ECE700.07: Game Theory with Engineering Applications

Lecture 2: Preferences and Utilities

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Overview

- Ordinal preferences
- Axioms of rational behavior
- Utility theorem
- Risk attitudes

- Readings
  - MAS Sec. 3.1

Ordinal Preferences

- Agents rank outcomes
  - a $\succ_i$ b means agent i strictly prefers a to b
  - a $\succsim_i$ b means agent i prefers a to b (a is at least as good as b)
  - a $\sim_i$ b means agent i is indifferent between a and b
- Lottery A defines probability distribution over outcomes $o \in \mathcal{L}$
  - $A = [p_1 : o_1, \ldots, p_k : o_K]$

Axioms of Rational Decision Making

- Completeness
  - For every A and B, either $A \succsim B$ or $B \succsim A$
- Transitivity
  - For every A, B, and C, if $A \succsim B$ and $B \succsim C$, then $A \succsim C$
- Independence of irrelevant alternatives
  - For every A, B, C, and p, $A \succsim B$ if and only if $pA + (1-p)C \succsim pB + (1-p)C$
- Continuity
  - For every A, B, and C, if $A \succsim B \succsim C$, then $\exists p$ such that $B \sim pA + (1-p)C$

von Neumann-Morgenstern Utility Theorem

- If all axioms are satisfied, then there exists function $u: \mathcal{L} \longmapsto \mathbb{R}$ such that
  - $u(o_1) \geq u(o_2)$ if and only if $o_1 \succcurlyeq o_2$
  - $u([p_1: o_1, \ldots, p_k: o_k]) = \sum_{i=1}^{k} p_i u(o_i)$
- Such function is called utility function
- What are units?
  - Doesn't really matter
  - Replacing $u(o)$ by $u'(o) = a + bu(o)$, doesn't change agent's preference
- Conversely, agents maximizing expectation of a function obey axioms

Are People "Rational" Decision Makers?

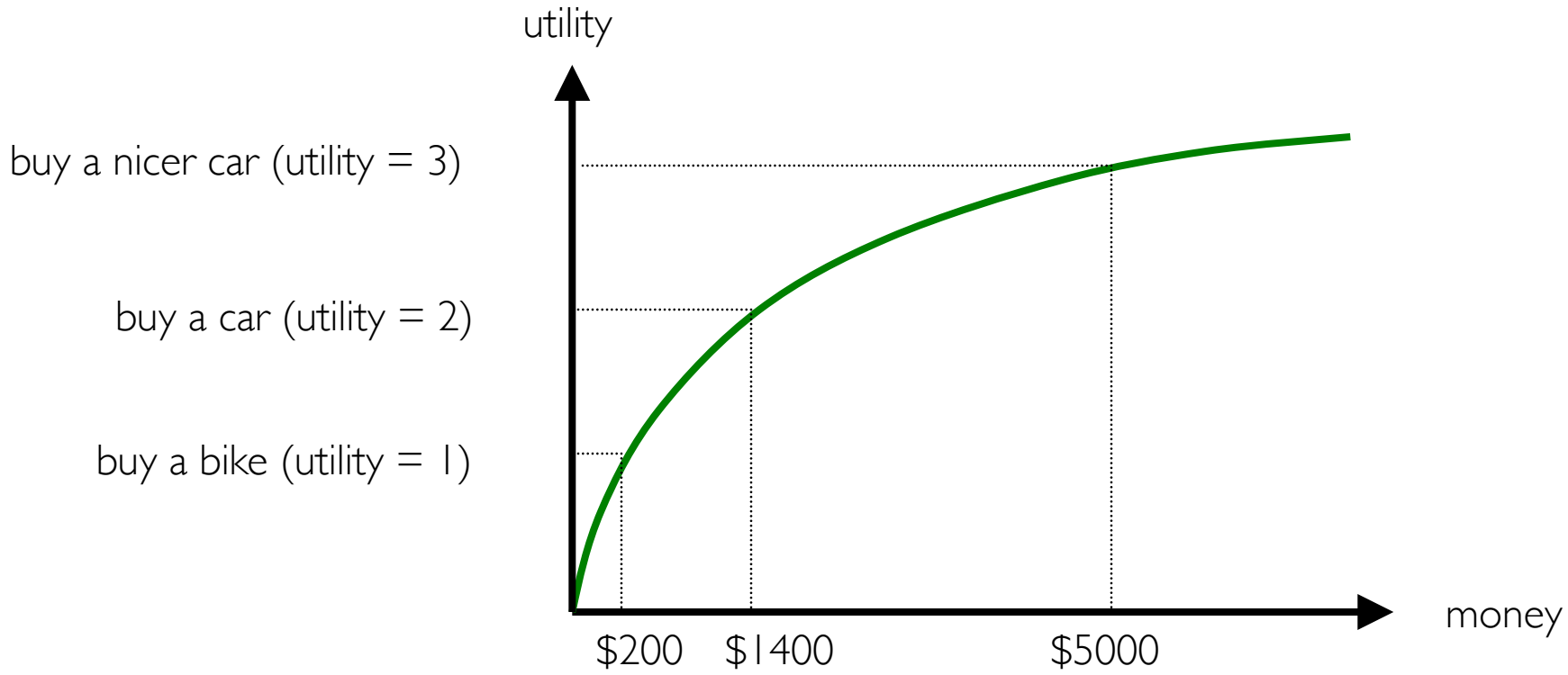
- Which one do you prefer?
  - Lottery ticket that pays out $10 with prob 0.5 and $0 otherwise
  - Lottery ticket that pays out $3 with prob 1
- How about these?
  - Lottery ticket that pays out $100,000,000 with prob 0.5 and $0 otherwise
  - Lottery ticket that pays out $30,000,000 with prob 1
- Usually, people do not simply go by expected value

