Formalization of Generic Universe Types

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Abstract

Ownership is a powerful concept to structure the object store and to control aliasing and modifications of objects. This paper presents an ownership type system for a Java-like programming language with generic types. Like our earlier Universe type system, Generic Universe Types enforce the owner-as-modifier discipline. This discipline does not restrict aliasing, but requires modifications of an object to be initiated by its owner. This allows owner objects to control state changes of owned objects, for instance, to maintain invariants. Generic Universe Types require a small annotation overhead and provide strong static guarantees. They are the first type system that combines the owner-as-modifier discipline with type genericity.

Contents

1 Introduction

The concept of object ownership allows programmers to structure the object store hierarchically and to control aliasing and access between objects. Ownership has been applied successfully to various problems, for instance, program verification [20, 22, 23], thread synchronization [6, 17], memory management [2, 8], and representation independence [3].

Existing ownership models share fundamental concepts: Each object has at most one owner object. The set of all objects with the same owner is called a *context*. The *root context* is the set of objects with no owner. The ownership relation is a tree order.

However, existing models differ in the restrictions they enforce. The original ownership types [11] and their descendants [7, 9, 10, 26] restrict aliasing and enforce the *owner-as-dominator* discipline: All reference chains from an object in the root context to an object `o` in a different context go through `o`'s owner. This severe restriction of aliasing is necessary for some of the applications of ownership, for instance, memory management and representation independence.

However, for applications such as program verification, restricting aliasing is not necessary. Instead, it suffices to enforce the *owner-as-modifier* discipline: An object `o` may be referenced by any other object, but reference chains that do not pass through `o`'s owner must not be used to modify `o`. This allows owner objects to control state changes of owned objects, and thus maintain invariants. The owner-as-modifier discipline is enforced by the Universe type system [13], in Spec#'s dynamic ownership model [20], and Effective Ownership Types [21]. The owner-as-modifier discipline imposes weaker restrictions than the owner-as-dominator discipline, which allows it to handle common implementations where objects are shared between objects, such as collections with iterators, shared buffers, or the Flyweight pattern [13, 24]. Some implementations can be slightly adapted to satisfy the owner-as-modifier discipline, for example an iterator can delegate modifications to the corresponding collection which owns the internal representation.

Although ownership type systems have covered all features of Java-like languages (including for example exceptions, inner classes, and static class members) there are only three proposals of ownership type systems that support generic types. SafeJava [5] supports type parameters and ownership parameters independently, but does not integrate both forms of parametricity. This leads to significant annotation overhead. Ownership Domains [1] combine type parameters and domain parameters into a single parameter space and thereby reduce the annotation overhead. However, their formalization does not cover type parameters. Ownership Generic Java (OGJ) [26] allows programmers to attach ownership information through type parameters. For instance, a collection of `Book` objects can be typed as "my collection of library books", expressing that the collection object belongs to the current `this` object, whereas the `Book` objects in the collection belong to an object "library". OGJ enforces the owner-as-dominator discipline. It piggybacks ownership information on type parameters. In particular, each class `C` has a type parameter to encode the owner of a `C` object. This encoding allows OGJ to use a slight adaptation of the normal Java type rules to also check ownership, which makes the formalization very elegant.

However, OGJ cannot be easily adapted to enforce the owner-as-modifier discipline.

For example, OGJ would forbid a reference from the iterator (object 6) in Fig. 1 to a node (object 5) of the map (object 3), because the reference bypasses the node's owner. However, such references are necessary, and are legal in the owner-as-modifier discipline. A type system can permit such references in two ways.

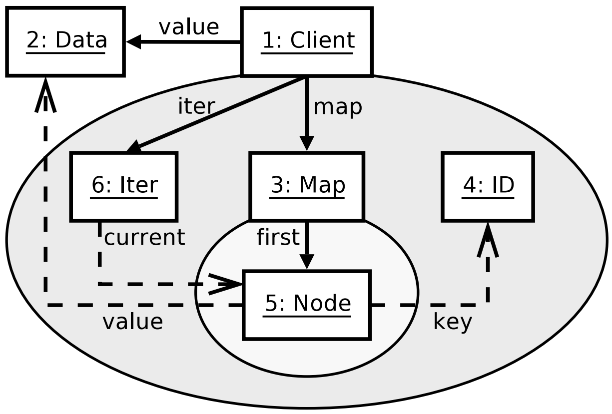

Figure 1: Object structure of a map from `ID` to `Data` objects. The map is represented by `Node` objects. The iterator has a direct reference to a node. Objects, references, and contexts are depicted by rectangles, arrows, and ellipses, respectively. Owner objects sit atop the context of objects they own. Arrows are labeled with the name of the variable that stores the reference. Dashed arrows depict references that cross context boundaries without going through the owner. Such references must not be used to modify the state of the referenced objects.

First, if the iterator contained a field `theMap` that references the associated map object, then path-dependent types [1, 7, 25] can express that the `current` field of the iterator points to a `Node` object that is owned by `theMap`. Unfortunately, path-dependent types require the fields on the path (here, `theMap`) to be final, which is too restrictive for many applications.

Second, one can loosen up the static ownership information by allowing certain references to point to objects in any context [13]. Subtyping allows values with specific ownership information to be assigned to "any" variables, and downcasts with runtime checks can be used to recover specific ownership information from such variables. In OGJ, this subtype relation between `any`-types and other types would require covariant subtyping, for instance, that `Node<This>` is a subtype of `Node<Any>`, which is not supported in Java (or C#). Therefore, piggybacking ownership on the standard Java type system is not possible in the presence of `any`.

In this paper, we present Generic Universe Types (GUT), an ownership type system for a programming language with generic types similar to Java 5 and C# 2.0. GUT enforces the owner-as-modifier discipline using an `any` ownership modifier (analogous to the `readonly` modifier in non-generic Universe types [13]). Our type system supports type parameters for classes, interfaces, and methods. The annotation overhead for programmers is as low as in OGJ, although the presence of `any` makes the type rules more involved. A particularly interesting aspect of our work is how generics and ownership can be combined in the presence of an `any` modifier, in particular, how a restricted form of ownership covariance can be permitted without runtime checks.

Outline. Sec. 2 of this paper illustrates the main concepts of Generic Universe Types by an example. Secs. 3 and 4 present the type rules and the runtime model of Generic Universe Types, respectively. Sec. 5 presents the type safety and the owner-as-modifier property theorems and proofs. Finally, Sec. 6 concludes.

2 Main Concepts

In this section, we explain the main concepts of Generic Universe Types informally by an example. Class `Map` (Fig. 2) implements a generic map from keys to values. Key-value pairs are stored in singly-linked `Node` objects. Class `Node` extends the superclass `Link` (both Fig. 3), which is used by the iterator class `Iter` (Fig. 4). The `main` method of class `Client` (Fig. 5) builds up the map structure shown in Fig. 1. For simplicity, we omit access modifiers from all examples. We chose an unusual iterator design to highlight several technical details of the type system. We will discuss this design and provide a more flexible solution at the end of this section.

Ownership Modifiers. A reference type in GUT is either a type variable or consists of an ownership modifier, a class name, and possibly type arguments. The *ownership modifier* expresses object ownership relative to the current receiver object `this`[1]. Programs may contain the ownership modifiers `peer`, `rep`, and `any`. `peer` expresses that an object has the same owner as the `this` object, `rep` expresses that an object is owned by `this`, and `any` expresses that an object may have any owner. `any` types are supertypes of the `rep` and `peer` types with the same class and type arguments because they convey less specific ownership information.

The use of ownership modifiers is illustrated by class `Map` (Fig. 2). A `Map` object owns its `Node` objects since they form the internal representation of the map and should, therefore, be protected from unwanted modifications. This ownership relation is expressed by the `rep` modifier of `Map`'s field `first`, which points to the first node of the map.

The owner-as-modifier discipline is enforced by disallowing modifications of objects through `any` references. That is, an expression of an `any` type may be used as receiver of field reads and calls to side-effect free (*pure*) methods, but not of field updates or calls to non-pure methods. To check this property, we require side-effect free methods to be annotated with the keyword `pure`.

Viewpoint Adaptation. Since ownership modifiers express ownership relative to `this`, they have to be adapted when this "viewpoint" changes. Consider the third parameter of `Node`'s method `init`. The `peer` modifier expresses that the parameter object must have the same owner as the receiver of the method. On the other hand, `Map`'s method `put` calls `init` on a `rep Node` receiver, that is, an object that is owned by `this`. Therefore, the

[1] We ignore static methods in this paper, but an extension is possible [22].

```
class Map<K,V> {
  rep Node<K,V> first;

  void put(K key, V value) {
    rep Node<K,V> newfirst = new rep Node<K,V>();
    newfirst.init(key, value, first);
    first = newfirst;
  }

  V get(K key) {
    peer Iter<rep Node<K,V>> i = iterator();
    while (i.hasNext()) {
      rep Node<K,V> mn = i.next();
      if (mn.key.equals(key)) return mn.value;
    }
    return null;
  }

  peer Iter<rep Node<K,V>> iterator() {
    peer Iter<rep Node<K,V>> res;
    res = new peer Iter<rep Node<K,V>>();
    res.init(first);
    return res;
  }
}
```

Figure 2: An implementation of a generic map. `Map` objects own their `Node` objects, as indicated by the `rep` modifier in all occurrences of class `Node`.

third parameter of the call to `init` also has to be owned by `this`. This means that from this particular call's viewpoint, the third parameter needs a `rep` modifier, although it is declared with a `peer` modifier. In the type system, this *viewpoint adaptation* is done by combining the type of the receiver of a call (here, `rep Node<K,V>`) with the type of the formal parameter (here, `peer Node<K,V>`). This combination yields the argument type from the caller's point of view (here, `rep Node<K,V>`).

