes the total sensing quality function, i.e.,

$$\mathcal{H}(\mathcal{P}, W) = \sum_{i=1}^{m} \sum_{j \in S_i} \sum_{v \in W_i^j} f^j(d(p_i, v))\phi_v^j. \qquad (2)$$

Observe that the objective functions is the adaptation of the objective function (1) to discrete environments. Also note that with $k = 1$, the quality measure $\mathcal{H}(\mathcal{P}, W)$ becomes the sensing quality for homogeneous robots as proposed in [17].

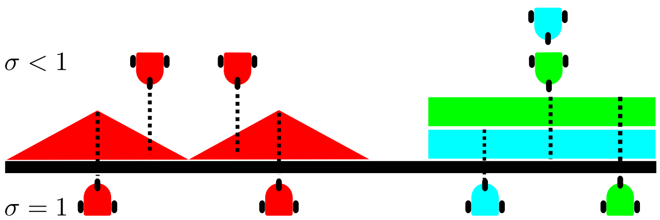


(a) Event density of event type 1, i.e., ϕ_1. The sensor capable of sensing this event is shown with a circle on the robots.

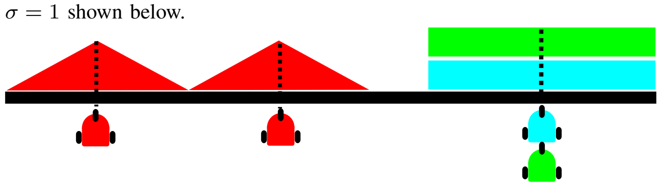


(b) Event density of event type 2. The sensor capable of sensing this event is shown with a triangle on the robots.

Fig. 1: The Voronoi partitions of an environment for each event type. A robot considered in a Voronoi partition if it is equipped with the required sensor for the event type.



(a) Locally optimal configuration for [11] with $\sigma < 1$ shown on top and $\sigma = 1$ shown below.



(b) Locally optimal configuration for Equation (1)

Fig. 2: Instance of coverage of three event types on a line. Triangles (resp. rectangle) show the density function for event type 1(resp. 2 and 3). Robots are capable of sensing the event types with the same color.

III. Gradient Ascent Algorithm for Continuous Environments

In this section, we provide a distributed control law for robots with a proof of convergence to a local maximum of $\mathcal{H}(\mathcal{P}, V)$ in Problem II.1.

A well-known approach to maximize the sensing quality for each robot to ascend the gradient of $\mathcal{H}$. Let δV_i^j be the boundary of the Voronoi cell V_i^j, and let δV_{ik}^j be the common boundary of the Voronoi cells V_i^j and V_k^j. Let $\mathbf{n}_{ik}^j(q)$ be the unit normal to δV_{ik}^j at q in the outward direction of V_i^j. Finally, we define the neighbors of a robot i in Voronoi partition j as $\mathcal{N}_i^j = \{l \in [n] | \delta V_i^j \cap \delta V_l^j \neq \emptyset\}$.

Then we establish the results on the derivative of $\mathcal{H}$ with respect to the position of robots in Lemma III.1. The results in Lemma III.1 is an extension to the homogeneous case in [5] and the proof follows closely.

Lemma III.1. *For differentiable sensing functions f^j, the derivative of $\mathcal{H}$ with respect to the position of robot i is*

$$\frac{\partial \mathcal{H}}{\partial p_i} = \sum_{i=1}^{m} \sum_{j \in S_i} \int_{V_i^j} \frac{\partial f^j(||q - p_i||)}{\partial p_i} \phi_j(q) dq.$$

Proof. By the Leibnitz theorem [18] we have,

$$\begin{aligned}\frac{\partial \mathcal{H}}{\partial p_i} =& \sum_{i=1}^{m} \sum_{j \in S_i} \Big(\int_{V_i^j} \frac{\partial f^j(||q - p_i||)}{\partial p_i} \phi_j(q) dq \\ &+ \sum_{l \in \mathcal{N}_i^j} \int_{V_i^j} f^j(||q - p_i||) \mathbf{n}_{il}^j(q) \frac{\partial (\delta V_{ik}^j)}{\partial p_i}(q) \phi_j(q) dq \\ &+ \sum_{l \in \mathcal{N}_i^j} \int_{V_i^j} f^j(||q - p_i||) \mathbf{n}_{li}^j(q) \frac{\partial (\delta V_{ik}^j)}{\partial p_i}(q) \phi_j(q) dq \Big).\end{aligned}$$

