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UNIVERSITY OF WATERLOO

Final Exam - Winter 2023 - ECE 750/493

1. Best response, NE, and SPE (20 points). Consider a normal-form game with two agents $N = \{1, 2\}$. Action of agent i is denoted by $a_i \in [0, \infty)$. Utilities are $u_1(A_1, A_2) = 2\ln(a_1 + a_2) - a_1$, and $u_2(a_1, a_2) = \ln(a_1 + a_2) - a_2$.

a. Find best response correspondence for agent 1 (5 points).

a. Find a pure strategy Nash equilibria of the game (5 points).

b. Suppose that agents play this game sequentially. First, assume that agent 1 takes an action, agent 2 observes agent 1's action and then, takes an action. Find subgame perfect equilibria of this new game (5 points).

c. Repeat b assuming that agent 2 takes an action first, and agent 1 takes an action after observing agent 2's decision (5 points).

2. Minmax and maxmin (20 points). Consider the following game.

		Agent 2: Left	Agent 2: Right
Agent 1	Up	$20, -20$	$-5, 0$
	Down	$10, 0$	$5, -10$

a. What is the minmax value of agent 1? Show your work (10 points).

Maxmin: ☐ Minmax: ☐

b. What is the maxmin value of agent 1 with pure strategies only (i.e., $\max_{a_1 \in A_1} \min_{a_2 \in A_2} u_1(a_1, a_2)$, where A_i is the set of agent i's pure strategies)? Show your work (5 points).

☐

c. Let S_i be the set of agent i's mixed strategies. Does the following equality always hold in any finite, 2-player, general-sum game? Why (5 points)?

$$\max_{s_1} \min_{a_2} u_1(s_1, a_2) = \max_{s_1} \min_{s_2} u_1(s_1, s_2)$$

☐

3. Correlated equilibrium (20 points) Consider the following game and the distribution π on its outcomes.

	A	B	C
A	$1, 1$ 35%	$-1, -1$ 0%	$0, 0$ 0%
B	$-1, -1$ 0%	$1, 1$ 35%	$0, 0$ 0%
C	$0, 0$ 0%	$0, 0$ 0%	$-1.5, -1.5$ 30%

a. Is π a coarse correlated equilibrium of the game? Why (10 points)?

☐

b. Is π a correlated equilibrium of the game? Why (10 points)

☐

4. Fictitious play (10 points) Consider the following game.

		Agent 2	
		L	R
Agent 1	U	$1, 1$	$2, 0$
	D	$0, 2$	$4, 4$

If both agents run the fictitious play algorithm to play this game repeatedly. Suppose that $\eta_1^1 = (1, 1)$ and $\eta_2^1 = (1, 2)$. Suppose that agent 1 takes U if both U and D have the same expected utility. Similarly, assume that agent 2 takes L if both L and R have the same expected utility. What outcome happens at round 3 of the game? Show your work.

5. BNE (16 points). Consider a game with two agents. For agent 1, $A_1 = \{U, D\}$, and for agent 2, $A_2 = \{L, R\}$. Agent 2 has a single type, but agent 1's types is H with probability 0.6 and L with probability 0.4 (these are common priors). Depending on agent 1's type, the game has the following two forms:

$\theta_1 = H$

	L	R
U	$0, -1$	$2, 0$
D	$2, 1$	$3, 0$

$\theta_1 = L$

	L	R
U	$3, -1$	$5, 0$
D	$2, 1$	$3, 0$

a. Complete the following table with the ex-ante expected utilities for both

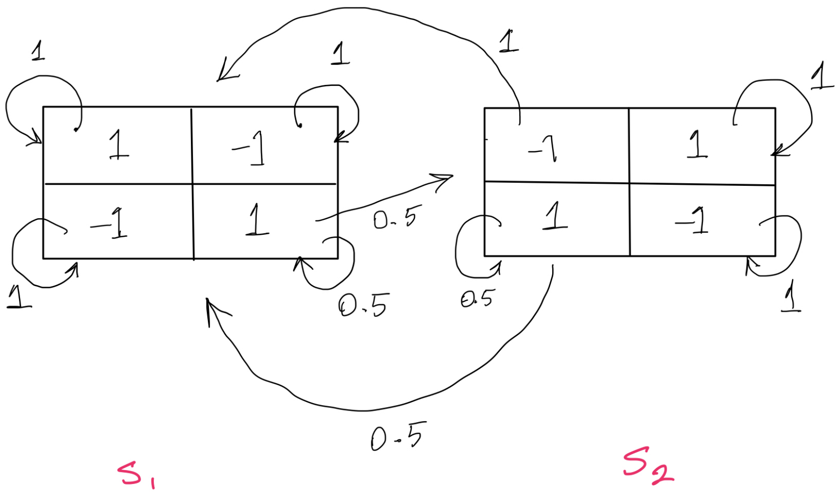
Agent 2

Agent 1		L	R
	UU		
	UD		
	DU		
	DD		

b. Consider the following mixed strategy profile s. Agent 2 mixes between L and R with equal probabilities. Agent 1 mixes between U and D with equal probabilities if $\theta_1 = H$ and takes U with probability 1 if $\theta_1 = L$. Calculate interim expected utility of agent 1 for each of agent 1's types (8 points).

$EU_1(s, H)$: ☐ $EU_1(s, L)$: ☐

6. Stochastic Games (14 points). Consider the following two-player, zero-sum, stochastic game (only utility of agent 1 is shown).



Suppose that $V_0(s_1) = V_0(s_2) = 1$, and $\delta = 0.5$. Compute the matrix game $G(s_1, V_0)$ and the value $V_1(s_1)$ for agent 1 according to the Shapley Algorithm.

$G(s_1, V_0)$

$V_1(s_1)$: