Assignment 1.1

Due on Tuesday Jan. 21^{st} (before the tutorial session)

Please read the rules for assignments on the course web page (`https://ece.uwaterloo.ca/~smzahedi/crs/ece750/`). Use Piazza (preferred) or directly contact Seyed (smzahedi@uwaterloo.ca) with any questions.

1. Utility Functions. (10 points). Alice is making plans for Spring Break. She most prefers to go to Cancun, a trip that would cost her \$3000. Another good option is to go to Miami, which would cost her only \$1000. Alice is really excited about Spring Break and cares about nothing else in the world right now. As a result, Alice's utility u as a function of her budget b is given by:

$$u(b) = \begin{cases} 0 & \text{if } b < \$1000, \\ 1 & \text{if } \$1000 \leq b < \$3000, \\ 2 & \text{if } b \geq \$3000. \end{cases}$$

Alice's budget right now is \$1500 (which would give her a utility of 1, for going to Miami).

Alice's wealthy friend Kathy is aware of Alice's predicament and wants to offer her a "fair gamble." Define a fair gamble to be *a random variable with expected value* \$0. An example of fair gamble (with two outcomes) is the following: \$-150 with probability 2/5, and \$100 with probability 3/5. If Alice were to accept this gamble, she would end up with \$1350 with probability 2/5, and with \$1600 with probability 3/5. In either case, Alice's utility is still 1, so Alice's expected utility for accepting this gamble is $(2/5)\cdot(1) + (3/5)\cdot(1) = 1$.

a. Find a fair gamble with two outcomes that would strictly increase Alice's expected utility (5 points).

b. Find a fair gamble with two outcomes that would strictly decrease Alice's expected utility (5 points).

2. Strict and Weak Dominance. (15 points).

a. The following game has a unique Nash equilibrium. Find it (5 points), and prove that it is unique (5 points). (Hint: look for strict dominance.)

(4,0)	(1,2)	(4,0)
(2,4)	(2,4)	(3,5)
(0,1)	(4,0)	(4,0)

b. Construct a single 2×2 normal-form game that simultaneously has all four of the following properties (5 points).

1. The game is not solvable by weak dominance (at least one agent does not have a weakly dominant strategy).
2. The game is solvable by iterated weak dominance (so that one pure strategy per agent remains).
3. In addition to the iterated weak dominance solution (which is a Nash equilibrium), there is a second pure-strategy Nash equilibrium.
4. Both agents strictly prefer the second equilibrium to the first.

(Hints: the second pure-strategy equilibrium should not be strict; the pure-strategy equilibria should be in opposite corners of the matrix.) If you cannot get all four properties, construct an example with as many of the properties as you can.