Observe that $\mathbf{n}_{il}^j = -\mathbf{n}_{li}^j$ for all $l \in \mathcal{N}_i^j$. Then the result follows immediately. □

An interesting observation from this is that unlike the homogeneous version of the coverage problem, two robots can share a location in $\mathcal{D}$ if they do not have sensors in common, i.e., the derivative is defined for any $\mathcal{P} \in \mathcal{D}^m \setminus Q_m$ where Q_m is the set of points in which robots with common sensor type overlap, i.e.,

$$\begin{aligned}Q_m = \{\{p_1, \dots, p_m\} \in \mathcal{D}^m | \exists i, j \in [m] \; p_i = p_j, \\ i \neq j, S_i \cap S_j \neq \emptyset\}.\end{aligned}$$

To derive a distributed control law, we first rewrite the derivative of $\mathcal{H}$ in the following form.

$$\frac{\partial \mathcal{H}}{\partial p_i} = 2 \sum_{j \in S_i} \int_{V_i^j} (p_i - q) \frac{d f^j(x)}{d(x^2)} |_{||q - p_i||} \phi_j(q) dq. \tag{3}$$

Let the mass and centroid of a Voronoi cell respectively be

$$\begin{aligned}M_{V_i} &= \int_{V_i} \frac{df}{d(x^2)} |_{||p_i - q||} \phi(q) dq, \\ C_{V_i} &= \frac{1}{M_{V_i}} \int_{V_i} q \frac{df}{d(x^2)} |_{||p_i - q||} \phi(q) dq.\end{aligned}$$

Then we have $\frac{\partial \mathcal{H}}{\partial p_i} = 2 \sum_{j \in S_i} M_{V_i^j}(p_i - C_{V_i^j})$. Now consider the following simple control law

$$u_i = -c_i \frac{1}{\sum_{j \in S_i} M_{V_i^j}} \sum_{j \in S_i} M_{V_i^j}(p_i - C_{V_i^j}) \quad \forall i \in [n], \tag{4}$$

where c_i is a positive gain such that $p_i + u_i$ remains in the convex set $\cap_{j \in S_i} V_i^j$. Robot i is the generator of a cell in each of the Voronoi partitions of the events in S_i, and the control law u_i moves the robot i to the average of the centroids of its Voronoi cells. Observe that computing this control law only requires the communication between robot i and its neighbors in each Voronoi partition. We assume that the communication range is sufficient for the robots to communicate with their neighboring robots.

The following shows the result on the convergence of the proposed distributed control law in Equation (4).

Proposition III.2 (Continuous-time Lloyd Ascent)**.** *Under control law* (4)*, the robots converge to the union of Q_m and the set of critical points of $\mathcal{H}$.*

Proof. Under control law (4), if c_i of robot i at some time instance becomes zero then robot i is on the boundary of $\cap_{j \in S_i} V_i^j$ and coinciding with another robot. Therefore, the robots have converged to Q_m. Otherwise, the robots starting from a collision free configuration in $\mathcal{D}^m \setminus Q_m$ remain inside $\mathcal{D}^m \setminus Q_m$ at any time instance, then the set $\mathcal{D}^m \setminus Q_m$ is a positively invariant set under control law (4). The derivative of $\mathcal{H}$ with respect to time is

$$\frac{d}{dt}\mathcal{H}(\mathcal{P}(t), V) = \sum_{i=1}^{m} \frac{\partial \mathcal{H}}{\partial p_i}\dot{p}_i = -\sum_{i=1}^{m} \frac{2c_i}{\sum_{j \in S_i} M_{V_i^j}}(\frac{\partial \mathcal{H}}{\partial p_i})^2.$$