Type Parameters. Ownership modifiers are also used in actual type arguments. For instance, `Map`'s method `get` instantiates class `Iter` with the type argument `rep Node<K,V>`. Thus, local variable `i` has type `peer Iter<rep Node<K,V>>`, which has two ownership modifiers. The *main modifier* `peer` expresses that the `Iter` object has the same owner as `this`, whereas the *argument modifier* `rep` expresses that the `Node` objects used by the iterator are owned by `this`. It is important to understand that this argument modifier again expresses ownership relative to the current `this` object (here, the `Map` object), and not relative to the instance of the generic class that contains the argument modifier (here, `i`).

```
class Link<X> {
  X next;
  void initLink(X n) { next = n; }
}

class Node<K,V> extends Link<peer Node<K,V>> {
  K key; V value;

  void init(K k, V v, peer Node<K,V> n) {
    initLink(n); key = k; value = v;
  }
}
```

Figure 3: Nodes form the internal representation of maps. Class `Link` implements rudimentary nodes for singly-linked lists. Its subclass `Node` instantiates `Link`'s type parameter to implement a list of nodes with the same owner. It also adds attributes to store the keys and values of the map.

Type variables have upper bounds, which default to `any Object`. In a class `C`, the ownership modifiers of an upper bound express ownership relative to the `C` instance `this`. However, when `C`'s type variables are instantiated, the modifiers of the actual type arguments are relative to the receiver of the method that contains the instantiation. Therefore, checking the conformance of a type argument to its upper bound requires a viewpoint adaptation. For instance, to check the instantiation `peer Iter<rep Node <K,V>>` in class `Map`, we adapt the upper bound of `Iter`'s type variable (`any Link<X>`) from viewpoint `peer Iter<rep Node <K,V>>` to the viewpoint `this`. With the appropriate substitutions, this adaptation yields `any Link<rep Node<K,V>>`. The actual type argument `rep Node<K,V>` is a subtype of the adapted upper bound. Therefore, the instantiation is correct. The `rep` modifier in the type argument and the adapted upper bound reflects correctly that the `current` node of this particular iterator is owned by `this`.

Type variables are not subject to the viewpoint adaptation that is performed for non-variable types. When type variables are used, for instance, in field declarations, the ownership information they carry stays implicit and does, therefore, not have to be adapted. The substitution of type variables by their actual type arguments happens in the scope in which the type variables were initially instantiated. Therefore, the viewpoint is the same as for the instantiation, and no viewpoint adaptation is required. For instance, the call expression `i.next()` in method `get` (Fig. 2) has type `rep Node<K,V>`, because the result type of `next()` is the type variable `X`, which gets substituted by the type argument of `i`'s type, `rep Node<K,V>`.

Therefore, even though an `Iter` object does not know the owner of the nodes it references (due to the `any` upper bound), clients of the iterator can recover the exact ownership information from the type argument. This illustrates that Generic Universe Types provide strong static guarantees similar to those of owner-parametric systems [11], even in the

```
class Iter<X extends any Link<X>> {
  X current;

  void init(X start) { setCurrent(start); }
  void setCurrent(X c) { current = c; }
  pure boolean hasNext() { return current != null; }

  X next() {
    X result = current;
    current = current.next;
    return result;
  }
}
```

Figure 4: Class `Iter` implements iterators over `Link` structures. The precise node type is passed as type parameter. The upper bound allows method `next` to access a node's `next` field.

presence of `any` types. The corresponding implementation in non-generic Universe types requires a downcast from the `any` type to a `rep` type with the corresponding runtime check [13].

Limited Covariance and Viewpoint Adaptation of Type Arguments. Subtyping with covariant type arguments is in general not type safe. For instance, if `List<String>` was a subtype of `List<Object>`, then clients that view a string list through type `List<Object>` could store `Object` instances in the string list, which breaks type safety. The same problem occurs for the ownership information encoded in types. If `peer Iter<rep Node<K,V>>` was a subtype of `peer Iter<any Node<K,V>>`, then clients that view the iterator through the latter type could use method `setCurrent` (Fig. 4) to set the iterator to a `Node` object with an arbitrary owner, even though the iterator requires a specific owner. The covariance problem can be prevented by disallowing covariant type arguments (like in Java and C#), by runtime checks, or by elaborate syntactic support [14].

However, the owner-as-modifier discipline supports a limited form of covariance without any additional checks. Covariance is permitted if the main modifier of the supertype is `any`. For example, `peer Iter<rep Node<K,V>>` is an admissible subtype of `any Iter<any Node<K,V>>`. This is safe because the owner-as-modifier discipline prevents mutations of objects referenced through `any` references. In particular, it is not possible to set the iterator to an `any Node` object, which prevents the unsoundness illustrated above.

Besides subtyping, GUT provides another way to view objects through different types, namely viewpoint adaptation. If the adaptation of a type argument yields an `any` type, the same unsoundness as covariance could occur. Therefore, when a viewpoint adaptation changes an ownership modifier of a type argument to `any`, it also changes the main modifier to `any`.

This behavior is illustrated by method `main` of class `Client` in Fig. 5. As illustrated by Fig. 1, the most precise type for the call expression `map.iterator()` would be `rep Iter<any Node<rep ID, any Data>>` because the `Iter` object is owned by the `Client` object `this` (hence, the main modifier `rep`), but the nodes referenced by the iterator are neither owned by `this` nor peers of `this` (hence, `any Node`). However, this viewpoint adaptation would change an argument modifier of `iterator`'s result type from `rep` to `any`. This would allow method `main` to use method `setCurrent` to set the iterator to an `any Node` object and is, thus, not type safe. The correct viewpoint adaptation yields `any Iter<any Node<rep ID, any Data>>`. This type is safe, because it prevents the `main` method from mutating the iterator, in particular, from calling the non-pure method `setCurrent`.

```
class ID { /* ... */ }
class Data { /* ... */ }

class Client {
  void main(any Data value) {
    rep Map<rep ID, any Data> map;
    map = new rep Map<rep ID, any Data>();
    map.put(new rep ID(), value);

    any Iter<any Node<rep ID, any Data>> iter;
    iter = map.iterator();
    // ...
  }
}
```

Figure 5: Main program for our example. The execution of method `main` creates the object structure in Fig. 1.

Unfortunately, since method `next` is also non-pure, `main` must not call `iter.next()` either, which renders `Iter` objects useless outside the associated `Map` object. However, this is not a severe restriction since an iterator that exposes internal nodes should not be available to clients in the first place. That is, `Map`'s `iterator` method should be private. Alternatively, the iterator could yield pairs of keys and values rather than internal nodes. Such an iterator is shown in Fig. 6. The adapted implementation of the `iterator` method looks as follows:

```
  peer PairIter<K,V> iterator() {
    return new peer PairIter<K,V>(first);
  }
```

Since the type arguments of `iterator`'s result type are type variables, they are not subject to viewpoint adaptation. With this implementation of `iterator`, the type of the call expression `map.iterator()` (Fig. 5) is determined by adapting the main modifier and by performing the appropriate substitutions, which yields `rep PairIter<rep ID, any Data>`.

Since the main modifier is `rep` rather than `any`, this type allows method `main` to call `next` on the iterator.

```
class Pair<X,Y> {
  X x; Y y;
  void init(X px, Y py) { x = px; y = py; }
}

class PairIter<K,V> {
  any Node<K,V> current;

  void init(any Node<K,V> start) { current = start; }
  boolean hasNext() { return current != null; }

  peer Pair<K,V> next() {
    peer Pair<K,V> result = new peer Pair<K,V>();
    result.init(current.key, current.value);
    current = current.next;
    return result;
  }
}
```

Figure 6: A more sensible iterator implementation yielding key-value pairs.

3 Static Checking

In this section, we formalize the compile time aspects of Generic Universe Types. We define the syntax of the programming language, formalize viewpoint adaptation, define subtyping and well-formedness conditions, and present the type rules.

3.1 Programming Language

We formalize Generic Universe Types for a sequential subset of Java 5 and C# 2.0 including classes and inheritance, instance fields, dynamically-bound methods, and the usual operations on objects (allocation, field read, field update, casts). For simplicity, we omit several features of Java and C# such as interfaces, exceptions, constructors, static fields and methods, inner classes, primitive types and the corresponding expressions, and all statements for control flow. We do not expect that any of these features is difficult to handle (see for instance [5, 12, 22] for a discussion of these features in ownership type systems for non-generic Java). The language we use is similar to Featherweight Generic Java [16]. We added field updates because the treatment of side effects is essential for ownership type systems and especially the owner-as-modifier discipline.