Uncertainty and Risk Attitudes

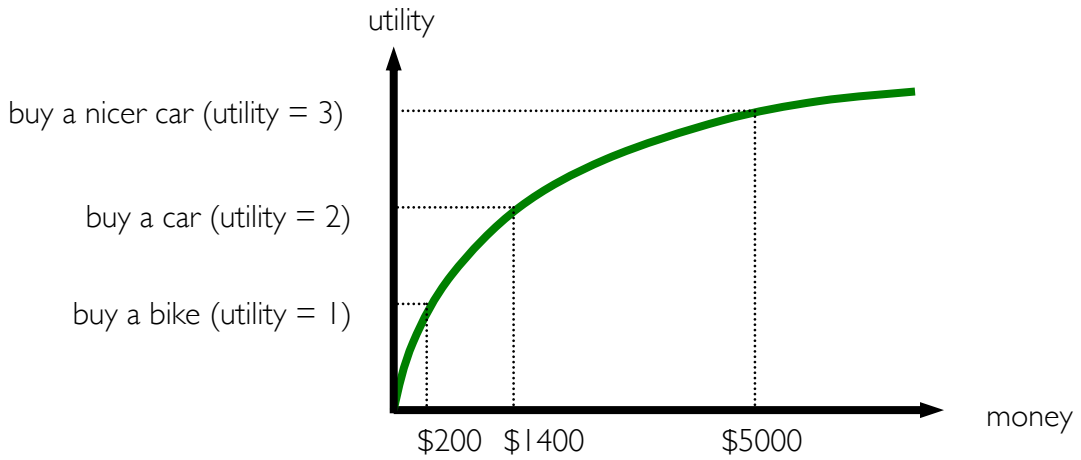
- Risk-neutral agent cares about expected value
- Risk-averse agent prefers expected value of lottery to the lottery ticket
  - Most of people are this way
- Risk-seeking agent prefers lottery ticket to expected value of the lottery

Example

- Typically, at some point, having one more dollar does not make people much happier (decreasing marginal utility)

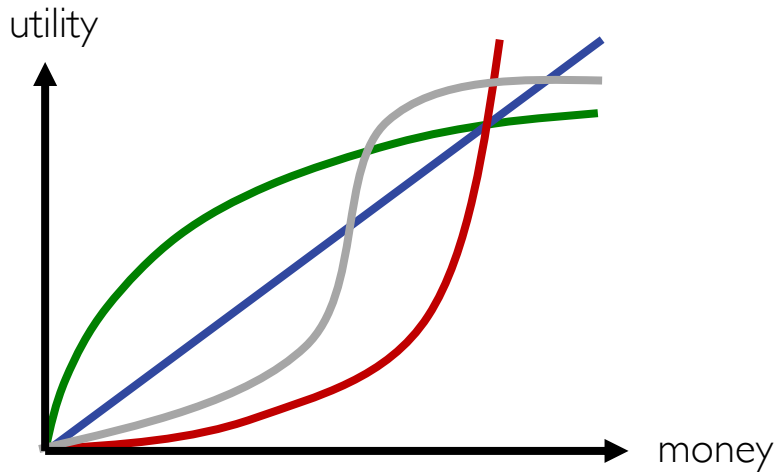


Example (cont.)



- Which one is better?
  - Lottery 1: get $1400 with prob 1
  - Lottery 2: get $5000 with prob 0.25 and $200 otherwise
- What about expected amount of money?

Risk Attitudes (revisited)



- Green has decreasing marginal utility ⟶ risk-averse
- Blue has constant marginal utility ⟶ risk-neutral
- Red has increasing marginal utility ⟶ risk-seeking
- Grey neither risk-averse (everywhere) nor risk-seeking (everywhere)

Questions?

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