Therefore, observe that the direction of control law (4) coincides with the gradient of $\mathcal{H}$. The rest of the proof follows from the proof of Proposition 3.1 in [19], which uses LaSalle's Invariance Principle to show at each step of the algorithm $\mathcal{H}$ monotonically increases and converges to the largest invariant set contained in

$$C_m = \{P \in D^m | (\frac{\partial \mathcal{H}(\mathcal{P})}{\partial p_i})^2 = 0 \quad \forall i \in [n]\} \qquad \square$$

Although control law (4) converges to a set consisting of Q_m, simulation results in Section V show that by resolving collisions with local a controller, the system remains in $\mathcal{D}^m \setminus Q_m$ and converges to the set of critical points of $\mathcal{H}$.

Remark III.3 (Unknown Density Functions). Consider the scenario where the robots do not have access to the density functions of the event types and they only observe events of different types arriving over time. A control law is proposed in [13] for the homogeneous case which drives the closest robot to the observed event and converge to a locally optimal positions. In the heterogeneous case, each observed event $Z = [z^d, z^c]$ is a random vector consisting of a discrete component $z^d \in [k]$, denoting the observed event type, and a continuous component $z^c \in \mathcal{D}$ representing the location of the event. The events of different types arrive with equal frequencies and according to their spatial density functions. We assume that the relative importance of the event types, denoted by $\Phi_j = \int_{\mathcal{D}} \phi_i(q)dq$ is known. By extending the results in [13], the control law become

$$p_{i,t+1} = \begin{cases} p_{i,t} - \gamma_t \Phi_j \frac{df^j}{dx^2}|_{||z_t^c - p_{i,t}||}(z_t^c - p_{i,t}) & \text{if } z_t^d \in S_i, z_t^c \in V_i^{z_t^d} \\ p_{i,t} & \text{otherwise.} \end{cases} \quad (5)$$

Observe that the expected value of the direction $\frac{df^j}{dx^2}|_{x=||z_t^c - p_{i,t}||}(z_t^c - p_{i,t})$ coincides with the direction of the derivative of $\mathcal{H}$ with respect to p_i in Equation (3). The control law (5) drives the closest robot with the required equipment towards the observed event. The convergence of the control law to locally optimal positions arrive from the proof of Theorem 3 in [13]. •

IV. Gradient Ascent Algorithm for Discrete Environments

In this section, we discuss the coverage problem for heterogeneous robots in a discrete environment with multiple event types (Problem II.2 in Section II). Given a graph, the goal is to position the robots on the vertices of the graph such that the total sensing function $\mathcal{H}$ is maximized.

We define two configurations of the robots on the graph. The first configuration $Q \subset \mathcal{V}$ is a subset of size m of the vertices of the graph which represents positioning the robots on the vertices. The second configuration $D \subset E \times [0, 1]$ represents the placement of the robots on the edges. For a placement of robot i on edge (u, v), i.e., $p_i = ((u, v), \gamma)$ (see Figure 3), parameter γ is the fraction of the path from p_i to v along the edge (u, v). The distance of robot i from vertex $w \in \mathcal{V}$ is $d'(p_i, w) = \min\{\gamma c(u, v) + d(u, w), (1 - \gamma)c(u, v) + d(v, w)\}$.

For a configuration D, each event is assigned to the closest robot with required equipment. Then total sensing function for configuration D and partition $W(D)$ is

$$\mathcal{H}'(D, W(D)) = \sum_{i=1}^{m} \sum_{j \in S_i} \sum_{v \in W_i^j} f^j(d'(p_i, v))\phi_v^j. \quad (6)$$

We provide a gradient ascent control law to position the robots in the graph maximizing Equation (2). To motivate the approach, first, we provide the following result on the optimal solution of the discrete problem.

