Waterloo Student ID Number:

WatIAM/Quest Login UserID:

UNIVERSITY OF WATERLOO

Midterm Exam - Fall 2019 - ECE 700T07

1. Best Response (10 points) Consider the following game. There are two agents, who each select $a_i \geq 0$. The utility of agent i is $u_i(a_i, a_j) = \sqrt{a_i + qa_j} - a_i$, where $q > -1$ measures the influence of agent j's action on agent i's utility. Calculate agent i's best response correspondence. Is it increasing or decreasing in a_j?

2. NE vs SPE (20 points) There are two (and only two) firms in a market, making identical products. Each firm decides on a quantity $q_i \in [0, 5]$. The price of the product is determined by the total production as $P = \max\{6 - (q_1 + q_2), 0\}$. Each firm's marginal cost is $c = 1$, so wishes to maximize profit: $q_i(P - 1)$.

a. (10 points) Suppose the firms make their choices simultaneously. Find the unique NE.

b. (10 points) Suppose that firm 1 first chooses q_1, which then firm 2 sees before choosing q_2. Is the NE outcome from part (a) still a NE outcome of this extensive-form game? Why?

3. Mixed Strategy (30 points) Consider the following game. There are $n \geq 2$ agents. Each one simultaneously decides whether to undertake some action, g, or not to. If no one chooses g, each agent's utility is zero. If one or more agents choose g, everyone gets a benefit of $x > 1$, and, additionally, each agent i choosing $a_i = g$ bears a cost of 1.

a. (10 points) Formulate the game.

b. (20 points) Find a symmetric mixed-strategy NE in which each agent chooses g with probably p^*. To do so:

- Formulate the "indifference condition" needed for agent i to be willing to play a mixed strategy.
- Solve the indifference condition to get p^* as it depends on x and n.

4. Short Answers (40) For each question:

- CIRCLE YOUR ANSWER
- No points for any explanation if true-false is not correct.
- No points for an explanation that exceeds 3 sentences.

Prisoner 1 \ Prisoner 2	Stay Silent	Confess	Suicide
Stay Silent	(-1, -1)	(-3, 0)	(0, -10)
Confess	(0, -3)	(-2, -2)	(-1, -10)
Suicide	(-10, 0)	(-10, -1)	(-10, -10)

Figure 1: Modified Prisoner's Dilemma.

a. (5 points) The game in Figure 1 has a dominant strategy equilibrium.

True False

Driver 1 \ Driver 2	S	D
S	(-5, -5) 20%	(1, -1) 40%
D	(-1, 1) 40%	(0, 0) 0%

Figure 2: Probability Distribution over Outcomes of the Game of Chicken.

b. (10 points) The probability distribution over states in Figure 2 forms a correlated equilibrium.

True False

c. (5 points) Checking for strict dominance by mixed strategies in a finite game is computationally hard (*i.e.* it is NP hard).

True False

d. (5 points) In a finite, general-sum game, finding the maximin strategy of each agent is computationally easy (*i.e.* it is not NP hard).

True False

e. (5 points) One-shot deviation principle gives us a mechanism to find at least an SPE for infinite-horizon games.

True False

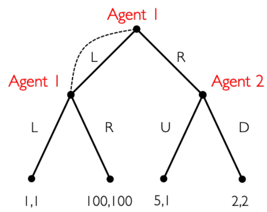

Figure 3: Game with Imperfect Recall.

b. (5 points) The game in Figure 3 has only one subgame.

True False

b. (5 points) The game in Figure 3 has a unique SPE.

True False