ECE 700T07 Midterm

Nov. 6^{th}, 2018

1. Consider the following game. There are two agents, who each select $a_i \geq 0$. The utility of agent i is $u_i(a_i, a_j) = \sqrt{a_i + qa_j} - a_i$, where $q > -1$ measures the influence of agent j's action on agent i's utility.

a. (10 points) Calculate agent i's best response function. Is it increasing or decreasing in a_j?

b. (10 points) Find the unique symmetric Nash equilibria of the game.

c. (Bonus Question - 10 points) Are there other, non-symmetric, equilibria? (hint: the answer depends on q.)

2. There are two (and only two) firms in a market, making identical products. Each firm decides on a quantity $q_i \in [0, 5]$. The price of the product is determined by the total production as $P = \max\{6 - (q_1 + q_2), 0\}$. Each firm's costs are zero, so wishes to maximize profit: $v_i = q_i P$.

a. (10 points) Suppose the firms make their choices simultaneously. Find the unique NE.

b. (10 points) Suppose that firm 1 first chooses q_1, which firm 2 sees and then selects q_2. Is the NE outcome from part (a) still a NE outcome of this extensive-form game?

c. (10 points) What is the subgame perfect equilibrium? Explain any difference with solution(s) for (b).

3. Consider the following game. There are $n \geq 2$ agents. Each one simultaneously decides whether to undertake some action, g, or not to. If no one chooses g, each agent's utility is zero. If one or more agents choose g, everyone gets a benefit of $x > 1$, and, additionally, each agent i choosing $a_i = g$ bears a cost of 1.

a. (5 points) Formulate the game.

b. (15 points) Find a symmetric mixed-strategy equilibrium in which each agent chooses g with probably p^*. To do so:

- Formulate the "indifference condition" needed for agent i to be willing to play a mixed strategy.
- Solve the indifference condition to get p^* as it depends on x and n.

4. (Bonus Question) There are three agents bargaining over the split of a dollar. The protocol is as follows. At the beginning of period 1, agent i is recognized to be the proposer with probability p_i where $p_1 + p_2 + p_3 = 1$ and $p_1 < p_2 < p_3$. The proposer makes a proposal (x_1, x_2, x_3), such that the amount proposed for j is $x_j \geq 0$ for all j and $x_1 + x_2 + x_3 \leq 1$. Then the agents vote on the proposal: if it receives two or more votes in favour (assume the proposer is bound to vote in favour), the proposal passes and the split is implemented; if not then we move to period 2. Period 2 follows the same procedure as period 1 (the probability of being the proposer is independent across the two periods). However, period 2 is the last period; if the proposal receives less than two votes in favour, all agents get utility 0. Each agent proportionally discounts utilities in period 2 by $\delta \in [0, 1]$.

a. (10 points) Find the subgame perfect equilibrium of the game.

b. (10 points) Calculate the expected utility for each agent before the game begins.