Fig. 7 summarizes the syntax of our language and our naming conventions for variables.

We assume that all identifiers of a program are globally unique except for `this` as well as method and parameter names of overridden methods. This can be achieved easily by preceding each identifier with the class or method name of its declaration (but we omit this prefix in our examples).

We use the superscript $^{\mathsf{s}}$ to distinguish the sorts for static checking from corresponding sorts that will be used to describe the runtime behavior. Whenever it is clear from context whether we refer to static or runtime entities, we omit the superscript $^{\mathsf{s}}$.

$\overline{\mathtt{T}}$ denotes a sequence of Ts. In such a sequence, we denote the i-th element by $\mathtt{T}_i$. We sometime use sequences of tuples $S = \overline{\mathtt{X}\ \mathtt{T}}$ as maps and use a function-like notation to access an element $S(\mathtt{X}_i) = \mathtt{T}_i$. A sequence $\overline{\mathtt{T}}$ can be empty. The empty sequence is denoted by ϵ.

A program (sort `Program`) consists of a sequence of classes, the identifier of a main class `C`, and a main expression `e`. A program is executed by creating an instance `o` of `C` and then evaluating `e` with `o` as `this` object. We assume that we always have access to the current program P, and keep P implicit in the notations. Each class (`Class`) has a class identifier (`ClassId`), type variables with upper bounds, a superclass with type arguments, a list of field declarations, and a list of method declarations. `FieldId` is the sort of field identifiers. Like in Java, each class directly or transitively extends the predefined class `Object`. The default upper bound for type variables is `any Object`.

A type ($^{\mathsf{s}}$`Type`) is either a non-variable type or a type variable identifier (`TVarId`). A non-variable type ($^{\mathsf{s}}$`NType`) consists of an ownership modifier, a class identifier, and a sequence of type arguments.

The sort `OM` of ownership modifiers contains $\mathtt{peer}_u$, $\mathtt{rep}_u$, and $\mathtt{any}_u$ as well as the modifier $\mathtt{this}_u$, which is used solely as main modifier for the type of `this`. The modifier $\mathtt{this}_u$ may not appear in programs, but is used internally by the type system to distinguish accesses through `this` from other accesses, which simplifies the type rules. We omit the subscript u if it is clear from context that we mean an ownership modifier.

A method (`Meth`) consists of a signature and an expression as body. The result of evaluating the expression is returned by the method. The signature of a method (`MethSig`) consists of the method type variables with their upper bounds, the purity annotation, the return type, the method identifier (`MethId`), and the formal method parameters (`ParId`) with their types. Sort `ParId` includes the implicit method parameter `this`.

To be able to enforce the owner-as-modifier discipline, we have to distinguish statically between side-effect free (pure) methods and methods that potentially have side effects. Pure methods are marked by the keyword `pure`. In our syntax, we mark all other methods by `nonpure`, although we omit this keyword in our examples. To focus on the essentials of the type system, we do not include purity checks, but they can be added easily [22].

The set of expressions (`Expr`) contains the `null` literal, method parameter access, field read, field update, method call, object creation, and cast.

Type checking is performed in a type environment ($^{\mathsf{s}}$`Env`), which maps the type variables of the enclosing class and the enclosing method to their upper bounds and method parameters to their types. Since the domains of both mappings are disjoint, we use an overloaded notation and simply write $^{\mathsf{s}}\Gamma(\mathtt{X})$ to refer to the upper bound of type variable `X` and $^{\mathsf{s}}\Gamma(\mathtt{x})$ to refer to the type of method parameter `x`.

$$
\begin{array}{rclcl}
\texttt{P} & \in & \texttt{Program} & ::= & \overline{\texttt{Class}}\ \texttt{ClassId}\ \texttt{Expr} \\
\texttt{Cls} & \in & \texttt{Class} & ::= & \texttt{class}\ \texttt{ClassId}\texttt{<}\overline{\texttt{TVarId}\ {}^{s}\texttt{NType}}\texttt{>}\ \texttt{extends}\ \texttt{ClassId}\texttt{<}\overline{{}^{s}\texttt{Type}}\texttt{>} \\
 & & & & \texttt{\{}\ \overline{\texttt{FieldId}\ {}^{s}\texttt{Type};}\ \overline{\texttt{Meth}}\ \texttt{\}} \\
{}^{s}\texttt{T} & \in & {}^{s}\texttt{Type} & ::= & {}^{s}\texttt{NType} \mid \texttt{TVarId} \\
{}^{s}\texttt{N} & \in & {}^{s}\texttt{NType} & ::= & \texttt{OM}\ \texttt{ClassId}\texttt{<}\overline{{}^{s}\texttt{Type}}\texttt{>} \\
\texttt{u} & \in & \texttt{OM} & ::= & \texttt{peer}_u \mid \texttt{rep}_u \mid \texttt{any}_u \mid \texttt{this}_u \\
\texttt{mt} & \in & \texttt{Meth} & ::= & \texttt{MethSig}\ \texttt{\{ return Expr \}} \\
 & & \texttt{MethSig} & ::= & \texttt{<}\overline{\texttt{TVarId}\ {}^{s}\texttt{NType}}\texttt{>}\ \texttt{Purity}\ {}^{s}\texttt{Type}\ \texttt{MethId}(\overline{\texttt{ParId}\ {}^{s}\texttt{Type}}) \\
\texttt{w} & \in & \texttt{Purity} & ::= & \texttt{pure} \mid \texttt{nonpure} \\
\texttt{e} & \in & \texttt{Expr} & ::= & \texttt{null} \mid \texttt{ParId} \mid \texttt{Expr.FieldId} \mid \texttt{Expr.FieldId=Expr} \mid \\
 & & & & \texttt{Expr.MethId}\texttt{<}\overline{{}^{s}\texttt{Type}}\texttt{>}(\overline{\texttt{Expr}}) \mid \texttt{new}\ {}^{s}\texttt{Type} \mid ({}^{s}\texttt{Type})\ \texttt{Expr} \\
{}^{s}\Gamma & \in & {}^{s}\texttt{Env} & ::= & \overline{\texttt{TVarId}\ {}^{s}\texttt{NType}};\overline{\texttt{ParId}\ {}^{s}\texttt{Type}}
\end{array}
$$

Figure 7: Syntax and type environments.

3.2 Viewpoint Adaptation

Since ownership modifiers express ownership relative to an object, they have to be adapted whenever the viewpoint changes. In the type rules, we need to *adapt a type* `T` *from a viewpoint* that is described by another type $\texttt{T}'$ *to the viewpoint* `this`. In the following, we omit the phrase "to the viewpoint `this`". To perform the viewpoint adaptation, we define an overloaded operator $\triangleright$ to: (1) Adapt an ownership modifier from a viewpoint that is described by another ownership modifier; (2) Adapt a type from a viewpoint that is described by an ownership modifier; (3) Adapt a type from a viewpoint that is described by another type.

Adapting an Ownership Modifier w.r.t. an Ownership Modifier. We explain viewpoint adaptation using a field access $\texttt{e}_1\texttt{.f}$. Analogous adaptations occur for method parameters and results as well as upper bounds of type arguments. Let `u` be the main modifier of $\texttt{e}_1$'s type, which expresses ownership relative to `this`. Let $\texttt{u}'$ be the main modifier of `f`'s type, which expresses ownership relative to the object that contains `f`. Then relative to `this`, the type of the field access $\texttt{e}_1\texttt{.f}$ has main modifier $\texttt{u}\triangleright\texttt{u}'$.

$$
\begin{array}{rclrcll}
\cdot \triangleright \cdot\ ::\ \texttt{OM} \times \texttt{OM} & \rightarrow & \texttt{OM} & & & & \\
\texttt{this}\triangleright\texttt{u}' & = & \texttt{u}' & \texttt{rep}\triangleright\texttt{peer} & = & \texttt{rep} & \\
\texttt{peer}\triangleright\texttt{peer} & = & \texttt{peer} & \texttt{u}\triangleright\texttt{u}' & = & \texttt{any} & \text{otherwise}
\end{array}
$$

The field access $\texttt{e}_1\texttt{.f}$ illustrates the motivation for this definition: (1) Accesses through `this` (that is, $\texttt{e}_1$ is the variable `this`) do not require a viewpoint adaptation since the ownership modifier of the field is already relative to `this`. (2) If the main modifiers of both $\texttt{e}_1$ and `f` are `peer`, then the object referenced by $\texttt{e}_1$ has the same owner as `this` and the object referenced by $\texttt{e}_1\texttt{.f}$ has the same owner as $\texttt{e}_1$ and, thus, the same owner as `this`. Consequently, the main modifier of $\texttt{e}_1\texttt{.f}$ is also `peer`. (3) If the main modifier of $\texttt{e}_1$ is `rep` and the main modifier of `f` is `peer`, then the main modifier of $\texttt{e}_1\texttt{.f}$ is `rep`, because the object referenced by $\mathtt{e}_1$ is owned by `this` and the object referenced by $\mathtt{e}_1$.f has the same owner as $\mathtt{e}_1$, that is, `this`. (4) In all other cases, we cannot determine statically that the object referenced by $\mathtt{e}_1$.f has the same owner as `this` or is owned by `this`. Therefore, in these cases the main modifier of $\mathtt{e}_1$.f is `any`.

