Dynamic Proportional Sharing
A Game-Theoretic Approach

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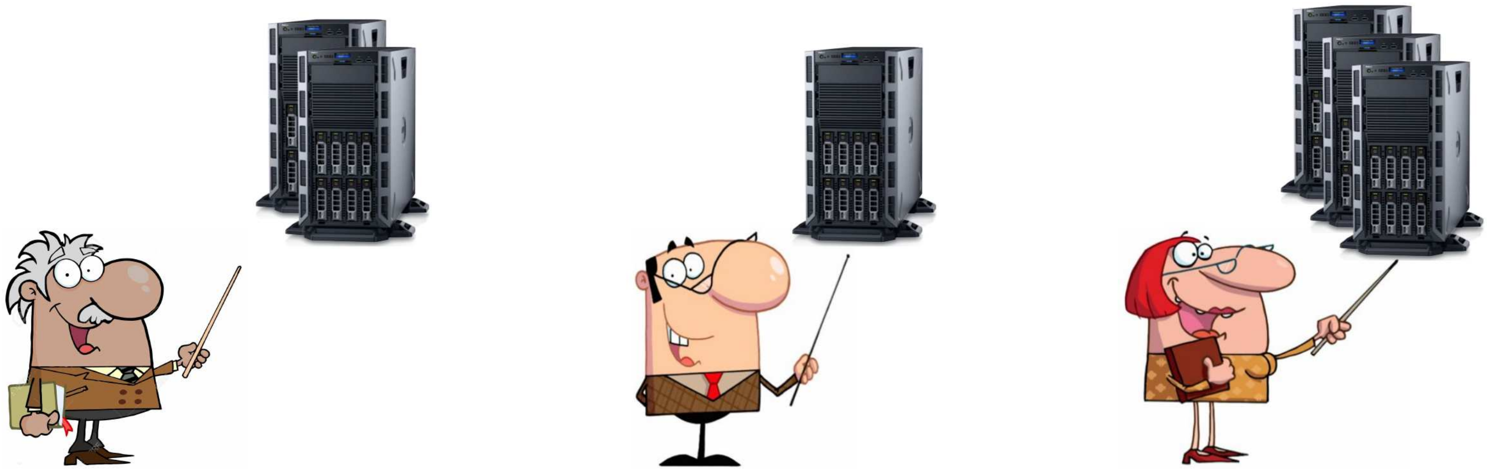


- Users **pool** resources in non-profit data centers
  - E.g., research groups within university
- Users are **entitled** to portion of resources
  - Based on contributions to shared pool

http://clipartmag.com rlv.zcache.com rlv.zcache.com.uk reviews.123rf.com

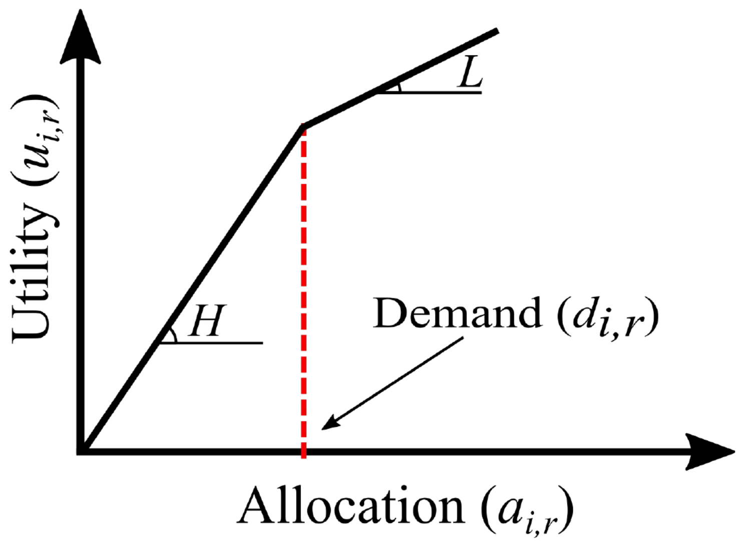
Resource Allocation over Time

- Users report their varying demand every round
- Allocator dynamically allocates resources at each round

