REF: Resource Elasticity Fairness with Sharing Incentives for Multiprocessors

Seyed Majid Zahedi, Benjamin C. Lee
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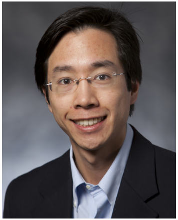
Motivation

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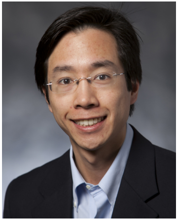

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 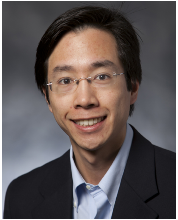

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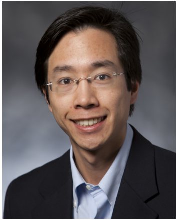

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- Each has $10K to buy clusters
- Alvy works on accelerators
- Ben works on memory systems

Strategic Behavior

- Alvy and Ben are strategic

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Conventional Wisdom in Computer Architecture

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- **Strategy-Proof (SP)**
  No user benefits from lying

Three Main Steps

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x_1, x_2	allocated memory bandwidth for users 1, 2
y_1, y_2	allocated cache size for users 1, 2

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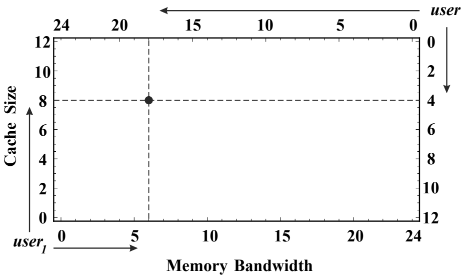
Possible Allocations

- 2 users
- 12MB cache
- 24GB/s bandwidth



Envy-Free (EF) Allocations

- Identify EF allocations for each user
- $\mathbf{u_1(A_1) \geq u_1(A_2)}$
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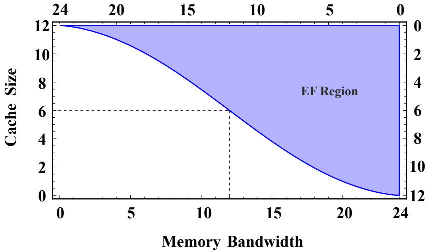

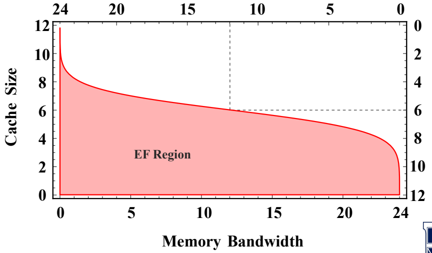

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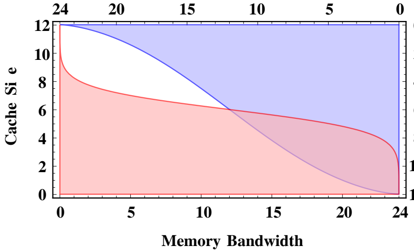

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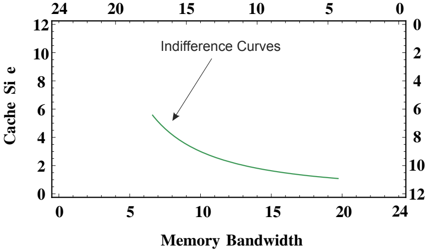

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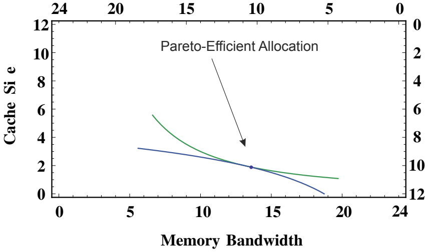

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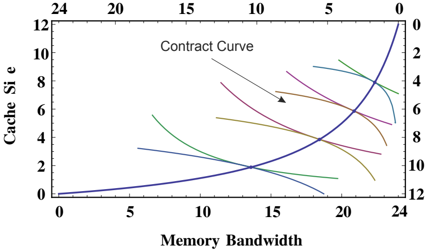

- Indifference curve: allocations that give same utility
- Contract curve: all Pareto-efficient allocations

