Assignment 1

Due on Thursday Feb. 5^{th}, 23:55 EST

- Please read the rules for assignments on the course webpage: (`https://ece.uwaterloo.ca/~smzahedi/crs/ece752/`).
- Use Piazza or directly contact the instructor (smzahedi@uwaterloo.ca) with any questions.
- For all questions, you must show your work. Final answers alone will not receive full credit.

1. (15 pints) Consider the following game. There are two agents, each of whom chooses an action $a_i \geq 0$. The utility of agent i is given by:

$$u_i(a_i, a_j) = \sqrt{a_i + qa_j} - a_i,$$

where $q > -1$ captures the extent to which agent j's action affects agent i's utility.

a. (5 points) Derive agent i's best-response function. Is it increasing or decreasing in a_j?

b. (5 points) Find the unique symmetric Nash equilibrium of the game.

c. (5 points) Do there exist other, asymmetric equilibria? Why? (Hint: The answer depends on q.)

2. (10 points) Consider the following game with $n \geq 2$ agents. Each agent simultaneously decides whether to undertake an action, denoted by g, or not. If no agent chooses g, each agent receives utility zero. If at least one agent chooses g, all agents receive a benefit of $x > 1$. In addition, each agent i who chooses $a_i = g$ incurs a cost of 1.

a. (5 points) Find all pure-strategy Nash equilibria.

b. (5 points) Find a symmetric mixed-strategy equilibrium in which each agent chooses g with probability p^*. (Hint: Proceed as follows: (i) Write down the indifference condition required for agent i to be willing to mix. (ii) Solve this condition to express p^* as a function of x and n.)

3. (5 points) Consider a normal-form game with two agents, $N = \{1, 2\}$. The action of agent i is denoted by $a_i \in [0, \infty)$. The utility functions are given by:

$$u_1(a_1, a_2) = 2\ln(a_1 + a_2) - a_1, \qquad u_2(a_1, a_2) = \ln(a_1 + a_2) - a_2.$$

Find a pure-strategy Nash equilibrium of the game. (Hint: Start by finding the best-response correspondence of agent 1.)

4. (15 points) Consider the following game.

		Agent 2: Left	Agent 2: Right
Agent 1	Up	$20, -20$	$-5, 0$
	Down	$10, 0$	$5, -10$

a. (5 points) What are the minmax and maxmin values for agent 1?

b. (5 points) What is the maxmin value for agent 1 when restricted to pure strategies only (i.e., $\max_{a_1 \in A_1} \min_{a_2 \in A_2} u_1(a_1, a_2)$), where A_i denotes the set of agent i's pure strategies)?

c. (5 points) Let S_i denote the set of agent i's mixed strategies. Does the following equality always hold in any finite two-player general-sum game? Explain your answer.

$$\max_{s_1} \min_{a_2} u_1(s_1, a_2) = \max_{s_1} \min_{s_2} u_1(s_1, s_2).$$

5. (5 points) Construct a single 2×2 normal-form game that simultaneously satisfies all four of the following properties (5 points):

1. The game is not solvable by weak dominance (i.e., at least one agent does not have a weakly dominant strategy).
2. The game is solvable by iterated weak dominance, so that exactly one pure strategy per agent remains.
3. In addition to the iterated weak-dominance solution (which is a Nash equilibrium), the game admits a second pure-strategy Nash equilibrium.
4. Both agents strictly prefer the second equilibrium to the first.

(Hints: The second pure-strategy equilibrium should not be strict; the two pure-strategy equilibria should appear in opposite corners of the payoff matrix.) If you are unable to satisfy all four properties, construct an example that satisfies as many of them as possible.