Adapting a Type w.r.t. an Ownership Modifier. As explained in Sec. 2, type variables are not subject to viewpoint adaptation. For non-variable types, we determine the adapted main modifier using the auxiliary function $\triangleright_m$ below and adapt the type arguments recursively:

$$\begin{array}{rcl} \cdot \triangleright \cdot \;::\; \mathtt{OM} \times {}^{s}\mathtt{Type} & \rightarrow & {}^{s}\mathtt{Type} \\ \mathtt{u} \triangleright \mathtt{X} & = & \mathtt{X} \\ \mathtt{u} \triangleright \mathtt{N} & = & (\mathtt{u} \triangleright_m \mathtt{N})\ \mathtt{C}\langle\overline{\mathtt{u} \triangleright \mathtt{T}}\rangle \text{ where } \mathtt{N} = \mathtt{u}'\ \mathtt{C}\langle\overline{\mathtt{T}}\rangle \end{array}$$

The adapted main modifier is determined by $\mathtt{u} \triangleright \mathtt{u}'$. However, we have to avoid the unsafe (covariance-like) viewpoint adaptations described in Sec. 2. They are prevented by setting the main modifier of $\mathtt{u} \triangleright \mathtt{N}$ to `any` if the adaptation promotes an ownership modifier in a type argument of `N` from `rep` or `peer` to `any`. Such an unsafe adaptation occurs only in the following situation: `u` is `rep` or `peer`, $\mathtt{u}'$ is `peer`, and at least one of `N`'s type arguments contains the modifier `rep`. In all other cases, $\mathtt{u} \triangleright \mathtt{u}'$ yields `any` anyway or no type argument of `N` is promoted to `any`. This leads to the following definition:

$$\begin{array}{rcl} \cdot \triangleright_m \cdot \;::\; \mathtt{OM} \times {}^{s}\mathtt{NType} & \rightarrow & \mathtt{OM} \\ \mathtt{u} \triangleright_m \mathtt{u}'\ \mathtt{C}\langle\overline{\mathtt{T}}\rangle & = & \begin{cases} \mathtt{any} & \text{if } (\mathtt{u} = \mathtt{rep} \vee \mathtt{u} = \mathtt{peer}) \wedge \mathtt{u}' = \mathtt{peer} \wedge \mathtt{rep} \in \overline{\mathtt{T}} \\ \mathtt{u} \triangleright \mathtt{u}' & \text{otherwise} \end{cases} \end{array}$$

The notation $\mathtt{u} \in \mathtt{T}$ expresses that type `T` or its (transitive) type arguments contain ownership modifier `u`. It is convenient to define this operator on sequences of types:

$$\begin{array}{rcl} \cdot \in \cdot \;::\; \mathtt{OM} \times \overline{{}^{s}\mathtt{Type}} & \rightarrow & \mathit{bool} \\ \mathtt{u} \in \mathtt{X} & = & \mathit{false} \\ \mathtt{u} \in \mathtt{u}'\ \mathtt{C}\langle\overline{\mathtt{T}}\rangle & = & \mathtt{u} = \mathtt{u}' \vee \mathtt{u} \in \overline{\mathtt{T}} \\ \mathtt{u} \in \overline{\mathtt{T}} & = & \exists i : \mathtt{u} \in \mathtt{T}_i \end{array}$$

Adapting a Type w.r.t. a Type. Now we are equipped to adapt a type `T` from a viewpoint that is described by another type, $\mathtt{u}\ \mathtt{C}\langle\overline{\mathtt{T}}\rangle$:

$$\begin{array}{rcl} \cdot \triangleright \cdot \;::\; {}^{s}\mathtt{NType} \times {}^{s}\mathtt{Type} & \rightarrow & {}^{s}\mathtt{Type} \\ \mathtt{u}\ \mathtt{C}\langle\overline{\mathtt{T}}\rangle \triangleright \mathtt{T} & = & (\mathtt{u} \triangleright \mathtt{T})[\overline{\mathtt{T}/\mathtt{X}}] \qquad \text{where } \overline{\mathtt{X}} = \mathrm{dom}(\mathtt{C}). \end{array}$$

The adaptation operator $\triangleright$ adapts the ownership modifiers of `T` and then substitutes the type arguments $\overline{\mathtt{T}}$ for the type variables $\overline{\mathtt{X}}$ of `C`. This substitution is denoted by $[\overline{\mathtt{T}/\mathtt{X}}]$. Since the type arguments already are relative to `this`, they are not subject to viewpoint adaptation. Therefore, the substitution of type variables happens after the viewpoint adaptation of `T`'s ownership modifiers. For a declaration $\mathtt{class}\ \mathtt{C}\langle\overline{\mathtt{X}\ \_}\rangle\ \ldots$, dom(`C`) denotes `C`'s type variables $\overline{\mathtt{X}}$. We use the character _ as wildcard, here, for the upper bounds of `X`.

Note that the first parameter is a non-variable type, because concrete ownership information `u` is needed to adapt the viewpoint and the actual type arguments $\overline{\mathtt{T}}$ are needed to substitute the type variables $\overline{\mathtt{X}}$. In the type rules, subsumption will be used to replace type variables by their upper bounds and thereby obtain a concrete type as first argument of $\triangleright$.

Example. The call `map.iterator()` in method `main` (Fig. 5) illustrates the most interesting viewpoint adaptation, which we discussed in Sec. 2. The type of this call is determined by adapting the return type of `iterator` (`peer Iter<rep Node<K,V>>`) from the type of the receiver expression (`rep Map<rep ID,any Data>`). According to the above definition, we first apply viewpoint adaption to the main modifier of the receiver type, `rep`, and the return type, and then substitute type variables.

The type arguments of the adapted type are obtained by applying viewpoint adaptation recursively to the type argument: `rep` $\triangleright$ `rep Node<K,V>`. This yields `any Node<K,V>` because `rep` $\triangleright$ `rep` $=$ `any` and because the type variables `K` and `V` are not subject to viewpoint adaptation. Note that here, an ownership modifier of a type argument is promoted from `rep` to `any`. Therefore, to avoid unsafe covariance-like adaptations, the main modifier of the adapted type must be `any`. This is actually the case because the main modifier is determined by `rep` $\triangleright_m$ `peer Iter<rep Node<K,V>>`, which yields `any`.

So far, the adaptation yields `any Iter<any Node<K,V>>`. Now we have to substitute the type variables `K` and `V` by the instantiations given in the receiver type, `rep ID` and `any Data`, to obtain the type of the call: `any Iter<any Node<rep ID,any Data>>`.

3.3 Subclassing and Subtyping

We use the term *subclassing* to refer to the relation on classes as declared in a program by the `extends` keyword, irrespective of ownership modifiers. *Subtyping* takes ownership modifiers into account.

Subclassing. The subclass relation $\sqsubseteq$ is defined on instantiated classes, which are denoted by $\mathtt{C}\langle\overline{\mathtt{T}}\rangle$. The subclass relation is the smallest relation satisfying the rules in Fig. 8. Each un-instantiated class is a subclass of the class it extends (SC-1). An antecedent of the form $\mathtt{class}\ \mathtt{C}\langle\overline{\mathtt{X}\ \mathtt{N}}\rangle\ \mathtt{extends}\ \mathtt{C}'\langle\overline{\mathtt{T}'}\rangle\ \{\ \overline{\mathtt{f}\ \mathtt{T};}\ \overline{\mathtt{m}}\ \}$ or a prefix thereof expresses that the program contains such a class declaration. Subclassing is reflexive (SC-2) and transitive (SC-3). Subclassing is preserved by substitution of type arguments for type variables (SC-4). Note that such substitutions may lead to ill-formed types, for instance when the upper bound of a substituted type variable is not respected. We prevent such types by well-formedness rules, which are presented below.

We illustrate subclassing by the classes `Link` and `Node` in Fig. 3. By rule SC-1, we obtain `Node<K,V>` $\sqsubseteq$ `Link<peer Node<K,V>>` from the `extends` clause of `Node`. Rule SC-4 allows us to instantiate the type variables, for instance, with the type arguments used in method `main`: `Node<rep ID,any Data>` $\sqsubseteq$ `Link<peer Node<rep ID,any Data>>`.

