Assignment 2.1

Due on Thursday Feb. 27^{th} (11:59 PM)

Please read the rules for assignments on the course web page (`https://ece.uwaterloo.ca/~smzahedi/crs/ece750/`). Use Piazza (preferred) or directly contact Seyed (smzahedi@uwaterloo.ca) with any questions.

1. (M)ILP Programming. (30 points). Calculate X = Your student ID mod 5.

- If $X \in \{0, 1\}$, solve problem (1.1).
- If $X = 2$, solve problem (1.2).
- If $X = 3$, solve problem (1.3).
- If $X = 4$, solve problem (1.4).

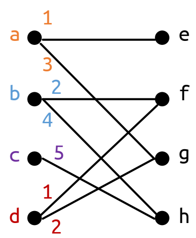

Figure 1: A bipartite matching problem.

1.1. Maximum Weighted Bipartite Matching. A bipartite graph is a graph whose vertices can be divided into two disjoint and independent sets. Given a bipartite graph with weights on the edges, subset of edges is a feasible matching if no two of the edges in the subset share a vertex. The value of a matching is the sum of the weights for matched edges. The goal is to find a feasible matching with the maximum value.

a. Formulate the maximum weighted bipartite matching problem as an integer linear program for the bipartite graph in Figure 1 (20 points).

b. Express your integer program of (a.) in a modeling language of your choice (e.g., CVXPY) and provide a solution with its optimal value (10 points).

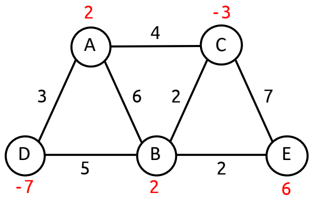

Figure 2: A transshipment problem.

1.2. Transshipment. Consider an undirected graph (V, E). Suppose that each vertex $v \in V$ has a demand d_v, which is the net amount of the commodity that it wants to receive. Negative d_v indicates that vertex v has a net supply rather than a net demand, that is, the vertex is a producer of the commodity. Suppose that we can transport an unlimited amount across each edge, but associated with each edge $e \in E$, there is a cost c_e of transporting one unit across e. Suppose further that transportation is possible with the same cost in both directions for each edge. Assuming that demand and supply are perfectly matched (*i.e.*, $\sum_{v \in V} d_v = 0$), the goal is to find a flow such that all demands are met (exactly), while incurring the minimum total transportation cost.

a. Suppose that only integer amounts of commodity can be transported over each edge. Formulate the minimum cost transshipment problem as an integer program for the graph in Figure 2, where black number over each edge indicate costs and red numbers indicate demands (20 points).

b. Express your integer program of (a.) in a modeling language of your choice (e.g., CVXPY) and provide a solution with its optimal value. Then, suppose that fractional amounts of commodity can also be transported over each edge. Solve the linear program and provide a solution with its optimal value (10 points).

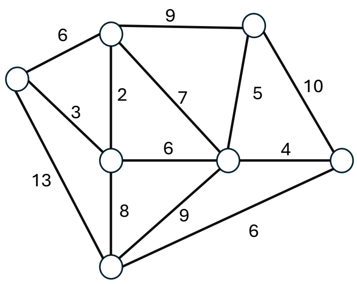

Figure 3: A heavy Hamiltonian cycle problem.

1.3. Heavy Hamiltonian Cycle. Let G be an undirected graph with weighted edges. A *heavy Hamiltonian cycle* is a cycle C that passes through each vertex of G **exactly once**, such that the total weight of the edges in C is at least half of the total weight of all edges in G.

a. Formulate the problem of finding a heavy Hamiltonian cycle as an integer linear program for the graph in Figure 3 (20 points).

b. Express your integer program of (a.) in a modeling language of your choice (e.g., CVXPY) and provide its solution (10 points).

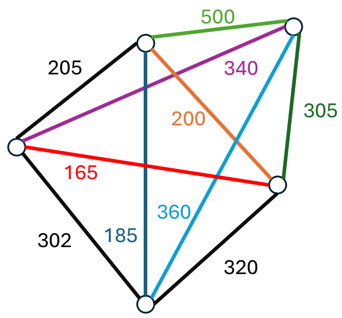

Figure 4: A traveling salesman problem.

1.4. Traveling Salesman Problem. The traveling salesman problem (TSP) asks the following question: "Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the origin city?"

a. Formulate the traveling salesman problem for the graph in Figure 4 as an integer linear program (20 points).

b. Express your integer program of (a.) in a modeling language of your choice (e.g., CVXPY) and provide its solution (10 points).