Lemma IV.1. *For any placement of the robots in the graph D, there exists another placement of the robots on the vertices Q such that*

$$\mathcal{H}'(D, W(D)) \leq \mathcal{H}(Q, W(Q)).$$

Proof. For any placement of the robots in the graph, if there exists a robot located on the edge of the graph, we create another placement without decreasing the total sensing quality. Figure 3 illustrates an instance in which a robot i is located on the edge (u, v). For each event type $j \in S_i$, we partition the vertices in W_i^j into two subset where the first subset $W_{i,u}^j$ consists of the vertices in W_i^j such that the shortest path from p_i contains u and similarly the second subset $W_{i,v}^j$ consists of the vertices in W_i^j such that the shortest path for the robot to reach the vertex passes through v. The total event mass on the first subset is $\sum_{j \in S_i} \sum_{w \in W_{i,u}^j} \phi_w^j$ and similarly for the second subset is $\sum_{j \in S_i} \sum_{w \in W_{i,v}^j} \phi_w^j$. Then we move the robot to the vertex with a larger total event mass. Observe that moving the robot to the vertex with a larger total event mass can only increase the total sensing quality. $\square$

This result motivates us to consider the cases where the robots are located on the vertices of the graph. The set of admissible controls for robot i, $\mathcal{U}_i$, is limited to $\cup_{j \in S_i} W_i^j(\mathcal{P}_t)$

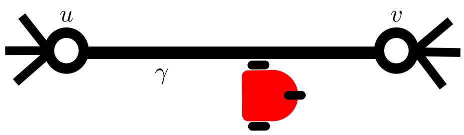


Fig. 3: Robot located on an edge of a graph.

to ensure that the robots are required to communicate with only their neighboring robots. The robots i, k are called neighbors if there exists j such that the sets W_i^j, W_k^j share in edge on graph G. Observe that the control set $\mathcal{U}_i$ is a non-empty set since it contains the current location of the robot. We say the robots are in a *locally optimal configuration* if there is no robot can take a control input in its control set to improve the sensing quality unilaterally.

Now we provide the control law on the graph as follows:

$$p_{i,t+1} = \underset{u \in \mathcal{U}_i}{\arg\max} \sum_{j \in S_i} \sum_{w \in W_i^j(\mathcal{P}_t)} f^j(d(u,w))\phi_w^j. \quad (7)$$

Control law (7) drives the robots to a configuration on the graph maximizing the total sensing quality of the events in the partitions dominated by the robot. This resembles the control law in Equation (4). The following lemma provides the results on the convergence of the control law (7).

Lemma IV.2. *Under the control law* (7)*, the robots converge to a set of locally optimal locations for the total sensing problem in discrete environments.*

Proof. By the definition of the control law in Equation (7) we move robot i to the new position such that it maximizes the sensing quality for the vertices in the partitions of robot i, i.e., $W_i^j(\mathcal{P}_t), j \in S_i$. Considering a fixed partition, by any robot relocating on the graph, the total sensing quality improves, i.e., $\mathcal{H}(\mathcal{P}_t, W(\mathcal{P}_t)) \leq \mathcal{H}(\mathcal{P}_{t+1}, W(\mathcal{P}_t))$.

Observe that the sensing quality is maximized when each event is assigned to the closest robot. Then by updating the partition for the new positions of the robots we have

$$\mathcal{H}(\mathcal{P}_{t+1}, W(\mathcal{P}_t))) \leq \mathcal{H}(\mathcal{P}_{t+1}, W(\mathcal{P}_{t+1})).$$

Therefore, the quality improves with each step of the control law and the algorithm converges to a locally optimal solution of the discrete total sensing quality problem. □

V. Experimental Results

In this section, we evaluate the performance of the control laws for both continuous and discrete environments.

A. Continuous Environment

The performance of the proposed control law for continuous environments is evaluated with four experiments. Each experiment consists of 8 GRITSBots [20] with different sensing capabilities. The proposed control law is implemented on the Robotarium [21] using both simulation and physical experiments. The density function for each event type j is given by a bivariate normal distribution, i.e.,

$$\phi_j(q) = \frac{\beta_j}{2\pi\sqrt{|\Sigma|}} \exp\Big(-\frac{1}{2}(q-\zeta_j)^T\Sigma^{-1}(q-\zeta_j)\Big),$$

	β_j	$\mathcal{S}$
Exp. 1	$\beta_j = 1\ j \in [k]$	$S_1 = S_7 = \{3,4\}, S_2 = \{2,4\}$, $S_3 = \{1,2,3\}, S_4 = \{1,3\}, S_8 = \{4\}$ $S_5 = \{1,2,3,4\}, S_6 = \{1,2\}$
Exp. 2	$\beta_j = 1\ j \in [k]$	$S_i = \{1,2\}\ i \leq 4,\ S_i = \{3,4\}\ i \geq 5$
Exp. 3	$\beta_j = 1\ j \in [k]$	$S_i = \{j \in [k] \| j <= i\}$
Exp. 4	$\beta_j = j\ j \in [k]$	$S_i = [k]$

TABLE I: Parameters of the experiments.