Roadmap

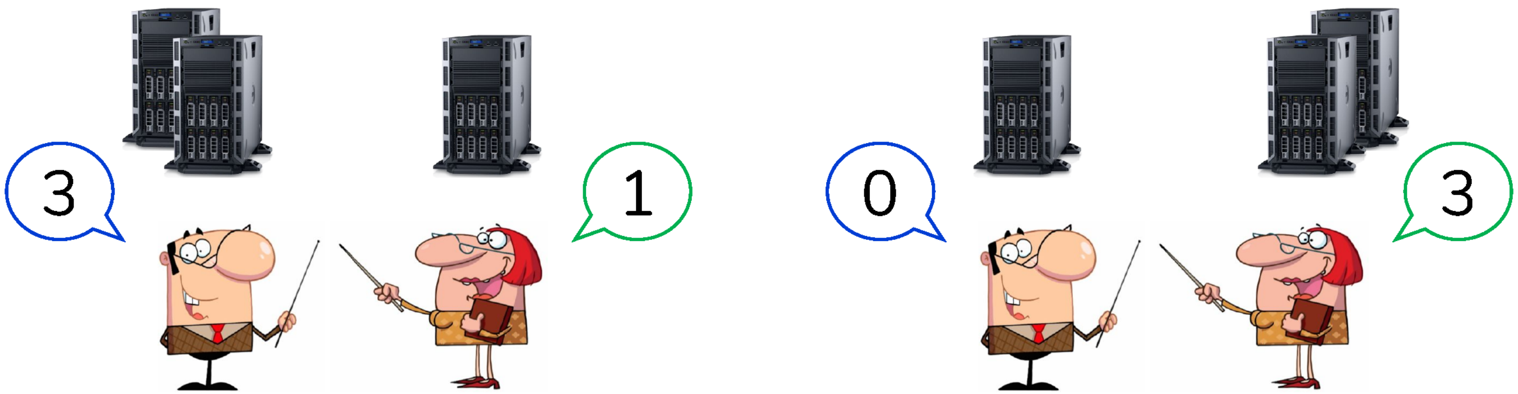
- Utility model
- Existing mechanisms
- Flexible lending mechanism
- T-period mechanism
- Performance evaluation

Utility Model

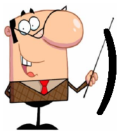


- **High** utility per unit up to **demand** and **low** utility afterwise
- E.g., **processor** allocation to job with **limited parallelism**
  - H to run critical tasks and L to run **replicate** tasks

Dynamic Allocation Example



Round 1 Round 2

$u($  $) = 2H + L$

$u($  $) = H + 2H$

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Max-Min Fairness

- Max-min is main component of modern schedulers
  - E.g., Hadoop schedulers, Spark and Mesos dynamic allocator
- There are two max-min mechanisms in dynamic settings
  - Maximize minimum allocations separately at each round
  - Maximize minimum cumulative allocations up to each round

Desirable Properties

- **Sharing incentives (SI)**

  Sharing should be (weakly) better than not sharing

- **Strategy-proofness (SP)**

  Truthful reporting should be (weakly) better than misreporting

- **Efficiency**

  H-valued resources should be allocated before L-valued resources

http://wallpaperswide.com

Properties of Max-Min Policies

- Theorem(s): Max-min policies **violate** SI and SP

- Theorem: **No** mechanism can satisfy both SI and efficiency
- Theorem: **No** mechanism can satisfy both SP and efficiency

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Flexible Lending Mechanism (Overview)

- Give users as many tokens as their **entitlements**
- Make users pay **one token** for each resource they receive
- Allocate **entire supply** (e.g. total entitlements) at each round
- Allocate **proportionally** to entitlements among users with tokens

