# Assignment 2

Due on Tuesday Mar. $12^{th}$ (before the class)

Please read the rules for assignments on the course web page (`https://ece.uwaterloo.ca/~smzahedi/crs/ece750/`). Use Piazza (preferred) or directly contact Seyed (smzahedi@uwaterloo.ca) with any questions.

**1. (M)ILP Programming. (30 points).** Calculate $X$ = Your student ID mod 2. If $X = 0$, solve problem (1.1). Otherwise, solve problem (1.2).

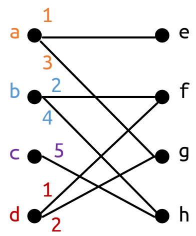


Figure 1: A bipartite matching problem.

**1.1. Maximum Weighted Bipartite Matching.** A bipartite graph is a graph whose vertices can be divided into two disjoint and independent sets. Given a bipartite graph with weights on the edges, subset of edges is a feasible matching if no two of the edges in the subset share a vertex. The value of a matching is the sum of the weights for matched edges. The goal is to find a feasible matching with the maximum value.

**a.** Formulate the maximum weighted bipartite matching problem as an integer linear program for the bipartite graph in Figure 1 (20 points).

**b.** Express your integer program of (a.) in a modeling language of your choice (e.g., CVXPY) and provide a solution with its optimal value (10 points).

**1.2. Transshipment.** Consider an undirected graph $(V, E)$. Suppose that each vertex $v \in V$ has a demand $d_v$, which is the net amount of the commodity that it wants to receive. Negative $d_v$ indicates that vertex $v$ has a net supply rather than a net demand, that is, the vertex is a producer of the commodity. Suppose that we can transport an unlimited amount across each edge, but associated with each edge $e \in E$, there is a cost $c_e$ of transporting one unit across $e$. Suppose further that transportation is possible with the same cost in both directions

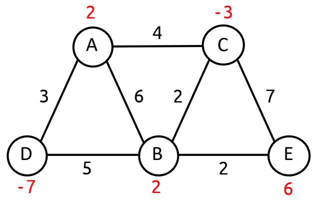


Figure 2: A transshipment problem.

for each edge. Assuming that demand and supply are perfectly matched (*i.e.*, $\sum_{v \in V} d_v = 0$), the goal is to find a flow such that all demands are met (exactly), while incurring the minimum total transportation cost.

**a.** Suppose that only integer amounts of commodity can be transported over each edge. Formulate the minimum cost transshipment problem as an integer program for the graph in Figure 2, where black number over each edge indicate costs and red numbers indicate demands (20 points).

**b.** Express your integer program of (a.) in a modeling language of your choice (e.g., CVXPY) and provide a solution with its optimal value. Then, suppose that fractional amounts of commodity can also be transported over each edge. Solve the linear program and provide a solution with its optimal value (10 points).

**2. (M)ILP for Game Theory (20 points).** Calculate $Y$ as follows.

$$Y = (\text{Your student ID} \mod 199) \mod 3.$$

Solve problem (2.1) if $Y = 0$, (2.2) if $Y = 1$, and (2.3) if $Y = 2$.

**2.1.** Express a mixed integer linear program to solve problem (2.a) from Assignment 1 in a modeling language of your choice (e.g., CVXPY) and provide a solution (20 points).

**2.2.** Express a linear program to solve problem (3.a) from Assignment 1 in a modeling language of your choice (e.g., CVXPY) and provide a solution with its optimal value (20 points).

**2.3.** Consider the game outlined in problem (4.c) of Assignment 1. Using a modeling language of your choice (e.g., CVXPY), find a correlated equilibrium that maximizes the social welfare (20 points).

**3. Teamwork (25 points).** Consider the following game-theoretic model of the equilibrium determination of the level of effort team members put into a team project, with team size of 2. In this game, the two team members simultaneously choose the level of effort, $e_1$ and $e_2$, to spend on the project. They each get utility from progress on the project (which is a function of the sum of the efforts) and dis-utility from the effort they personally expend. Agent 1, values the achievement on the project more. Specifically, assume that $e_1$ and $e_2$ are chosen from the set of non-negative real numbers and for $k > 1$, we have:

$$u_1(e_1, e_2) = k. \log(e_1 + e_2) - e_1,$$
$$u_2(e_1, e_2) = \log(e_1 + e_2) - e_2.$$

**a.** Find best response correspondences for each of the agents (5 points).

**b.** Find the pure strategy Nash equilibria of the game. How does the distribution of effort in equilibrium reflect the difference in the agent's preferences (5 points)?

**c.** Now, suppose that agents play this game sequentially. First, assume that agent 1 decides how much effort she wants to put into the project. Agent 2 observes agent 1's level of effort, and then she decides how much effort to put into the project. Model this new game as an extensive form game and find its subgame perfect equilibria (10 points).

**d.** This time, repeat c. assuming that agent 2 decides first and agent 1 decides after observing agent 2's decision. From b, c, and d, which outcome is more fair in your opinion (5 points)?

**4. Cournot Competition (25 points).** There are two (and only two) firms in a market, making identical products. Each firm decides on a quantity $q_i \in [0, 5]$. The price of the product is determined by the total production as $P = \max\{6 - (q_1 + q_2), 0\}$. Each firm's costs are zero, so wishes to maximize profit: $v_i = q_i P$ .

**a.** Suppose the firms make their choices simultaneously. Find the unique NE (5 points).

**b.** Suppose that firm 1 first chooses $q_1$, which firm 2 sees and then selects $q_2$. Is the NE outcome from part (a) still a NE outcome of this extensive form game? Why? If so, is it unique? Why (10 points)?

**c.** What is the subgame perfect equilibrium? Explain any differences (10 points).