Subtyping. The subtype relation $<:$ is defined on types. The judgment $\Gamma \vdash \mathtt{T} <: \mathtt{T}'$ expresses that type `T` is a subtype of type $\mathtt{T}'$ in type environment Γ. The environment is needed since types may contain type variables. The rules for this subtyping judgment are presented in Fig. 9. Two types with the same main modifier are subtypes if the

$$\text{SC-1}\;\frac{\mathtt{class}\ \mathtt{C}{<}\overline{\mathtt{X}\ \_}{>}\ \mathtt{extends}\ \mathtt{C}'{<}\overline{\mathtt{T}'}{>}}{\mathtt{C}{<}\overline{\mathtt{X}}{>} \sqsubseteq \mathtt{C}'{<}\overline{\mathtt{T}'}{>}} \qquad \text{SC-2}\;\frac{}{\mathtt{C}{<}\overline{\mathtt{T}}{>} \sqsubseteq \mathtt{C}{<}\overline{\mathtt{T}}{>}}$$

$$\text{SC-3}\;\frac{\begin{array}{c}\mathtt{C}{<}\overline{\mathtt{T}}{>} \sqsubseteq \mathtt{C}''{<}\overline{\mathtt{T}''}{>}\\ \mathtt{C}''{<}\overline{\mathtt{T}''}{>} \sqsubseteq \mathtt{C}'{<}\overline{\mathtt{T}'}{>}\end{array}}{\mathtt{C}{<}\overline{\mathtt{T}}{>} \sqsubseteq \mathtt{C}'{<}\overline{\mathtt{T}'}{>}} \qquad \text{SC-4}\;\frac{\mathtt{C}{<}\overline{\mathtt{T}}{>} \sqsubseteq \mathtt{C}'{<}\overline{\mathtt{T}'}{>}}{\mathtt{C}{<}\overline{\mathtt{T}}[\overline{\mathtt{T}''/\mathtt{X}''}]{>} \sqsubseteq \mathtt{C}'{<}\overline{\mathtt{T}'}[\overline{\mathtt{T}''/\mathtt{X}''}]{>}}$$

Figure 8:

$$\text{ST-1}\;\frac{\mathtt{C}{<}\overline{\mathtt{T}}{>} \sqsubseteq \mathtt{C}'{<}\overline{\mathtt{T}'}{>}}{\Gamma \vdash \mathtt{u}\ \mathtt{C}{<}\overline{\mathtt{T}}{>} <: \mathtt{u} \rhd (\mathtt{peer}\ \mathtt{C}'{<}\overline{\mathtt{T}'}{>})} \qquad \text{ST-2}\;\frac{}{\Gamma \vdash \mathtt{this}_u\ \mathtt{C}{<}\overline{\mathtt{T}}{>} <: \mathtt{peer}\ \mathtt{C}{<}\overline{\mathtt{T}}{>}}$$

$$\text{ST-3}\;\frac{\begin{array}{c}\Gamma \vdash \mathtt{T} <: \mathtt{T}''\\ \Gamma \vdash \mathtt{T}'' <: \mathtt{T}'\end{array}}{\Gamma \vdash \mathtt{T} <: \mathtt{T}'} \qquad \text{ST-4}\;\frac{}{\Gamma \vdash \mathtt{X} <: \Gamma(\mathtt{X})} \qquad \text{ST-5}\;\frac{\mathtt{T} <:_{\mathtt{a}} \mathtt{T}'}{\Gamma \vdash \mathtt{T} <: \mathtt{T}'}$$

$$\text{TA-1}\;\frac{}{\mathtt{T} <:_{\mathtt{a}} \mathtt{T}} \qquad \text{TA-2}\;\frac{\overline{\mathtt{T} <:_{\mathtt{a}} \mathtt{T}'}}{\mathtt{u}\ \mathtt{C}{<}\overline{\mathtt{T}}{>} <:_{\mathtt{a}} \mathtt{any}\ \mathtt{C}{<}\overline{\mathtt{T}'}{>}}$$

Figure 9: Rules for subtyping and limited covariance.

corresponding classes are subclasses (ST-1). Ownership modifiers in the `extends` clause ($\overline{\mathtt{T}'}$) are relative to the instance of class `C`, whereas the modifiers in a type are relative to `this`. Therefore, $\overline{\mathtt{T}'}$ has to be adapted from the viewpoint of the `C` instance to `this`. Since both $\mathtt{this}_u$ and `peer` express that an object has the same owner as `this`, a type with main modifier $\mathtt{this}_u$ is a subtype of the corresponding type with main modifier `peer` (ST-2). This rule allows us to treat `this` as an object of a `peer` type. Subtyping is transitive (ST-3). A type variable is a subtype of its upper bound in the type environment (ST-4). Two types are subtypes, if they obey the limited covariance described in Sec. 2 (ST-5). Covariant subtyping is expressed by the relation $<:_{\mathtt{a}}$. Covariant subtyping is reflexive (TA-1). A supertype may have more general type arguments than the subtype if the main modifier of the supertype is `any` (TA-2). Note that the sequences $\overline{\mathtt{T}}$ and $\overline{\mathtt{T}'}$ in rule TA-2 can be empty, which allows one to derive, for instance, `peer Object` $<:_{\mathtt{a}}$ `any Object`. Reflexivity of $<:$ follows from TA-1 and ST-5.

In our example, we can use TA-1 twice (for `K` and `V`) and TA-2 to obtain `rep Node<K,V>` $<:_{\mathtt{a}}$ `any Node<K,V>`. Using this result, TA-2, and ST-5, we derive `peer Iter<rep Node<K,V>>` $<:$ `any Iter<any Node<K,V>>`, which is an example for limited covariance. Note that our rules do not allow us to derive `peer Iter<rep Node<K,V>>` $<:$ `peer Iter<any Node<K,V>>`, which would be unsafe covariant subtyping as discussed in Sec. 2.

3.4 Lookup Functions

In this subsection, we define the functions to look up the type of a field or the signature of a method.

Field Lookup. The function ${}^{s}\!fType(\mathtt{C},\mathtt{f})$ yields the type of field `f` as declared in class `C`. The result is undefined if `f` is not declared in `C`. Since identifiers are assumed to be globally unique, there is only one declaration for each field identifier.

$$\text{SFT}\frac{\mathtt{class\ C<\_>\ extends\ \_<\_>}\ \{\ \ldots\mathtt{T\ f}\ldots;\ \_\ \}}{{}^{s}\!fType(\mathtt{C},\mathtt{f}) = \mathtt{T}}$$

The function $fields(\mathtt{C})$ yields the identifiers of all fields that are declared in or inherited by class `C`.

$$\text{SF-1}\frac{}{fields(\mathtt{Object}) = \epsilon} \qquad \text{SF-2}\frac{\mathtt{class\ C<\_>\ extends\ C'<\_>}\ \{\ \overline{\mathtt{T\ f}};\ \_\ \}}{fields(\mathtt{C}) = \overline{\mathtt{f}} \circ fields(\mathtt{C'})}$$

Method Lookup. The function $mType(\mathtt{C},\mathtt{m})$ yields the signature of method `m` as declared in class `C`. The result is undefined if `m` is not declared in `C`. We do not allow overloading of methods; therefore, the method identifier is sufficient to uniquely identify a method.

$$\text{SMT}\frac{\mathtt{class\ C<\_>\ extends\ \_<\_>}\ \{\ \_;\ \ldots<\overline{\mathtt{X}_m\ \mathtt{N}_b}>\ \mathtt{w}\ \mathtt{T}_r\ \mathtt{m}(\overline{\mathtt{x}\ \mathtt{T}_p})\ldots\ \}}{mType(\mathtt{C},\mathtt{m}) = <\overline{\mathtt{X}_m\ \mathtt{N}_b}>\ \mathtt{w}\ \mathtt{T}_r\ \mathtt{m}(\overline{\mathtt{x}\ \mathtt{T}_p})}$$

3.5 Well-Formedness

In this subsection, we define well-formedness of types, methods, classes, programs, and type environments. The well-formedness rules are summarized in Fig. 10 and explained in the following.

Well-Formed Types. The judgement $\Gamma \vdash \mathtt{T\ ok}$ expresses that type `T` is well-formed in type environment Γ. Type variables are well-formed, if they are contained in the type environment (WFT-1). A non-variable type $\mathtt{u\ C}<\overline{\mathtt{T}}>$ is well-formed if its type arguments $\overline{\mathtt{T}}$ are well-formed and for each type variable the actual type argument is a subtype of the upper bound, adapted from the viewpoint $\mathtt{u\ C}<\overline{\mathtt{T}}>$ (WFT-2). The viewpoint adaptation is necessary because the type arguments describe ownership relative to the `this` object where $\mathtt{u\ C}<\overline{\mathtt{T}}>$ is used, whereas the upper bounds are relative to the object of type $\mathtt{u\ C}<\overline{\mathtt{T}}>$.

An interesting aspect of rule WFT-2 is that it permits type variables of a class `C` to be used in upper bounds of `C`. For instance in class `Iter` (Fig. 4), type variable `X` is used in its own upper bound, `any Link<X>`. To illustrate how rule WFT-2 works, we show that the instantiation of class `Iter` in method `main`, `any Iter<any Node<rep ID,any Data>>` is well-formed.

