ECE750T-28:
Computer-aided Reasoning for Software Engineering

Lecture 5: Conflict-driven Clause Learning SAT solving (Part 2)

Vijay Ganesh
(Original notes from Isil Dillig)
Announcements

- Posted two papers related to this lecture on the course webpage
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▶ One is about a SAT solver called GRASP and the other about the Chaff SAT solver
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- Posted papers on concolic testing and applications to automatic exploit construction
Overview

- **Last lecture**: Basic CDCL algorithm for deciding satisfiability in boolean logic
- **Focus was on overall architecture, conflict analysis and conflict-clause learning**

- Many competitive solvers based on DPLL, but extend it in three important ways:
  1. Non-chronological backtracking
  2. Learning from past "mistakes"
  3. Heuristics for choosing variables and assignments

- In addition, some implementation tricks to perform BCP fast
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- In non-chronological backtracking, we don’t have to go back to most recent decision level
Learning

- Learning = acquisition of new clauses that prevent bad assignments similar to those already explored
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- For instance, suppose the SAT solver makes an assignment and discovers that $p_5 = \top$, $p_{32} = \bot$, $p_{100} = \top$ is inconsistent

Thus, we can add this clause without changing $\varphi$'s satisfiability (why?)

Such clauses "learned" by SAT solvers called conflict clauses

SAT solvers maintain a database of conflict clauses to prevent bad future assignments
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$$\phi \implies \neg(p_5 \land \neg p_{32} \land p_{100})$$

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Decision Heuristics

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- This is something of a black art, but one of the most important elements in SAT solving . . .
Architecture of DPLL-Based SAT Solvers

Decide

BCP

Sat

Analyse Conflict

UNSAT

backtrack if d > 0

conflict

no conflict
Architecture of DPLL-Based SAT Solvers

- **Search**
- **Decide**
- **BCP**
- **Analyze Conflict**

**Processing Flow**:
- **SAT**
- **UNSAT**
- **backtrack if d > 0**

**Decision Points**:
- **no conflict**
- **conflict**
Architecture of DPLL-Based SAT Solvers

- Search
- Deduction
- Decide
- BCP
- Analyze Conflict
- SAT
- UNSAT
- backtrack if $d > 0$
- yes conflict
- no conflict

If $d > 0$, backtrack.
Recall: BCP is all possible applications of unit resolution
BCP in SAT Solvers

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- In addition to performing BCP, SAT solvers also remember deductions performed in the BCP process
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- Thus, BCP process recorded as **implication graph**
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Necessary for analyzing conflicts, inferring conflict clauses

Thus, BCP process recorded as implication graph

First some terminology . . .
Some Terminology and Conventions

- **Decision variable**: variable assigned in the Decide step
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- **Important note**: Think of assignments as literals: Assignment $p = \top$ is literal $p$; assignment $p = \bot$ as literal $\neg p$
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- **Important note**: Think of assignments as literals: Assignment $p = \top$ is literal $p$; assignment $p = \bot$ as literal $\lnot p$

- **Also**: An assignment corresponds to a new unit clause added to our set of clauses
Decision Level Example

\((\neg x_1 \lor x_2) \land (\neg x_3 \lor \neg x_4)\)
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- Is \(x_2\) a decision variable?
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- BCP yields: $x_2 = \top$
- Is $x_2$ a decision variable? No
- Decision level of $x_2$?
Decision Level Example

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(\neg x_1 \lor x_2) \land (\neg x_3 \lor \neg x_4)
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- Decide assigns \( x_1 = \top \) \( \Rightarrow \) \( x_1 \) decision var at level 1

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- Decision level of \( x_2 \)? 1
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- Decide next assigns \(x_4 = \top\). BCP deduces:
\[(\neg x_1 \lor x_2) \land (\neg x_3 \lor \neg x_4)\]

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(¬x₁ ∨ x₂) ∧ (¬x₃ ∨ ¬x₄)

- Decide assigns \( x₁ = \top \) ⇒ \( x₁ \) decision var at level 1

- BCP yields: \( x₂ = \top \)

- Is \( x₂ \) a decision variable? No

- Decision level of \( x₂ \)? 1

- Decide next assigns \( x₄ = \top \). BCP deduces: \( x₃ = \bot \)

- \( x₄ \) decision variable with decision level:
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- Decide next assigns \(x_4 = \top\). BCP deduces: \(x_3 = \bot\)
- \(x_4\) decision variable with decision level: 2
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- \( x_4 \) decision variable with decision level: 2

- \( x_3 \)'s decision level:
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- \(x_4\) decision variable with decision level: 2
- \(x_3\)’s decision level: 2
Implication Graph

- An implication graph is a labeled directed acyclic graph.
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- **Nodes**: literals in the current partial assignment.

