Assignment 1.2

Due on Thursday Feb. 6^{th} (11:59 PM)

Please read the rules for assignments on the course web page (`https://ece.uwaterloo.ca/~smzahedi/crs/ece750/`). Use Piazza (preferred) or directly contact Seyed (smzahedi@uwaterloo.ca) with any questions.

3. Correlated Equilibrium. (35 points). Consider the following game.

(5,5)	(1,6)	(-6,0)
(6,1)	(-6,-6)	(0,-6)
(0,-6)	(-6,0)	(-1,-1)

a. Find the correlated equilibrium that achieves the highest symmetric utilities (10 points).

b. Consider a distribution that assigns a probability of 1/3 to the outcomes (6,1), (1,6), and (-1,-1), with zero probability assigned to all other outcomes. Is this distribution a coarse correlated equilibrium? (10 points)

c. Is the distribution from part (b) a correlated equilibrium? (5 points).

d. Suppose that $f_i : A_i \to A_i$ is a function that maps recommended strategies (i.e., signals) to agent i's playing strategies. In other works, suppose that upon receiving a recommendation to play r_i, agent i plays $f_i(r_i)$. Using one of the definitions provided in the slides, prove that a joint distribution $\pi \in \Delta(A)$ is a correlated equilibrium of a finite game **if and only if**

$$\sum_{r \in A} \pi(r)[u_i(r_i, r_{-i}) - u_i(f_i(r_i), r_{-i})] \geq 0,$$

for all i and for all functions $f_i : A_i \to A_i$ (10 points).

4. (Pure and Mixed Nash Equilibrium. (45 points).

a. Each of n candidates chooses a position to take on the real line in the interval $[0, 1]$. There is a continuum of citizens, whose favorite positions are uniformly distributed between $[0, 1]$. A candidate attracts votes of citizens whose favorite positions are closer to his position than to the position of any other candidate; if k candidates choose the same position, then each receives the fraction $1/k$ of the votes that the position attracts. The pay-off of each candidate is his vote share.

- Model the game and find all pure strategy Nash equilibria when n = 2 (5 points).

- Show that there does not exist a pure strategy Nash equilibrium when n = 3 (10 points).
- Find a mixed strategy Nash equilibrium when n = 3 (10 points).

b. Two agents are involved in a dispute over an object. The value of the object to agent i is $v_i > 0$. Time is modeled as a continuous variable that starts at 0 and runs indefinitely. Each agent chooses when to concede the object to the other agent; if the first agent to concede does so at time t, the other agent obtains the object at that time. If both agents concede simultaneously, the object is split equally between them, agent i receiving a utility of $v_i/2$. Time is valuable: until the first concession each agent loses one unit of utility per unit of time.

- Formulate this situation as a strategic game (5 points).
- Show that in all Nash equilibria one of the agents concedes immediately (10 points).

c. Consider the following game (Aumann, 1974).

(0,0,3)	(0,0,0)
(1,0,0)	(0,0,0)

(2,2,2)	(0,0,0)
(0,0,0)	(2,2,2)

(0,0,0)	(0,0,0)
(0,1,0)	(0,0,3)

Agents 1, 2, and 3 pick the row, column, and matrix respectively. Find a mixed strategy Nash equilibrium that is not a pure-strategy Nash equilibrium (5 points).