The types `rep ID` and `any Data` are trivially well-formed. To show that the type argument `any Node<rep ID,any Data>` is well-formed, we have to show that the type arguments are subtypes of the adapted upper bounds (WFT-2). The (un-adapted and adapted) upper bounds are `any Object`, which is a supertype of `rep ID` and `any Data`.

$$\text{WFT-1}\ \frac{\mathtt{X} \in \mathrm{dom}(\Gamma)}{\Gamma \vdash \mathtt{X}\ \ \mathtt{ok}} \qquad \text{WFT-2}\ \frac{\begin{array}{c}\mathtt{class\ C}\texttt{<}\overline{\_\ \mathtt{N}}\texttt{>}\ \ldots \\ \Gamma \vdash \overline{\mathtt{T}}\ \ \mathtt{ok} \qquad \Gamma \vdash \overline{\mathtt{T}} <: ((\mathtt{u\ C}\texttt{<}\overline{\mathtt{T}}\texttt{>}) \rhd \overline{\mathtt{N}})\end{array}}{\Gamma \vdash \mathtt{u\ C}\texttt{<}\overline{\mathtt{T}}\texttt{>}\ \ \mathtt{ok}}$$

$$\text{WFM-1}\ \frac{\begin{array}{c}\Gamma = \overline{\mathtt{X}_m\ \mathtt{N}_b}, \overline{\mathtt{X\ N}}; \mathtt{this}\ (\mathtt{this}_u\ \mathtt{C}\texttt{<}\overline{\mathtt{X}}\texttt{>}), \overline{\mathtt{x\ T}_p} \\ \Gamma \vdash \mathtt{T}_r, \overline{\mathtt{N}_b}, \overline{\mathtt{T}_p}\ \ \mathtt{ok} \qquad \Gamma \vdash \mathtt{e} : \mathtt{T}_r \\ \mathit{override}(\mathtt{C}, \mathtt{m}) \qquad \mathtt{w} = \mathtt{pure}\ \Rightarrow\ \overline{\mathtt{T}_p} = \overline{\mathtt{any} \rhd \mathtt{T}_p}\end{array}}{\texttt{<}\overline{\mathtt{X}_m\ \mathtt{N}_b}\texttt{>}\ \mathtt{w}\ \mathtt{T}_r\ \mathtt{m}(\overline{\mathtt{x\ T}_p})\ \{\ \mathtt{return\ e}\ \}\ \ \mathtt{ok\ in\ C}\texttt{<}\overline{\mathtt{X\ N}}\texttt{>}}$$

$$\text{WFM-2}\ \frac{\begin{array}{c}(\forall \mathtt{class\ C'}\texttt{<}\overline{\mathtt{X'\ N'}}\texttt{>} : \mathtt{C}\texttt{<}\overline{\mathtt{X}}\texttt{>} \sqsubseteq \mathtt{C'}\texttt{<}\overline{\mathtt{T'}}\texttt{>}\ \wedge\ \mathrm{dom}(\mathtt{C}) = \overline{\mathtt{X}} \Rightarrow \\ \mathit{mType}(\mathtt{C'}, \mathtt{m})\ \text{is undefined}\ \vee \mathit{mType}(\mathtt{C}, \mathtt{m}) = \mathit{mType}(\mathtt{C'}, \mathtt{m})[\overline{\mathtt{T'}/\mathtt{X'}}])\end{array}}{\mathit{override}(\mathtt{C}, \mathtt{m})}$$

$$\text{WFC}\ \frac{\begin{array}{c}\overline{\mathtt{X\ N}}; \_ \vdash \overline{\mathtt{N}}, \overline{\mathtt{T}}, (\mathtt{this}_u\ \mathtt{C'}\texttt{<}\overline{\mathtt{T'}}\texttt{>})\ \ \mathtt{ok} \\ \overline{\mathtt{mt}}\ \ \mathtt{ok\ in}\ \ \mathtt{C}\texttt{<}\overline{\mathtt{X\ N}}\texttt{>} \qquad \mathtt{rep} \notin \overline{\mathtt{N}}\end{array}}{\mathtt{class\ C}\texttt{<}\overline{\mathtt{X\ N}}\texttt{>}\ \mathtt{extends\ C'}\texttt{<}\overline{\mathtt{T'}}\texttt{>}\ \{\ \overline{\mathtt{f\ T}};\ \overline{\mathtt{mt}}\ \}\ \ \mathtt{ok}}$$

$$\text{WFP}\ \frac{\begin{array}{c}\overline{\mathtt{Cls}}\ \ \mathtt{ok} \\ \mathtt{class}\ \ \mathtt{C}\texttt{<>} \ldots \in \overline{\mathtt{Cls}} \\ \epsilon;\ \mathtt{this}\ (\mathtt{this}_u\ \mathtt{C}\texttt{<>}) \vdash \mathtt{e} : \mathtt{N}\end{array}}{\overline{\mathtt{Cls}}, \mathtt{C}, \mathtt{e}\ \ \mathtt{ok}} \qquad \text{SWFE}\ \frac{\begin{array}{c}\Gamma = \overline{\mathtt{X\ N}}, \overline{\mathtt{X'\ N'}}\ ;\ \mathtt{this}\ (\mathtt{this}_u\ \mathtt{C}\texttt{<}\overline{\mathtt{X}}\texttt{>}), \overline{\mathtt{x\ T}} \\ \mathtt{class\ C}\texttt{<}\overline{\mathtt{X\ N}}\texttt{>}\ \ \ldots \qquad \Gamma \vdash \overline{\mathtt{N}}, \overline{\mathtt{N'}}, \overline{\mathtt{T}}\ \ \mathtt{ok}\end{array}}{\Gamma\ \ \mathtt{ok}}$$

Figure 10: Well-formedness rules.

Finally, we have to show that `any Node<rep ID, any Data>` is a subtype of the adaptation of the upper bound `any Link<X>`. As illustrated in Sec. 3.2, we first perform the adaptation of ownership modifiers, which yields `any Link<X>`. Then we substitute the type variable `X` by the actual type argument, which yields `any Link<any Node<rep ID, any Data>>`. It is this substitution that makes it possible to use `X` in its own upper bound. It is easy to show that `any Node<rep ID, any Data>` is a subtype of `any Link<peer Node<rep ID, any Data>>` (ST-1, ST-5, see Sec. 3.3), which in turn is a subtype of `any Link<any Node<rep ID,any Data>>` (TA-1, TA-2, ST-6). By transitivity (ST-3), we have the desired subtype relation.

Well-Formed Methods. The judgement $\mathtt{m}\ \ \mathtt{ok\ in\ C}\texttt{<}\overline{\mathtt{X\ N}}\texttt{>}$ expresses that method `m` is well-formed in a class `C` with type parameters $\overline{\mathtt{X\ N}}$. According to rule WFM-1, `m` is well-formed if: (1) the return type, the upper bounds of `m`'s type variables, and the parameter types are well-formed in the type environment that maps `m`'s and `C`'s type variables to their upper bounds as well as `this` and the explicit method parameters to their types. The type of `this` is the enclosing class, $\mathtt{C}\texttt{<}\overline{\mathtt{X}}\texttt{>}$, with main modifier $\mathtt{this}_u$; (2) the method body, expression `e`, is well-typed with `m`'s return type; (3) `m` respects the rules for overriding, see below; (4) if m is pure then the only ownership modifier that occurs in a parameter type is any. We will motivate the last requirement when we explain the type rule for method calls.

Method m respects the rules for overriding if it does not override a method or if all overridden methods have the identical signatures after substituting type variables of the superclasses by the instantiations given in the subclass (WFM-2). For simplicity, we require that overrides do not change the purity of a method, although overriding non-pure methods by pure methods would be safe.

Well-Formed Classes. The judgement Cls ok expresses that class declaration Cls is well-formed. According to rule WFC, this is the case if: (1) the upper bounds of Cls's type variables, the types of Cls's fields, and the instantiation of the superclass are well-formed in the type environment that maps Cls's type variables to their upper bounds; (2) Cls's methods are well-formed; (3) Cls's upper bounds do not contain the rep modifier.

Note that Cls's upper bounds express ownership relative to the current Cls instance. If such an upper bound contains a rep modifier, clients of Cls cannot instantiate Cls because none of the ownership modifiers peer, rep, or any for an actual type argument, which are relative to the client's viewpoint, expresses the required ownership relation. Therefore, we forbid upper bounds with rep modifiers by Requirement (3).

Well-Formed Programs. The judgement P ok expresses that program P is well-formed. According to rule WFP, this is the case if all classes in P are well-formed, the main class C is a non-generic class in P, and the main expression e is well-typed in an environment where this is an instance of C.

Well-Formed Type Environments. The judgement Γ ok expresses that type environment Γ is well-formed. According to rule SWFE, this is the case if all upper bounds of type variables and the types of method parameters are well-formed. Moreover, this must be mapped to a non-variable type with main modifier $\texttt{this}_u$ and an uninstantiated class.

3.6 Type Rules

We are now ready to present the type rules (Fig. 11). The judgement $\Gamma \vdash \texttt{e} : \texttt{T}$ expresses that expression e is well-typed with type T in environment Γ. Our type rules implicitly require types to be well-formed, that is, a type rule is applicable only if all types involved in the rule are well-formed in the respective environment. We also omit checks for valid appearances of the ownership modifier $\texttt{this}_u$. As explained earlier, $\texttt{this}_u$ must not occur in the program and is only used as main modifier of the type of this.