A special node $C$ is called the conflict node. Edge to conflict node labeled with $c$: current partial assignment contradicts clause $c$. 
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- **Example**: Node labeled $\neg x : 3$ means variable $x$ was assigned to $\bot$ at decision level 3.
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- **Edges from** $l_1, \ldots, l_k$ **to** $l$ **labeled with** $c$: Assignments $l_1, \ldots, l_k$ caused assignment $l$ due to clause $c$ during BCP
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- Edges from \( l_1, \ldots, l_k \) to \( l \) labeled with \( c \): Assignments \( l_1, \ldots, l_k \) caused assignment \( l \) due to clause \( c \) during BCP.
- A special node \( C \) is called the conflict node.
- Edge to conflict node labeled with \( c \): current partial assignment contradicts clause \( c \).
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- Consider the following set of clauses:

\[ c_1 : (\neg a \lor c) \quad c_2 : (\neg a \lor \neg b) \quad c_3 : (\neg c \lor b) \]
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Assume *Decide* assigned \( a = \top \) at decision level 2.
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- Assignment contradicts \( c_3 \)!
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Consider the following clauses:

\[ c_1 : (\neg a \lor c) \quad c_2 : (\neg c \lor \neg a \lor b) \quad c_3 : (\neg c \lor d) \quad c_4 : (\neg d \lor \neg b) \]
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Suppose \textit{Decide} assigned \( a = \top \) at decision level 1.
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- Using clause \( c_1 \), BCP yields:
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Consider the following clauses:

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\begin{align*}
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- Using clause \( c_2 \), BCP yields:
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Using clause \( c_2 \), BCP yields: \( b = \top \)
Consider the following clauses:

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Using clause \( c_2 \), BCP yields: \( b = \top \)

Using clause \( c_3 \), BCP yields: \( d = \top \)

Assignment \( b = \top, d = \top \) contradicts: \( c_4 : (\neg d \lor \neg b) \)
Consider the following clauses:

\[ c_1 : (\neg a \lor c) \quad c_2 : (\neg c \lor \neg a \lor b) \quad c_3 : (\neg c \lor d) \quad c_4 : (\neg d \lor \neg b) \]

Suppose \textit{Decide} assigned \( a = \top \) at decision level 1

Resulting implication graph:
Example 3

- Based on this implication graph, what is $c_4$?
Example 3

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- What is $c_3$? $\neg x_2 \lor x_4$

- What is $c_1$? 
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- What is $c_3$? $\neg x_2 \lor x_4$
- What is $c_1$? $\neg x_1 \lor x_2$
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- Based on this implication graph, what is \( c_4 \)? \( \neg x_3 \lor \neg x_4 \)

- What is \( c_3 \)? \( \neg x_2 \lor x_4 \)

- What is \( c_1 \)? \( \neg x_1 \lor x_2 \)

- What is \( c_2 \)?
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What is $c_3$? $\neg x_2 \lor x_4$

What is $c_1$? $\neg x_1 \lor x_2$

What is $c_2$? $\neg x_1 \lor x_5 \lor x_3$
Implication Graph Properties

- Root nodes in the implication graph correspond to what kind of variables?
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  decision variables
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Analyzing Conflicts

- So far: Implication graph used to record history of choices and subsequent BCP
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- But whole point of recording this history is to **analyze conflict**
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▶ AnalyzeConflict has two goals:
Analyzing Conflicts

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  1. Learn new conflict clauses
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- **AnalyzeConflict** has two goals:
  1. Learn new conflict clauses
  2. Figure out what level to backtrack to

- **Next:** How to use the implication graph to derive conflict clauses and choose backtracking level
Conflict Clauses

- A conflict clause is a clause (disjunct) implied by the original formula.
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- The implication graph is very useful for deriving small clauses implied by the original formula!
What can we say about source of conflict based on this (partial) implication graph?

Partial assignment $x_1, x_8, \neg x_7$ leads to conflict!

Are other decision variables relevant to conflict? No!