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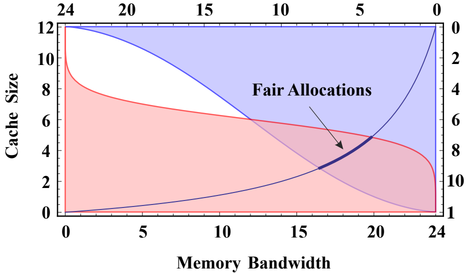

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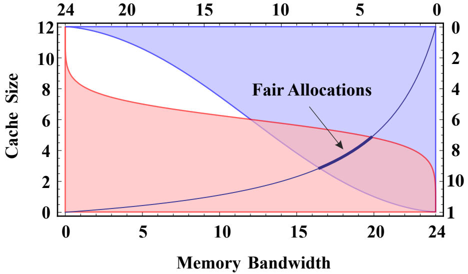

Many possible fair allocations!

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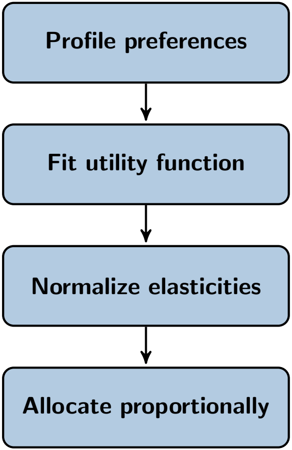
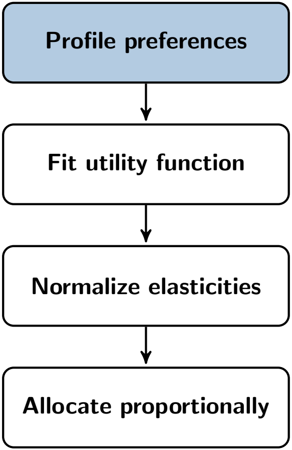
Resource Elasticity Fairness Mechanism



- REF: fair allocation mechanism
- Guarantees desiderata
  - Sharing incentives
  - Envy-freeness
  - Pareto-efficiency
  - Strategy-proofness in large

Profiling for REF



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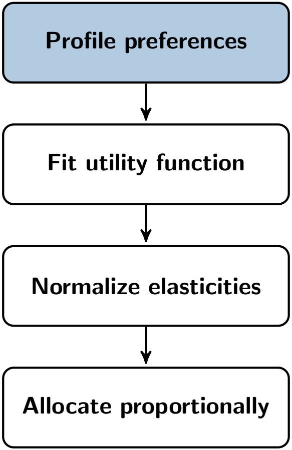

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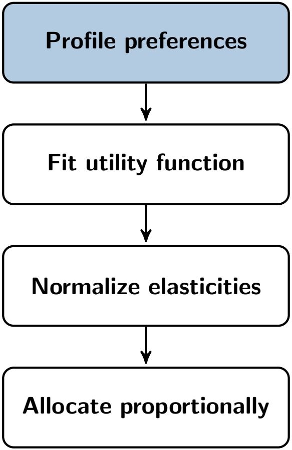

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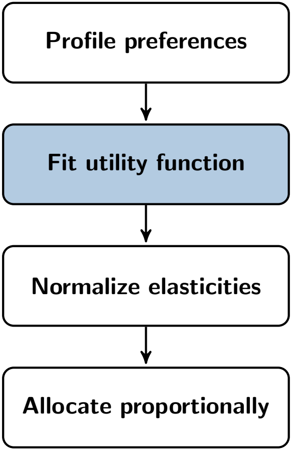
  - Various hardware
- On-line profiling
  - $\alpha = 0.5$, then update
  - Statistical machine learning

Fitting Utilities



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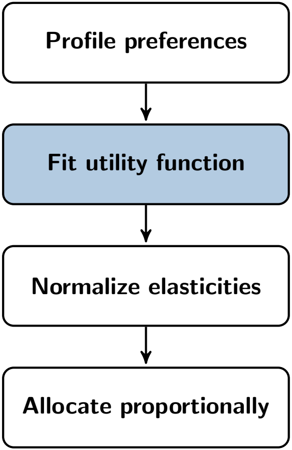

