Assignment 3

Due on Wednesday April 3^{rd} (before the class)

Please read the rules for assignments on the course web page (`https://ece.uwaterloo.ca/~smzahedi/crs/ece750/`). Use Piazza (preferred) or directly contact Seyed (smzahedi@uwaterloo.ca) with any questions.

1. Two-party Bargaining (15 points). There are two agents bargaining over the split of a dollar. In odd rounds, agent 1 proposes the split, in even rounds, agent 2 does. The number of rounds, R, is finite and odd. The game ends when either (i) the proposer's split is accepted in round r, or (ii) all proposals are rejected, in which case payoffs are zero for both. If the game ends in round r with agent i receiving amount y, then $v_i = \delta_i^{(r-1)} y$.

a. Find an SPE for any $R = 2k + 1$, where $k \in \{1, 2, \dots\}$, when $\delta_1 < \delta_2 \in (0, 1)$ (5 points).

b. Find all SPEs for any $R = 2k + 1$, where $k \in \{1, 2, \dots\}$, when $\delta_1 = \delta_2 \in (0, 1)$ (5 points).

c. Find an SPE of the game when $R = \infty$ and verify it using one-shot deviation principle (5 points).

2. BNE (15 points). Consider a game with two agents. For agent 1, $A_1 = \{U, D\}$, and for agent 2, $A_2 = \{L, R\}$. Agent 2 has a single type, but agent 1's types is H with probability 0.6 and L with probability 0.4 (these are common priors). Depending on agent 1's type, the game has the following two forms:

	L	R
U	$1, -1$	$3, 2$
D	$3, 3$	$2, 1$

$\theta_1 = H$

	L	R
U	$2, -2$	$2, 0$
D	$2, 3$	$3, 1$

$\theta_1 = L$

a. Complete the following table with the ex-ante expected utilities for both agents (4 points).

		Agent 2	
		L	R
Agent 1	UU		
	UD		
	DU		
	DD		

b. Complete the table in (a) with the interim expected utilities for both agents when $\theta_1 = H$ (4 points).

c. Calculate agent 1's best response to agent 2's strategy, s_2, which mixes between L and R with equal probabilities (7 points).

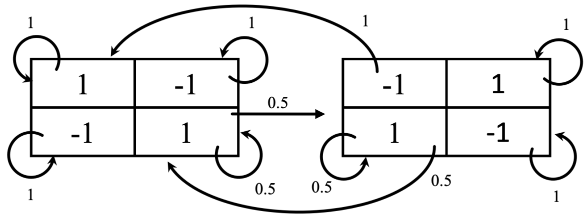
3. All-pay Auction (20 points). In an all-pay auction, bidders submit bids, and the object is allocated to bidder submitting the highest bid. The unusual feature is that ALL bidders pay their bids (regardless of who wins). Assume we are in the symmetric environment with independent private values.

a. Find the unique candidate for a symmetric, increasing equilibrium (10 points).

b. Verify that it is indeed an equilibrium for the case of $F(v) = v$ on $[0, 1]$ (5 points).

c. Is the all-pay auction "standard"? Maintain $F(v) = v$ on $[0, 1]$ and let n=2. Find the bid of a bidder with $v = 1/2$ in the symmetric equilibria or the first-price, second-price, and all-pay auctions. Explain intuitively the reason for their ranking (5 points).

4. Stochastic Games (15 points). Consider the following two-player, zero-sum, stochastic game (only utility of agent 1 is shown).



Suppose that $V_0(s_1) = V_0(s_2) = 1$, and $\delta = 0.5$. Compute the value $V_t(s_1)$ for agent 1 for $t = 1, 2$, and 3 according to the Shapley Algorithm. Show your work.

5. Disclosure Game (15 points). Consider the following game. There are two agents: sender (S) and receiver (R). Sender has a type θ from set Θ, over which there is a common prior μ_0. Sender takes action by sending a message $m \in M_\theta = [0, \theta]$. Receiver observes m, updates belief about θ to μ, and selects action by responding $y \in Y$. Receiver's utility is $u_R = (m, y, \theta) = -(y - \theta)^2$ and sender's utility is $u_S(m, y, \theta) = y$.

a. What is receiver's best response given m and μ (i.e., what is $BR_R(\mu, m)$ (5 points)?

b. Prove that in any PBE, receiver perfectly infers θ from m (i.e, prove that in any PBE, sender sends a unique message for each type) (10 points).

6. Modified Disclosure Game (20 points). Consider the disclosure game outlined in (5). Suppose now that the sender does not always know θ. In particular, sender knows θ only with probability $p < 1$. When the sender does not know θ, then the sender cannot report θ and can only send a null message $\emptyset$. Suppose that $\Theta = \{0, 1, 2, \ldots, 10\}$, μ_0 is uniform on Θ. Let $M_\theta = \{\emptyset, \theta\}$ be the two actions available to the sender when the sender knows θ, and $M_\emptyset = \{\emptyset\}$ be the only action available to the sender when the sender does not know θ.

a. Show that if the sender knows $\theta = 0$, then the sender has strict incentive to send $m = \emptyset$ (5 points).

b. Suppose that there is i for which the sender weakly prefers $m = \emptyset$ to $m = i$ if the sender knows $\theta = i$. Show then for any $j < i$, the sender strictly prefers $m = \emptyset$ to $m = j$ if the sender knows $\theta = j$ (5 points).

c. Suppose we are looking for PBE in which the sender sends $m = \emptyset$ if the sender knows that $\theta = \{0, ..., k\}$ and sends $m = \theta$ if the sender knows that $\theta > k$. Calculate the sender's utility, π_k, for sending $m = \emptyset$ (5 points).

d. Show that PBE in (c) exists if and only if $k \leq \pi_k \leq k + 1$, and conclude that such PBE exists only if $k \leq 4$ (5 points).