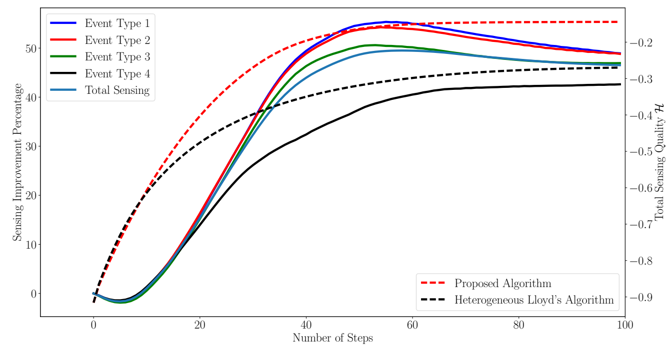


Fig. 4: Performance of the proposed control law for Experiment 3

where ζ_j is the mean and $\Sigma = [0.1, 0; 0, 0.1]$ is the covariance matrix. The parameter β_j is a scaling factor to represent the importance of each event type. The sensing function is for all event types $f^j(x) = -x^2, j \in [k]$. Table I shows the parameters of each experiment.

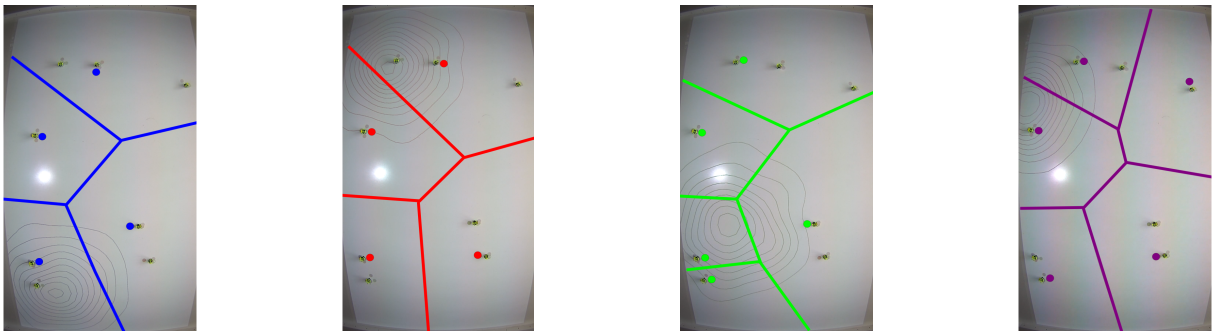
In Figure 4, the sensing quality of the proposed control law is compared to that of the Heterogeneous Lloyd's algorithm proposed in [11]. The results are the average of 100 instances of Experiment 3 with different uniformly random initial locations for the robots. The mean values $\zeta_j, j \in [k]$ of density functions for each instance are generated randomly in a 1×1 environment. The dashed lines represent the total sensing quality in Equation (1) for both algorithms. The solid lines in Figure 4 show the significant improvement of the sensing quality for each event type with respect to the Heterogeneous Lloyd's algorithm. The percentage improvement is the ratio of the difference between the total sensing quality of the Heterogeneous Lloyds algorithm to the total sensing quality of the proposed algorithm. The comparison of the two algorithms on different experiments are given in Table II. The proposed control law outperforms the existing algorithm in total sensing quality and in the sensing quality of each event type. Figure 5 illustrates the final configuration of the robots in the physical implementation of the proposed algorithm in the Robotarium with 8 robots for an instance of Experiment 1. In the experiment, the proposed algorithm improved the total sensing quality by 87.7% in 1 minute.