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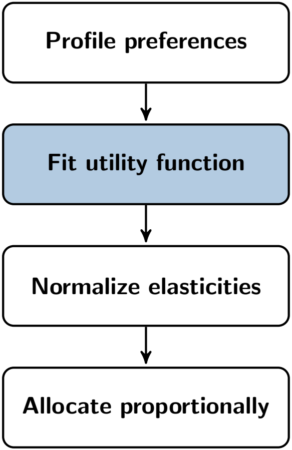
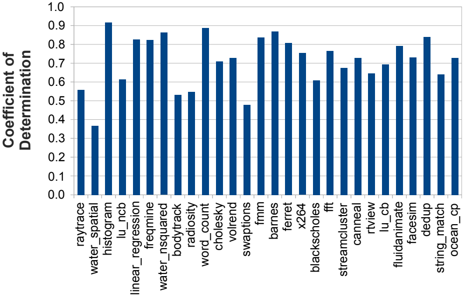


- $u = \prod_{r=1}^{R} x_r^{\alpha_r}$
- $\log(u) = \sum \alpha_r \log(x_r)$
- Use linear regression to find α_r

Cobb-Douglas Accuracy

- IPC as utility
- Cache size and memory bandwidth
- R-squared $\rightarrow$ 1 as fit improves



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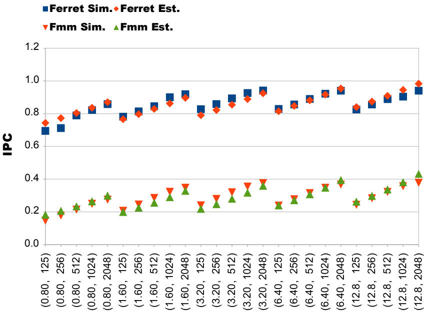

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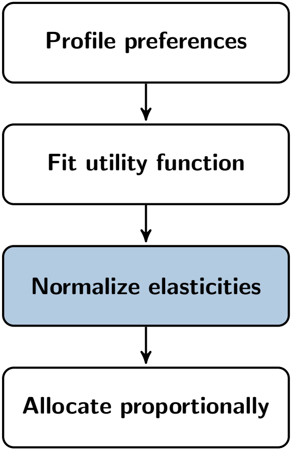

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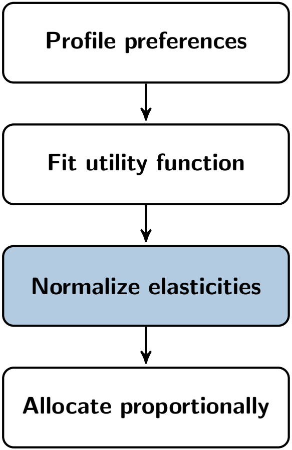

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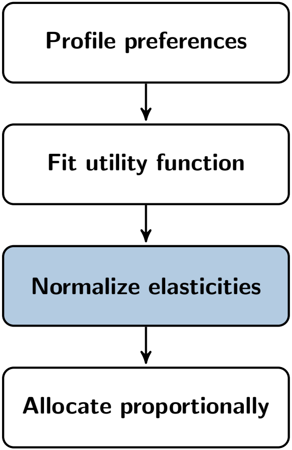

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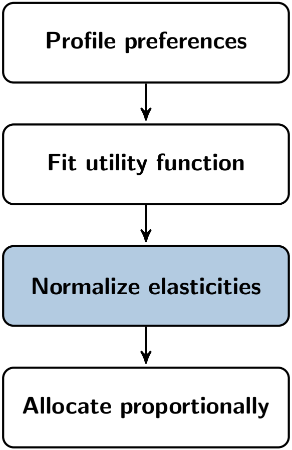
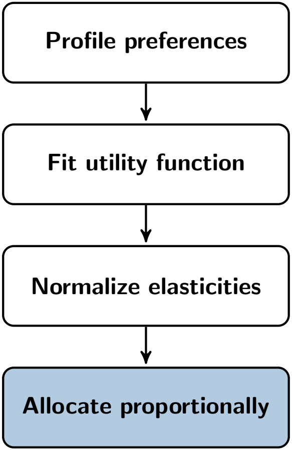


- Normalize elasticities to sum to one
- $u = x^{0.2} y^{0.3} \rightarrow u = x^{0.4} y^{0.6}$
- Compare users' elasticities on same scale