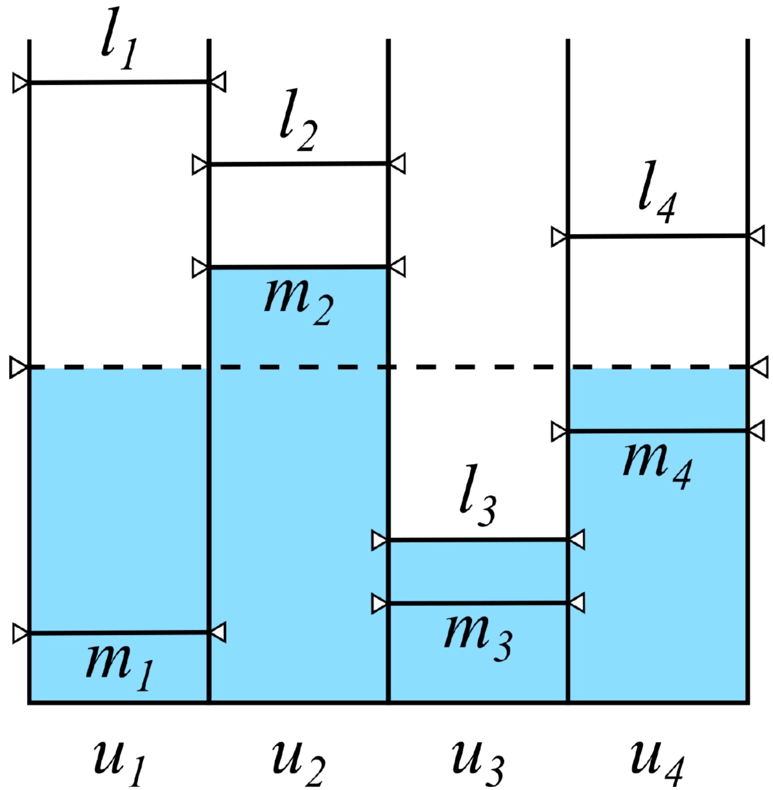
Fig. http://db.tradestreaming.com

Proportional Sharing With Constraints

- PSWC(A, **w**, **l**, **m**)
  - A = amount to allocate
  - **w** = weights
  - **m** = minimum allocations
  - **l** = limit allocations

- Solvable in O(n log(n))

[Divvy alg. Gulati et al. 2012]



Flexible Lending Mechanism (Details)

- Calculate **allocatable demand** for each user
  - min(reported demand, number of tokens)
- Allocate using PSWC based on **total allocatable demand**
  - Total allocatable demand ≥ supply
  - Total allocatable demand < supply
- Make users **pay** one token per unit of allocated resources

Flexible Lending Mechanism (Details ...)

- If total allocatable demand ≥ supply, call PSWC with
  - ***m*** = 0
  - ***l*** = allocatable demands

- If total allocatable demand < supply, call PSWC with
  - ***m*** = allocatable demands
  - ***l*** = number of tokens

Properties of Flexible Lending Mechanism

- Theorem:

  FLM satisfies (tight) **0.5 approx. to sharing incentives**

- Theorem:

