Assignment 2

Due on Friday, Oct. 25^{th} (before the class)

Please read the rules for assignments on the course web page (`https://ece.uwaterloo.ca/~smzahedi/crs/ece700t7/`). Use Piazza (preferred) or directly contact me (smzahedi@uwaterloo.ca) with any questions. **Please note that you are required to email me your codes before the deadline.**

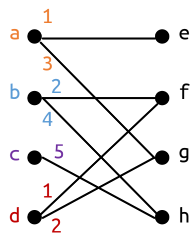

Figure 1: A bipartite matching problem.

1. Maximum Weighted Bipartite Matching. (30 points). A bipartite graph is a graph whose vertices can be divided into two disjoint and independent sets. Given a bipartite graph with weights on the edges, subset of edges is a feasible matching if no two of the edges in the subset share a vertex. The value of a matching is the sum of the weights for matched edges. The goal is to find a feasible matching with the maximum value.

a. Formulate the maximum weighted bipartite matching problem as an integer linear program for the bipartite graph in Figure 1 (20 points).

b. Express your integer program of (a.) in a modelling language of your choice and provide a solution with its optimal value (10 points).

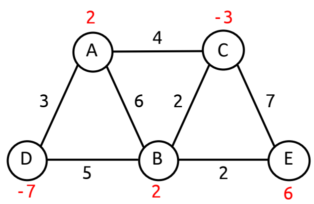

Figure 2: A transshipment problem.

2. Transshipment. (30 points). Consider an undirected graph (V, E). Suppose that each vertex $v \in V$ has a demand d_v, which is the net amount of the commodity that it wants to receive. Negative d_v indicates that vertex v has a net supply rather than a net demand, that is, the vertex is a producer of the commodity. Suppose that we can transport an unlimited amount across each edge, but associated with each edge $e \in E$, there is a cost c_e of transporting one unit across e. Suppose further that transportation is possible with the same cost in both directions for each edge. Assuming that demand and supply are perfectly matched (*i.e.*, $\sum_{v \in V} d_v = 0$), the goal is to find a flow such that all demands are met (exactly), while incurring the minimum total transportation cost.

a. Suppose that only integer amounts of commodity can be transported over each edge. Formulate the minimum cost transshipment problem as an integer program for the graph in Figure 2, where black number over each edge indicate costs and red numbers indicate demands (20 points).

b. Express your integer program of (a.) in a modelling language of your choice and provide a solution with its optimal value. Then, suppose that fractional amounts of commodity can also be transported over each edge. Solve the linear program and provide a solution with its optimal value (10 points).

3. Game Theory. (40 points).

a. Express a mixed integer linear program to solve (2.a) from Assignment 1 in a modelling language of your choice and provide a solution (20 points).

b. Express a linear program to solve (3.a) from Assignment 1 in a modelling language of your choice and provide a solution with its optimal value (20 points).