Allocating Proportional Shares



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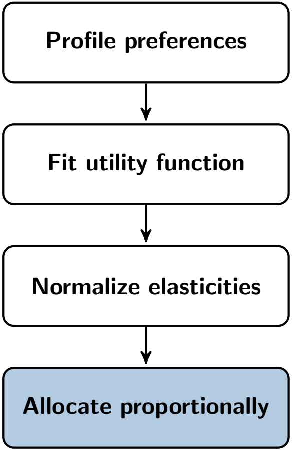

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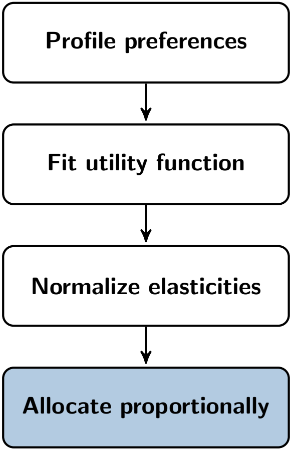

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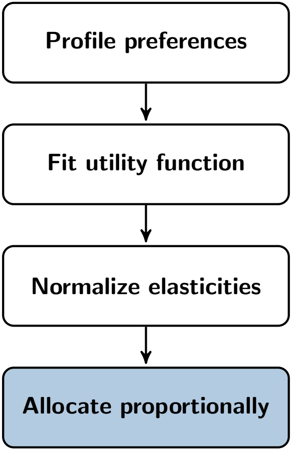

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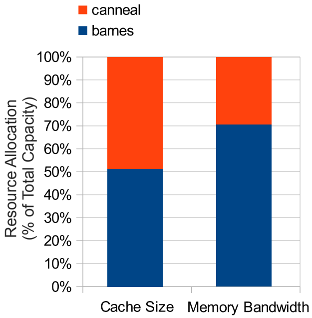
Fairness versus Equal Slowdown



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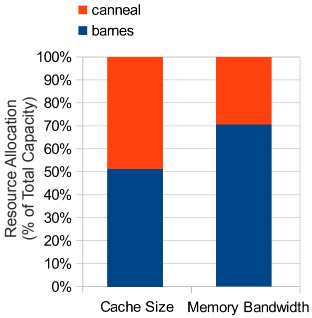

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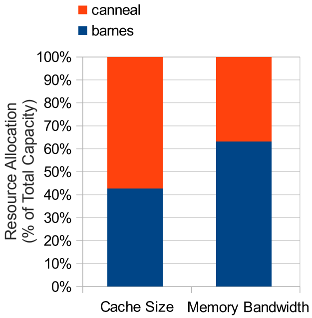
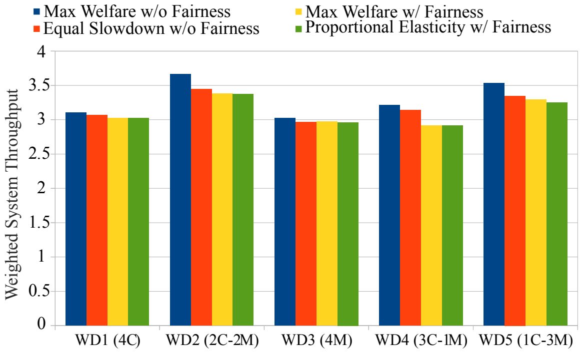


- Resource elasticity fairness provides both SI and EF
- Canneal receives more cache, less memory

Fairness versus Performance



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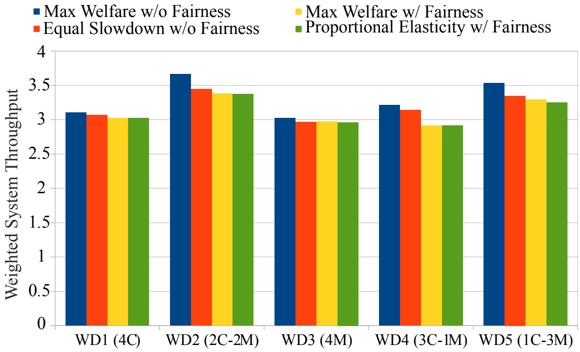

- Measure weighted throughput
- REF incurs $< 10\%$ penalty

Summary

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Thank you

Questions?