Implication graph allows us to identify a minimal set of "choices" (assignments) relevant to conflict!
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One Strategy to Derive Conflict Clause

- **One way to derive conflict clause:** Conjoin all literals associated with root nodes reaching conflict node, use negation as conflict clause.

Why is this correct?

- Literals associated with root nodes forming a partial assignment $A$ is sufficient to derive contradiction.

Thus, $\phi \land A$ is unsatisfiable; hence $\neg A$ is implied by the formula!

Question: Ok, $\neg A$ is valid conflict clause, but why is it better than taking the negation of the whole partial assignment?

Answer: Because it only includes literals relevant to contradiction; thus resulting clause much smaller!
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▶ In this example, this would yield:
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Using Implication Graph to Analyze Conflicts

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\[ c' \] prevents the same partial assignment in the next step.
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- A key concept is unique implication points
A node $N$ in the implication graph is a unique implication point (UIP) if all paths from current decision node to the conflict node must go through $N$. 

Same concept as dominator.

Is the current decision node a UIP? 

Yes

Can there be multiple unique implication points? 

Yes

First unique implication point: UIP closest to conflict node.
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- **First unique implication point**: UIP closest to conflict node
Which nodes are UIP’s?
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\( \neg x_7 : 8, x_4 : 8 \)
UIP Example

Which nodes are UIP’s? \( \neg x_7 : 8, x_4 : 8 \)

Which node is first UIP?
Which nodes are UIP’s? $\neg x_7 : 8$, $x_4 : 8$

Which node is first UIP? $x_4 : 8$
Inferring better conflict clauses: Start with clause labeling incoming edge to conflict node, derive new clauses via resolution until we find literal in first UIP
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- Resolve \( c \) and \( c' \).
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- Pick any incoming edge to $l$ labeled with clause $c'$.

- Resolve $c$ and $c'$.

- Set current clause be resolvent of $c$ and $c'$.

- Repeat until current clause contains negation of the first UIP literal (as the single literal at current decision level)
Analyzing Conflict via Resolution Example

First UIP

Vijay Ganesh (Original notes from Isil Dillig), ECE750T-28: Computer-aided Reasoning for Software Engineering Lecture 5: Conflict-driven Clause Learning SAT solving (Part 2) 26/46
What is $c_1$?
Analyzing Conflict via Resolution Example

What is $c_1$? ($x_2 \lor x_3$)
Analyzing Conflict via Resolution Example

What is $c_1$? ($x_2 \lor x_3$)

Last assigned literal in $c_1$: 

First UIP
Analyzing Conflict via Resolution Example

- What is $c_1$? $(x_2 \lor x_3)$

- Last assigned literal in $c_1$: $(\neg x_3)$
Analyzing Conflict via Resolution Example

- What is $c_1$? $(x_2 \lor x_3)$
- Last assigned literal in $c_1$: $\neg x_3$
- Clause $c_3$ labeling incoming edge:
Analyzing Conflict via Resolution Example

- What is $c_1$? $(x_2 \lor x_3)$
- Last assigned literal in $c_1$: $(\neg x_3)$
- Clause $c_3$ labeling incoming edge: $(\neg x_3 \lor \neg x_4)$
Analyzing Conflict via Resolution Example

▶ What is $c_1$? $(x_2 \lor x_3)$

▶ Last assigned literal in $c_1$: $(\neg x_3)$

▶ Clause $c_3$ labeling incoming edge: $(\neg x_3 \lor \neg x_4)$

▶ Resolve $c_1$ and $c_3$: 
Analyzing Conflict via Resolution Example

- What is $c_1$? $(x_2 \lor x_3)$
- Last assigned literal in $c_1$: $\neg x_3$
- Clause $c_3$ labeling incoming edge: $(\neg x_3 \lor \neg x_4)$
- Resolve $c_1$ and $c_3$: $x_2 \lor \neg x_4$
Analyzing Conflict via Resolution Example

▶ What is $c_1$? $(x_2 \lor x_3)$

▶ Last assigned literal in $c_1$: $(-x_3)$

▶ Clause $c_3$ labeling incoming edge: $(-x_3 \lor -x_4)$

▶ Resolve $c_1$ and $c_3$: $x_2 \lor -x_4$

▶ $-x_4$ only literal from decision level 8 $\Rightarrow x_2 \lor -x_4$ conflict clause
**Another Example**