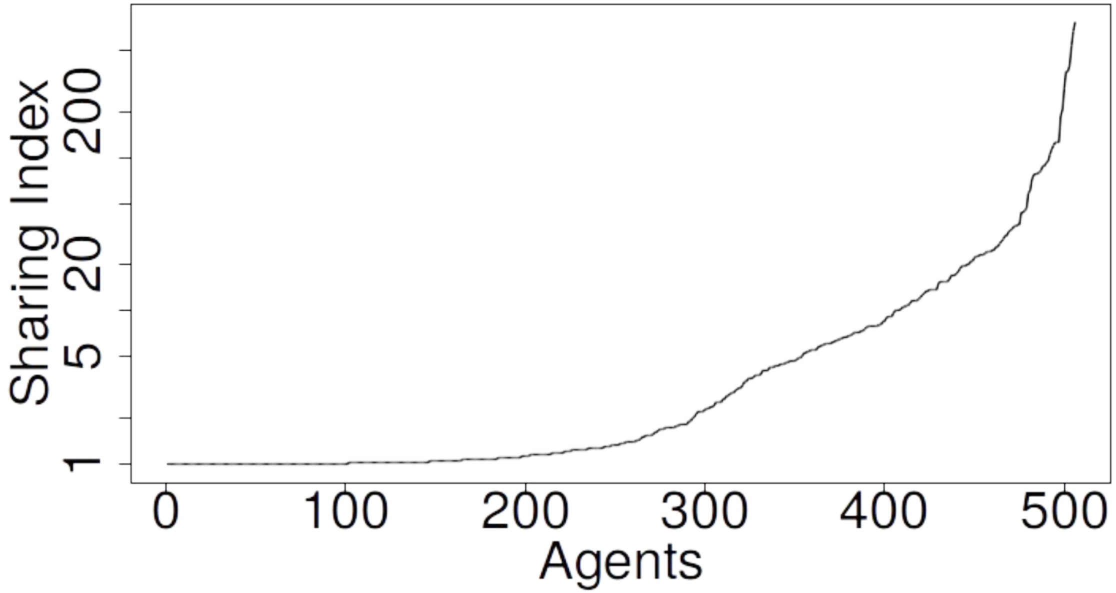
  FLM satisfies **strategy-proofness**

- Theorem:

  FLM **approaches efficiency** as rounds grow for **symm. users**

Evaluation on Google Traces

[Reiss et al. 2011]



- Define **sharing index**
  - utility from sharing / utility from not sharing
- Achieve **high performance**
  - Minimum of **0.98** and average of **15**

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T-Period Mechanism

- **T-Period mechanism**

  Rounds are divided to borrowing and payback periods

- **Theorem(s)**:

  T-Period mechanism satisfy **SP and SI** for **T = 1 and 2**

- Unfortunately, **T >= 3 breaks** strategy-proofness

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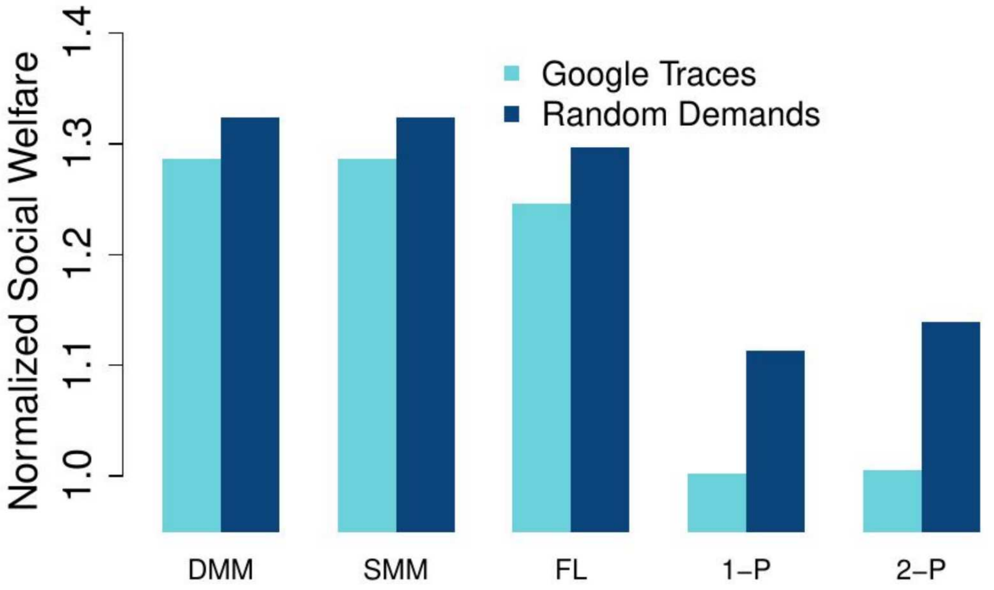
Performance Evaluation



- Flexible lending mechanism achieves 97% of full efficiency

	DMM	SMM	FLM	1-P	2-P
SP	⊖	⊖	✓	✓	✓
SI	⊖	⊖	Approx.	✓	✓

Recap

- Flexible lending mechanism satisfies
  - Minimum **0.5 sharing incentives**
  - **Strategy-proofness**
  - **Efficiency** for symmetric users as number of rounds grows

- (1 & 2)-Period mechanisms satisfy
  - **Strategy-proofness**
  - **Sharing incentives**

Thank You!