Assignment 3

Due on Friday, Nov. 15^{th} (before the class)

Please read the rules for assignments on the course web page (`https://ece.uwaterloo.ca/~smzahedi/crs/ece700t7/`). Use Piazza (preferred) or directly contact me (smzahedi@uwaterloo.ca) with any questions.

1. Teamwork (20 points). Consider the following game-theoretic model of the equilibrium determination of the level of effort team members put into a team project, with team size of 2. In this game, the two team members simultaneously choose the level of effort, e_1 and e_2, to spend on the project. They each get utility from progress on the project (which is a function of the sum of the efforts) and dis-utility from the effort they personally expend. Agent 1, values the achievement on the project more. Specifically, assume that e_1 and e_2 are chosen from the set of non-negative real numbers and for $k > 1$, we have:

$$\begin{aligned} u_1(e_1, e_2) =& k. \log(e_1 + e_2) - e_1, \\ u_2(e_1, e_2) =& \log(e_1 + e_2) - e_2. \end{aligned}$$

a. Find best response functions for each of the agents (5 points).

b. Find the pure strategy Nash equilibria of the game. How does the distribution of effort in equilibrium reflect the difference in the agent's preferences (5 points)?

c. Now, suppose that agents play this game sequentially. First, assume that agent 1 decides how much effort she wants to put into the project. Agent 2 observes agent 1's level of effort, and then she decides how much effort to put into the project. Model this new game as an extensive form game and find its subgame perfect equilibria (5 points).

d. This time, repeat c. assuming that agent 2 decides first and agent 1 decides after observing agent 2's decision. From b, c, and d, which outcome is more fair in your opinion (5 points)?

2. Cournot Competition (20 points). There are two (and only two) firms in a market, making identical products. Each firm decides on a quantity $q_i \in [0, 5]$. The price of the product is determined by the total production as $P = \max\{6 - (q_1 + q_2), 0\}$. Each firm's costs are zero, so wishes to maximize profit: $v_i = q_i P$.

a. Suppose the firms make their choices simultaneously. Find the unique NE (5 points).

b. Suppose that firm 1 first chooses q_1, which firm 2 sees and then selects q_2. Is the NE outcome from part (a) still a NE outcome of this extensive form game? Why? If so, is it unique? Why (8 points)?

c. What is the subgame perfect equilibrium? Explain any difference (7 points).

3. Two-party Bargaining (30 points). There are two agents bargaining over the split of a dollar. In odd rounds, agent 1 proposes the split, in even rounds, agent 2 does. The number of rounds, R, is finite and odd. The game ends when either (i) the proposer's split is accepted in round r, or (ii) all proposals are rejected, in which case payoffs are zero for both. If the game ends in round R with agent i receiving amount y, then $v_i = \delta^{(r-1)}y$, where $\delta \in (0, 1)$.

a. Find the subgame perfect equilibrium for any R and prove that it is *unique* (5 points).

b. In what round is agreement reached and what is the utility of each agent (5 points)?

c. Fix R and take the limit of utilities as $\delta \to 1$. Explain the intuition behind the answer (5 points).

d. Fix δ and take the limit of utilities as $R \to \infty$ (5 points).

e. Use the answer to part (d) to find an equilibrium in the game where $R = \infty$ (hint: verify using one-shot deviation principle) (10 points).

4. Three-party Bargaining (30 points). There are three agents bargaining over the split of a dollar. The protocol is as follows. At the beginning of round 1, agent i is recognized to be the proposer with probability p_i where $p_1+p_2+p_3 = 1$ and $p_1 < p_2 < p_3$. The proposer makes a proposal (x_1, x_2, x_3), such that the amount proposed for j is $x_j \geq 0$ for all j and $x_1 + x_2 + x_3 \leq 1$. Then the agents vote on the proposal: if it receives two or more votes in favor (assume the proposer is bound to vote in favor), the proposal passes and the split is implemented; if not then they move to round 2. Round 2 follows the same procedure as round 1 (the probability of being the proposer is independent across the two rounds). However, round 2 is the last round–if the proposal receives less than two votes in favor, all agents get utility 0. Each agent proportionally discounts payoffs in round 2 by the same $\delta \in [0, 1]$.

a. Formulate the game as an extensive form game and find a subgame perfect equilibrium of the game. Is it unique? Why (10 points)?

b. Calculate the expected utility of each agent before the game begins. How does each change with δ? Why (10 points)?

c. Let $p_1 = \frac{4}{15}$, $p_2 = \frac{5}{15}$, and $p_3 = \frac{6}{15}$. Is it better to have a higher probability of being the proposer? Why or why not (10 points)?