- What is the first UIP?
Another Example

What is the first UIP? $x_4 \oplus 5$
Another Example

- What is the first UIP? $x_4@5$

- Start with clause $c_4$:
Another Example

▶ What is the first UIP? $x_4 \oplus 5$

▶ Start with clause $c_4$: $\neg x_6 \lor x_7$
Another Example

- What is the first UIP? $x_4 \oplus 5$
- Start with clause $c_4$: $\neg x_6 \lor x_7$
- Suppose $\neg x_7$ assigned later than $x_6$, so pick $\neg x_7$
Another Example

- What is the first UIP? $x_4 \oplus 5$

- Start with clause $c_4$: $\neg x_6 \lor x_7$

- Suppose $\neg x_7$ assigned later than $x_6$, so pick $\neg x_7$

- Clause on incoming edge to $\neg x_7$:
Another Example

- What is the first UIP? $x_4 @ 5$

- Start with clause $c_4$: $\neg x_6 \lor x_7$

- Suppose $\neg x_7$ assigned later than $x_6$, so pick $\neg x_7$

- Clause on incoming edge to $\neg x_7$: $c_3: (\neg x_5 \lor \neg x_6 \lor \neg x_7)$
Another Example

- What is the first UIP? \( x_4 @ 5 \)
- Start with clause \( c_4: \neg x_6 \lor x_7 \)
- Suppose \( \neg x_7 \) assigned later than \( x_6 \), so pick \( \neg x_7 \)

- Clause on incoming edge to \( \neg x_7 \): \( c_3: (\neg x_5 \lor \neg x_6 \lor \neg x_7) \)
- Resolve \( c_3, c_4: \)
Another Example

- What is the first UIP? $x_4 @ 5$
- Start with clause $c_4$: $\neg x_6 \lor x_7$
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- Clause on incoming edge to $\neg x_7$: $c_3 : (\neg x_5 \lor \neg x_6 \lor \neg x_7)$
- Resolve $c_3, c_4$: $\neg x_5 \lor \neg x_6$
Another Example

- What is the first UIP? \(x_4 @ 5\)
- Start with clause \(c_4: \neg x_6 \lor x_7\)
- Suppose \(\neg x_7\) assigned later than \(x_6\), so pick \(\neg x_7\)

- Clause on incoming edge to \(\neg x_7\): \(c_3: (\neg x_5 \lor \neg x_6 \lor \neg x_7)\)
- Resolve \(c_3, c_4: \neg x_5 \lor \neg x_6\)
- Suppose \(x_6\) assigned later, pick \(x_6\)
Another Example

- What is the first UIP? $x_4@5$

- Start with clause $c_4$: $\neg x_6 \lor x_7$

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- Clause on incoming edge to $\neg x_7$: $c_3 : (\neg x_5 \lor \neg x_6 \lor \neg x_7)$

- Resolve $c_3, c_4$: $\neg x_5 \lor \neg x_6$

- Suppose $x_6$ assigned later, pick $x_6$

- Clause on incoming edge:
Another Example

- What is the first UIP? $x_4 \land 5$
- Start with clause $c_4$: $\neg x_6 \lor x_7$
- Suppose $\neg x_7$ assigned later than $x_6$, so pick $\neg x_7$

- Clause on incoming edge to $\neg x_7$: $c_3: (\neg x_5 \lor \neg x_6 \lor \neg x_7)$
- Resolve $c_3, c_4$: $\neg x_5 \lor \neg x_6$
- Suppose $x_6$ assigned later, pick $x_6$

- Clause on incoming edge: $c_2: \neg x_4 \lor x_{10} \lor x_6$
Another Example

- What is the first UIP? $x_4 \oplus 5$

- Start with clause $c_4$: $\neg x_6 \lor x_7$

- Suppose $\neg x_7$ assigned later than $x_6$, so pick $\neg x_7$

- Clause on incoming edge to $\neg x_7$: $c_3 : (\neg x_5 \lor \neg x_6 \lor \neg x_7)$

- Resolve $c_3, c_4$: $\neg x_5 \lor \neg x_6$

- Suppose $x_6$ assigned later, pick $x_6$

- Clause on incoming edge: $c_2 : \neg x_4 \lor x_{10} \lor x_6$

- Resolve current clause with $c_2$:
Another Example

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- Start with clause $c_4$: $\neg x_6 \lor x_7$
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- Clause on incoming edge to $\neg x_7$: $c_3: (\neg x_5 \lor \neg x_6 \lor \neg x_7)$
- Resolve $c_3, c_4$: $\neg x_5 \lor \neg x_6$
- Suppose $x_6$ assigned later, pick $x_6$

- Clause on incoming edge: $c_2: \neg x_4 \lor x_{10} \lor x_6$
- Resolve current clause with $c_2$: $\neg x_4 \lor x_{10} \lor \neg x_5$
Another Example, cont.