B. Discrete Environment

In the discrete environment, we evaluate the performance of the proposed control law in a deployment problem motivated by transportation applications. Figure 6 shows 100 randomly generated pick-up locations in Manhattan. Requests for rides arrive at the pick-up locations. Each request has a capacity requirement in $\{2, 3, 4, 5\}$, giving the size of the

	Event 1		Event 2		Event 3		Event 4		Total Sensing	
	Avg.	σ	Avg.	σ	Avg.	σ	Avg.	σ	Avg.	σ
Exp. 1	57.3	7.1	56.3	8.6	56.3	7.9	42.2	6.5	55.3	5.0
Exp. 2	57.8	4.3	57.1	3.9	57.8	4.3	57.9	4.4	58.1	2.1
Exp. 3	47.9	5.5	58.2	4.4	46.3	5.0	41.5	7.3	46.3	3.2
Exp. 4	54.8	5.0	55.5	4.5	57.6	4.2	56.0	3.2	56.6	1.8

TABLE II: Average percentage improvement of the sensing quality for the proposed algorithm compared to Heterogeneous Lloyd's algorithm over 100 instances for each experiment. The mean deviation from average is denoted by σ.



(a) Event type 1 (b) Event type 2 (c) Event type 3 (d) Event type 4

Fig. 5: Eight robots with different sensing capabilities in the Robotarium with the environment size of 2×3.2 meters. The density functions of each event type are shown via contours. The colored circles next to the robots show the generators of each Voronoi partition.

group seeking a ride. There are 5 vehicles of each capacity $\{2, 3, 4, 5\}$ requirement. There are then four event types, one for events with each capacity requirement. Observe that a vehicle with capacity i can serve any request with a group size of $k \leq i$. The objective is to position the vehicles in the environment to minimize the time to respond to a request. Note, here we are considering only the initial deployment problem, which is to place the vehicles at locations to best respond to an initial request.

Figure 7 shows the improvement in the total service quality for different event types and the total service quality. The results are the average of 1000 randomly generated mass functions. The lines show the mean value and the shaded area represent the first and third quartiles of each data set. Notice that the event type with capacity 2 is serviced by all the vehicles, and the algorithm shows the best improvement for this event type.

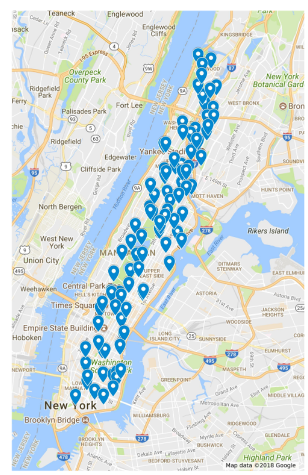


Fig. 6: Pick-up locations in Manhattan N.Y.

VI. Conclusion

This paper considers the problem of coverage control of multiple robots with heterogeneous sensing capabilities in continuous and discrete environments. A new formulation is introduced for measuring the coverage of multiple event types with different event distributions, and a distributed control law is presented for maximizing the coverage of the robots in the environment, which only requires communication between neighboring robots. The extensive results show significant improvement in the sensing quality of the events compared to the existing studies. For future work, we plan to extend the analysis to capture different sensing functions for the same event type on different robots, and more complex event types such as pick-up-and-delivery tasks in ride-sharing applications.

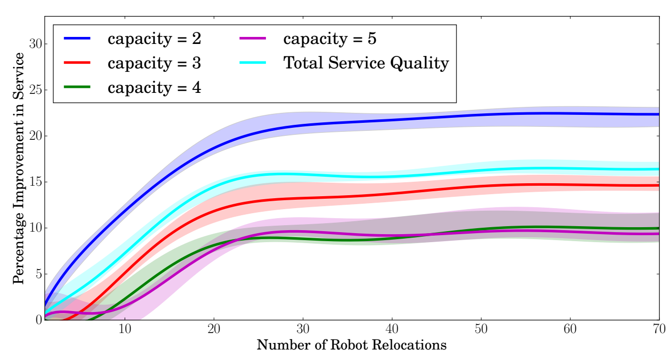


Fig. 7: Percentage improvement of the service time

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