Assignment 2.2

Due on Wednesday Mar. 13^{th} (11:59 PM)

Please read the rules for assignments on the course web page (`https://ece.uwaterloo.ca/~smzahedi/crs/ece750/`). Use Piazza (preferred) or directly contact Seyed (smzahedi@uwaterloo.ca) with any questions.

2. (M)ILP for Game Theory (40 points).

2.1. Express a mixed integer linear program to solve problem (2.a) from Assignment 1 in a modeling language of your choice (e.g., CVXPY) and provide a solution (20 points).

2.2. Express a linear program to solve problem (3.a) from Assignment 1 in a modeling language of your choice (e.g., CVXPY) and provide a solution with its optimal value (20 points).

3. Teamwork (15 points). Consider the following game-theoretic model of the equilibrium determination of the level of effort team members put into a team project, with team size of 2. In this game, the two team members simultaneously choose the level of effort, e_1 and e_2, to spend on the project. They each get utility from progress on the project (which is a function of the sum of the efforts) and dis-utility from the effort they personally expend. Agent 1, values the achievement on the project more. Specifically, assume that e_1 and e_2 are chosen from the set of non-negative real numbers and for $k > 1$, we have:

$$u_1(e_1, e_2) = k.\log(e_1 + e_2) - e_1,$$
$$u_2(e_1, e_2) = \log(e_1 + e_2) - e_2.$$

a. Find best response correspondences for each of the agents (show your work - 5 points).

b. Find the pure strategy Nash equilibria of the game (show your work). How does the distribution of effort in equilibrium reflect the difference in the agent's preferences (5 points)?

c. Now, suppose that agents play this game sequentially. First, assume that agent 1 decides how much effort she wants to put into the project. Agent

2 observes agent 1's level of effort, and then she decides how much effort to put into the project. Model this new game as an extensive form game and find its subgame perfect equilibria (show your work - 5 points).

4. Cournot Competition (15 points). There are two (and only two) firms in a market, making identical products. Each firm decides on a quantity $q_i \in [0, 5]$. The price of the product is determined by the total production as $P = \max\{6 - (q_1 + q_2), 0\}$. Each firm's costs are zero, so wishes to maximize profit: $v_i = q_i P$.

a. Suppose the firms make their choices simultaneously. Find the unique NE (show your work - 5 points).

b. Suppose that firm 1 first chooses q_1, which firm 2 sees and then selects q_2. Is the NE outcome from part (a) still a NE outcome of this extensive form game? Why? If so, is it unique? Why (show your work - 5 points)?

c. What is the subgame perfect equilibrium? Explain any differences (show your work - 5 points).