Current clause:

\[ \neg x_3 \lor x_5 \lor \neg x_6 \lor x_7 \lor \neg x_8 \lor x_9 \lor \neg x_{10} \]

Are we done?
No (because \( x_5 \) is also from current decision level)

Pick last assigned literal: \( x_5 \)

Incoming edge to \( x_5 \):
\[ x_2 \lor \neg x_4 \lor x_5 \]

Resolve with current clause:
\[ x_2 \lor \neg x_4 \lor x_{10} \]

Are we done?
Yes!

New conflict clause:
\[ x_2 \lor \neg x_4 \lor x_{10} \]
Another Example, cont.

- Current clause: $\neg x_4 \lor x_{10} \lor \neg x_5$

Vijay Ganesh (Original notes from Isil Dillig)
Another Example, cont.

- Current clause: $\neg x_4 \lor x_{10} \lor \neg x_5$

- Are we done?
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- Current clause: \( \neg x_4 \lor x_{10} \lor \neg x_5 \)
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Another Example, cont.

- Current clause: \( \neg x_4 \lor x_{10} \lor \neg x_5 \)

- Are we done? No (because \( x_5 \) is also from current decision level)

- Pick last assigned literal: \( x_5 \)
Another Example, cont.

- Current clause: $\lnot x_4 \lor x_{10} \lor \lnot x_5$

- Are we done? No (because $x_5$ is also from current decision level)

- Pick last assigned literal: $x_5$

- Incoming edge to $x_5$: 
Another Example, cont.

- **Current clause**: \( \neg x_4 \lor x_{10} \lor \neg x_5 \)

- **Are we done?** No (because \( x_5 \) is also from current decision level)

- **Pick last assigned literal**: \( x_5 \)

- **Incoming edge to \( x_5 \)**: \( x_2 \lor \neg x_4 \lor x_5 \)
Another Example, cont.

- Current clause: \( \neg x_4 \lor x_{10} \lor \neg x_5 \)

- Are we done? No (because \( x_5 \) is also from current decision level)

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- Resolve with current clause:
Another Example, cont.

- Current clause: $\neg x_4 \lor x_{10} \lor \neg x_5$

- Are we done? No (because $x_5$ is also from current decision level)

- Pick last assigned literal: $x_5$

- Incoming edge to $x_5$: $x_2 \lor \neg x_4 \lor x_5$

- Resolve with current clause: $x_2 \lor \neg x_4 \lor x_{10}$
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- Current clause: \( \neg x_4 \lor x_{10} \lor \neg x_5 \)

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Current clause: \( \neg x_4 \lor x_{10} \lor \neg x_5 \)

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Another Example, cont.

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- Incoming edge to $x_5$: $x_2 \lor \neg x_4 \lor x_5$

- Resolve with current clause: $x_2 \lor \neg x_4 \lor x_{10}$

- Are we done? Yes!

- New conflict clause: $x_2 \lor \neg x_4 \lor x_{10}$
Why is this correct?

- **Observe**: At each step, we perform resolution between a clause $c$ on incoming edge of node $l$ and a clause $c'$ on outgoing edge of $l$.
Why is this correct?

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- Furthermore, since \( c \) and \( c' \) are clauses from original formula, any clause we derive is implied by the original formula.
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- Thus, final conflict clause is implied by the original formula!

- It's unclear whether there is a deep reason this works well.

- Empirical results show this strategy is effective . . .
Backtracking

▶ Recall: AnalyzeConflict has two goals.
Recall: AnalyzeConflict has two goals.

First goal: Deriving conflict clauses √
Backtracking

- **Recall**: AnalyzeConflict has two goals.

- **First goal**: Deriving conflict clauses ✓

- **Second goal**: Figure out what level to backtrack to
Recall: AnalyzeConflict has two goals.