An expression of type T can also be typed with T's supertypes (GT-Subs). The type of method parameters (including this) is determined by a lookup in the type environment (GT-Var). The null-reference can have any type (GT-Null). The rules for object creation (GT-New) and cast (GT-Cast) are straightforward. Note that in combination with subsumption, GT-Cast allows up and down casts. The rule could be strengthened to prevent more cast errors statically, but we omit this check since it is not strictly needed.

As explained in detail in Sec. 3.2, the type of a field access is determined by adapting the declared type of the field from the viewpoint described by the type of the receiver

$$\text{GT-Subs}\ \frac{\Gamma \vdash \texttt{e} : \texttt{T} \qquad \Gamma \vdash \texttt{T} <: \texttt{T}'}{\Gamma \vdash \texttt{e} : \texttt{T}'} \qquad \text{GT-Var}\ \frac{\texttt{x} \in \mathrm{dom}(\Gamma)}{\Gamma \vdash \texttt{x} : \Gamma(\texttt{x})} \qquad \text{GT-Null}\ \frac{}{\Gamma \vdash \texttt{null} : \texttt{T}}$$

$$\text{GT-New}\ \frac{}{\Gamma \vdash \texttt{new T} : \texttt{T}} \qquad \text{GT-Cast}\ \frac{\Gamma \vdash \texttt{e}_0 : \texttt{T}_0}{\Gamma \vdash (\texttt{T})\ \texttt{e}_0 : \texttt{T}}$$

$$\text{GT-Read}\ \frac{\Gamma \vdash \texttt{e}_0 : \texttt{N}_0 \qquad \texttt{N}_0 = \_\ \texttt{C}_0\texttt{<}\_\texttt{>}}{\Gamma \vdash \texttt{e}_0.\texttt{f} : \texttt{N}_0 \rhd \mathit{fType}(\texttt{C}_0, \texttt{f})} \qquad \text{GT-Write}\ \frac{\begin{array}{c}\Gamma \vdash \texttt{e}_0 : \texttt{N}_0 \qquad \texttt{N}_0 = \texttt{u}_0\ \texttt{C}_0\texttt{<}\_\texttt{>} \\ \texttt{T}_1 = \mathit{fType}(\texttt{C}_0, \texttt{f}) \\ \Gamma \vdash \texttt{e}_2 : \texttt{N}_0 \rhd \texttt{T}_1 \\ \texttt{u}_0 \neq \texttt{any} \qquad \mathit{rp}(\texttt{u}_0, \texttt{T}_1)\end{array}}{\Gamma \vdash \texttt{e}_0.\texttt{f}{=}\texttt{e}_2 : \texttt{N}_0 \rhd \texttt{T}_1}$$

$$\text{GT-Invk}\ \frac{\begin{array}{c}\Gamma \vdash \texttt{e}_0 : \texttt{N}_0 \qquad \texttt{N}_0 = \texttt{u}_0\ \texttt{C}_0\texttt{<}\_\texttt{>} \\ \mathit{mType}(\texttt{C}_0, \texttt{m}) = \texttt{<}\overline{\texttt{X}_m\ \texttt{N}_b}\texttt{>}\ \texttt{w}\ \texttt{T}_r\ \texttt{m}(\overline{\texttt{x}\ \texttt{T}_p}) \\ \Gamma \vdash \overline{\texttt{T}} <: (\texttt{N}_0 \rhd \overline{\texttt{N}_b})[\overline{\texttt{T}/\texttt{X}_m}] \qquad \Gamma \vdash \overline{\texttt{e}_2} : (\texttt{N}_0 \rhd \overline{\texttt{T}_p})[\overline{\texttt{T}/\texttt{X}_m}] \\ (\texttt{u}_0 {=} \texttt{any} \Rightarrow \texttt{w} {=} \texttt{pure}) \qquad \mathit{rp}(\texttt{u}_0, \overline{\texttt{T}_p} \circ \overline{\texttt{N}_b})\end{array}}{\Gamma \vdash \texttt{e}_0.\texttt{m}\texttt{<}\overline{\texttt{T}}\texttt{>}(\overline{\texttt{e}_2}) : (\texttt{N}_0 \rhd \texttt{T}_r)[\overline{\texttt{T}/\texttt{X}_m}]}$$

Figure 11: Type rules.

(GT-Read). If this type is a type variable, subsumption is used to go to its upper bound because *fType* is defined on class identifiers. Subsumption is also used for inherited fields to ensure that `f` is actually declared in $\texttt{C}_0$. (Recall that $\mathit{fType}(\texttt{C}_0, \texttt{f})$ is undefined otherwise.)

For a field update, the right-hand side expression must be typable as the viewpoint adapted field type, which is also the type of the whole field update expression (GT-Write). The rule is analogous to field read, but has two additional requirements. First, the main modifier $\texttt{u}_0$ of the type of the receiver expression must not be `any`. With the owner-as-modifier discipline, a method must not update fields of objects in arbitrary contexts. Second, the requirement $\mathit{rp}(\texttt{u}_0, {}^{s}\texttt{T}_1)$ enforces that `f` is updated through receiver `this` if its declared type contains a `rep` modifier. In that case, the viewpoint adaptation $\texttt{N}_0 \rhd {}^{s}\texttt{T}_1$ yields an `any` type, but it is obviously unsafe to update `f` with an object with an arbitrary owner. It is convenient to define *rp* for sequences of types. The definition uses the fact that the ownership modifier $\texttt{this}_u$ is only used for the type of `this`:

$$\begin{array}{rcl}\mathit{rp} :: \texttt{OM} \times \overline{{}^{s}\texttt{Type}} & \rightarrow & \mathit{bool} \\ \mathit{rp}(\texttt{u}, \overline{\texttt{T}}) & = & \texttt{u} = \texttt{this}_u \vee (\forall i : \texttt{rep} \notin \texttt{T}_i)\end{array}$$

The rule for method calls (GT-Invk) is in many ways similar to field reads (for result passing) and updates (for argument passing). The method signature is determined using the receiver type $\texttt{N}_0$ and subsumption. The type of the invocation expression is determined by viewpoint adaptation of the return type $\texttt{T}_r$ from the receiver type $\texttt{N}_0$. Modulo subsumption, the actual method parameters must have the formal parameter types, adapted from $\texttt{N}_0$ and with actual type arguments $\overline{\texttt{T}}$ substituted for the method's type variables $\texttt{X}_m$. For instance, in the call first.init(key, value, first) in method put (Fig. 2), the adapted third formal parameter type is rep Node<K,V> $\triangleright$ peer Node<K,V>. This adaptation yields rep Node<K,V>, which is also the type of the third actual method argument.

To enforce the owner-as-modifier discipline, only pure methods may be called on receivers with main modifier any. This requirement prevents method main (Fig. 5) from calling iter.next() as discussed in Sec. 2. For a call on a receiver with main modifier any, the viewpoint-adapted formal parameter type contains only the modifier any. Consequently, arguments with arbitrary owners can be passed. For this to be type safe, pure methods must not expect arguments with specific owners. This is enforced by rule WFM-1 (Fig. 10). Finally, if the receiver is different from this, then neither the formal parameter types nor the upper bounds of the method's type variables must contain rep.

4 Runtime Model

In this section, we explain the runtime model of Generic Universe Types. We present the heap model, the runtime type information, well-formedness conditions, and an operational semantics. The runtime model is used to prove type safety in the next section.

4.1 Heap Model

Fig. 12 summarizes our heap model. To distinguish sorts of the runtime model from their static counterparts, we use the prefix $^{\texttt{r}}$.

$$
\begin{array}{rcll}
\texttt{h} & \in & \texttt{Heap} & = \texttt{Addr} \rightarrow \texttt{Obj} \\
\iota & \in & \texttt{Addr} & = \text{Set of Addresses} \cup \{\texttt{null}_a\} \\
\texttt{o} & \in & \texttt{Obj} & = {}^{\texttt{r}}\texttt{Type, Fields} \\
{}^{\texttt{r}}\texttt{T} & \in & {}^{\texttt{r}}\texttt{Type} & = \texttt{OwnerAddr ClassId}<\overline{{}^{\texttt{r}}\texttt{Type}}> \\
\texttt{Fs} & \in & \texttt{Fields} & = \texttt{FieldId} \rightarrow \texttt{Addr} \\
\iota & \in & \texttt{OwnerAddr} & = \texttt{Addr} \cup \{\texttt{any}_a\} \\
{}^{\texttt{r}}\Gamma & \in & {}^{\texttt{r}}\texttt{Env} & = \overline{\texttt{TVarId}\ {}^{\texttt{r}}\texttt{Type}};\ \overline{\texttt{ParId Addr}}
\end{array}
$$

Figure 12: Definitions for the heap model.

A heap (sort Heap) maps addresses to objects. The set of addresses (Addr) contains the special null-reference $\texttt{null}_a$. An object (Obj) consist of its runtime type and a mapping from field identifiers to the addresses stored in the fields.