First goal: Deriving conflict clauses ✔

Second goal: Figure out what level to backtrack to

Backtrack to level d means delete all variable assignments made after level d (but assignments at level d not deleted)
Backtracking

- **Recall:** AnalyzeConflict has two goals.

- **First goal:** Deriving conflict clauses ✓

- **Second goal:** Figure out what level to backtrack to

- **Backtrack to level d** means delete all variable assignments made after level $d$ (but assignments at level $d$ not deleted)

- **Next:** Talk about how to infer a good level to backtrack to
Backtracking and Asserting Clauses

▶ **A good strategy:** We want to backtrack to a level that makes conflict clause $c$ an **asserting clause** in the next step.
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- Why do we want to make $c$ an asserting clause?

- BCP will force an assignment to unassigned literal $l$ in $c$. 
Choosing Backtracking Level

▶ Question: If we want to make conflict clause $c$ an asserting clause in the next step, what level do we need to backtrack to?

If $l$ is the literal in $c$ with second highest decision level $d$, backtrack to the level $d$.

Why? Since conflict clause contains only one literal, say $l'$, from the first highest decision level, backtracking to $d$ will assert $l'$!
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What level do we backtrack to?
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- **Different assignment than before!**
Recall: SAT Solver Architecture

- **Decide**
  - SAT
  - no conflict
  - backtrack if \( d > 0 \)
- **BCP**
  - conflict
- **Analyze Conflict**
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Recall: SAT Solver Architecture

- **Decision heuristics for choosing variable order and truth assignment**
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  2. variable state independent decaying sum (VSIDS)
Dynamic Largest Individual Sum (DLIS)

- This heuristic chooses the literal that satisfies the **largest number of currently unsatisfied clauses**.

\[(x_1 \lor \neg x_2) \land (\neg x_2 \lor x_3) \land (\neg x_1 \lor x_2 \lor \neg x_3)\]

What assignment would DLIS pick for this formula? (assuming no assignments so far)
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- Thus, overhead can be high and must be implemented carefully to minimize bookkeeping
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- Introduced in the CHAFF SAT solver from Princeton, written by undergrads!
Implementation Tricks

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- We’ll talk about two issues:
  1. number of conflict clauses
  2. trick to perform BCP fast: watch literals
Conflict Clauses

- **Recall**: After analyzing conflict, we add new conflict clause to our clause database
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⇒ Tradeoff between conflict prevention and minimizing overhead
Conflict Clauses, cont.

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Conflict Clauses, cont.

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▶ Typically, keep most recent conflict clauses since they are most relevant to current part of search space
Implementing BCP

- Implementing BCP efficiently is very important because SAT solvers spend a lot of time doing BCP

Naive implementation of BCP: Requires scanning all currently unsatisfied clauses

But industrial instances of boolean SAT problems contain hundreds of thousands of clauses

Thus, scanning all unsatisfied clauses too expensive!

A more intelligent implementation:
Keep mapping from each literal to all clauses in which each literal appears (because we perform unit resolution after each variable assignment)

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- **Idea:** Since a clause will not imply new variable assignment unless it has only two literals left, we only need to look at clauses that have at most two unassigned literals!
Watch Literals

- To efficiently detect clauses with at most two unassigned literals, select two unassigned literals in each unsatisfied clause as **watch literals**
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- If a watch literal is assigned a truth value and there are no other unassigned non-watch literals left, BCP implies an assignment to the only remaining watch literal!
Watch Literals, cont.

▶ **Question:** Given this invariant, if we make assignment \( l \), which clauses can imply new variable assignments?

▶ Only those clauses in which \( \neg l \) appears as watch literal

▶ If \( \neg l \) does not appear, we can't perform unit resolution

▶ If \( \neg l \) appears but is not a watch literal, then clause has more than two unassigned literals \( \Rightarrow \) won't imply new assignment!

▶ Watch literal trick makes BCP much faster because much fewer clauses contain negation of current literal as a watch literal!

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Watch Literals, cont.

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- Most competitive solvers today are based on DPLL

- But they extend DPLL in three ways: non-chronological backtracking, conflict clause learning, and decision heuristics

- In addition, clever implementation tricks like watch literals

- Some competitive DPLL-based SAT solvers: ZChaff, MiniSAT, PicoSAT

- There are also other kinds of SAT solvers not based on DPLL, for instance, perform stochastic search (e.g., WalkSAT)

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