The runtime type (${}^{\texttt{r}}\texttt{Type}$) of an object o consists of the address of o's owner object, of o's class, and of runtime types for the type arguments of this class. We store the runtime type arguments including the associated ownership information explicitly in the heap because this information is needed in the runtime checks for casts. In that respect, our runtime model is similar to that of the .NET CLR [18]. The owner address of objects in the root context is $\texttt{null}_a$. The special owner address $\texttt{any}_a$ is used when the corresponding static type has the $\texttt{any}_u$ modifier. Consider for instance an execution of method main

(Fig. 5), where the address of `this` is ι. The runtime type of the object stored in `map` is $\iota\ \texttt{Map<}\iota\ \texttt{ID}, \texttt{any}_{\texttt{a}}\ \texttt{Data>}$.

The first component of a runtime environment ($^{\texttt{r}}\texttt{Env}$) maps method type variables to their runtime types. The second component is the stack, which maps method parameters to the addresses they store.

Operations on Heaps and Objects. Updating a field `f` of the object at address ι in heap `h` with an address ι' is denoted by $\texttt{h}[\iota.\texttt{f} := \iota']$. $\mathit{owner}(\texttt{h}, \iota)$ yields the owner address of the object at address ι in heap `h`, whereas $\mathit{owners}(\texttt{h}, \iota)$ yields the set of all transitive owners of that object.

$$
\begin{array}{lcl}
\cdot[\cdot.\cdot := \cdot] \; :: \; \texttt{Heap} \times \texttt{Addr} \times \texttt{FieldId} \times \texttt{Addr} \rightarrow \texttt{Heap} & & \\
\quad \texttt{h}[\iota.\texttt{f} := \iota'] & = & \texttt{h}[\iota \mapsto (\texttt{h}(\iota)\downarrow_1, \texttt{h}(\iota)\downarrow_2 [\texttt{f} \mapsto \iota'])] \\
 & & \\
\mathit{owner} :: \texttt{Heap} \times \texttt{Addr} \rightarrow \texttt{OwnerAddr} & & \\
\quad \mathit{owner}(\texttt{h}, \iota) & = & \texttt{h}(\iota)\downarrow_1\downarrow_1 \\
 & & \\
\mathit{owners} :: \texttt{Heap} \times \texttt{Addr} \rightarrow \mathcal{P}(\texttt{OwnerAddr}) & & \\
\quad \mathit{owner}(\texttt{h}, \iota) & \in & \mathit{owners}(\texttt{h}, \iota) \\
\quad \iota' \in \mathit{owners}(\texttt{h}, \iota) \wedge \iota' \notin \{\texttt{any}_a, \texttt{null}_a\} & \Rightarrow & \mathit{owner}(\texttt{h}, \iota') \in \mathit{owners}(\texttt{h}, \iota)
\end{array}
$$

We use projection $\downarrow_i$ to select the i-th component of a tuple, for instance, the runtime type and field mapping of an object.

Subtyping on Runtime Types. Judgment $\iota \vdash {}^{\texttt{r}}\texttt{T}\ {}^{\texttt{r}}\!<:\ {}^{\texttt{r}}\texttt{T}'$ expresses that the runtime type $^{\texttt{r}}\texttt{T}$ is a subtype of $^{\texttt{r}}\texttt{T}'$ from the viewpoint of address ι. The viewpoint, ι, is required in order to give meaning to the ownership modifier `rep`.

Subtyping for runtime types is defined in Fig. 13. According to RT-1, subtyping follows subclassing if (1) the runtime types have the same owner address, (2) in the type arguments, the ownership modifiers $\texttt{this}_u$ and `peer` are substituted by the owner address ι' of the runtime types (we use the same owner address for both modifiers since they both express ownership by the owner of `this`), (3) `rep` is substituted by the viewpoint address ι, (4) $\texttt{any}_u$ is substituted by $\texttt{any}_a$, and (5) the type variables $\overline{\texttt{X}}$ of the subclass `C` are substituted consistently by $\overline{^{\texttt{r}}\texttt{T}}$. Note that in a well-formed program, $\texttt{this}_u$ never occurs in a type argument; nevertheless we include the substitution for consistency. According to RT-2, subtyping is transitive.

Like for subtyping on static types, we have limited covariance for runtime types. Covariant subtyping is expressed by the relation ${}^{\texttt{r}}\!<:_{\texttt{a}}$. Two runtime types are subtypes if they are covariant subtypes (RT-3). The rules for limited covariance, RTA-1 and RTA-2, are analogous to the rules TA-1 and TA-2 for static types (Fig. 9). Reflexivity of ${}^{\texttt{r}}\!<:$ follows from RTA-1 and RT-3.

The judgement $\texttt{h} \vdash \iota : {}^{\texttt{r}}\texttt{T}'$ expresses that in heap `h`, the address ι has type $^{\texttt{r}}\texttt{T}'$. The type of ι is determined by the type of the object at ι and the subtype relation (RT-2). The `null` reference can have any type (RT-3).

$$\text{RT-1}\ \frac{\mathtt{C}<\overline{\mathtt{X}}> \sqsubseteq \mathtt{C}'<\overline{{}^{s}\mathtt{T}}> \qquad \mathrm{dom}(\mathtt{C}) = \overline{\mathtt{X}}}{\iota \vdash \iota'\ \mathtt{C}<\overline{{}^{r}\mathtt{T}}>\ {}^{r}\!<:\ \iota'\ \mathtt{C}'<\overline{{}^{s}\mathtt{T}[\iota'/\mathtt{this}_u, \iota'/\mathtt{peer}, \iota/\mathtt{rep}, \mathtt{any}_a/\mathtt{any}_u, \overline{{}^{r}\mathtt{T}/\mathtt{X}}]}>}$$

$$\text{RT-2}\ \frac{\begin{array}{c}\iota \vdash {}^{r}\mathtt{T}\ {}^{r}\!<:\ {}^{r}\mathtt{T}'' \\ \iota \vdash {}^{r}\mathtt{T}''\ {}^{r}\!<:\ {}^{r}\mathtt{T}'\end{array}}{\iota \vdash {}^{r}\mathtt{T}\ {}^{r}\!<:\ {}^{r}\mathtt{T}'} \qquad \text{RT-3}\ \frac{{}^{r}\mathtt{T}\ {}^{r}\!<:_{\mathtt{a}}\ {}^{r}\mathtt{T}'}{\iota \vdash {}^{r}\mathtt{T}\ {}^{r}\!<:\ {}^{r}\mathtt{T}'}$$

$$\text{RTA-1}\ \frac{}{{}^{r}\mathtt{T}\ {}^{r}\!<:_{\mathtt{a}}\ {}^{r}\mathtt{T}} \qquad \text{RTA-2}\ \frac{\overline{{}^{r}\mathtt{T}\ {}^{r}\!<:_{\mathtt{a}}\ {}^{r}\mathtt{T}'}}{\iota'\ \mathtt{C}<\overline{{}^{r}\mathtt{T}}>\ {}^{r}\!<:_{\mathtt{a}}\ \mathtt{any}_a\ \mathtt{C}<\overline{{}^{r}\mathtt{T}'}>}$$

$$\text{RT-4}\ \frac{\begin{array}{c}h(\iota) = {}^{r}\mathtt{T}, \_ \\ \iota \vdash {}^{r}\mathtt{T}\ {}^{r}\!<:\ {}^{r}\mathtt{T}'\end{array}}{\mathtt{h} \vdash \iota : {}^{r}\mathtt{T}'} \qquad \text{RT-5}\ \frac{}{\mathtt{h} \vdash \mathtt{null}_a : {}^{r}\mathtt{T}'}$$

Figure 13: Rules for subtyping on runtime types.

In Sec. 3.3, we have derived `Node<K,V>` $\sqsubseteq$ `Link<peer Node<K,V>>`. By rule RT-1, we get for instance: $\iota \vdash \iota'\ \mathtt{Node}<{}^{r}\mathtt{T}_1, {}^{r}\mathtt{T}_2>\ {}^{r}\!<:\ \iota'\ \mathtt{Link}<\iota'\ \mathtt{Node}<{}^{r}\mathtt{T}_1, {}^{r}\mathtt{T}_2>>$.

From Static Types to Runtime Types. Static types and runtime types are related by the *dynamization function dyn*. This function maps a static type ${}^{s}\mathtt{T}$ to the corresponding runtime type. The viewpoint is described by an address ι and a runtime type ${}^{r}\mathtt{T}$ (usually the address and runtime type of the `this` object). In ${}^{s}\mathtt{T}$, *dyn* substitutes `rep` by ι, `peer` and $\mathtt{this}_u$ by the owner in ${}^{r}\mathtt{T}$, ι', and $\mathtt{any}_u$ by $\mathtt{any}_a$. It also substitutes all type variables in ${}^{s}\mathtt{T}$ by the instantiations given in $\iota'\ \mathtt{C}<\overline{{}^{r}\mathtt{T}}>$, a supertype of ι's runtime type, or in the runtime environment. The substitutions performed by *dyn* are analogous to the ones in rule RT-1 (Fig. 13), which also involves mapping static types to runtime types. We do not use *dyn* in RT-1 to avoid that the definitions of ${}^{r}\!<:$ and *dyn* are mutually recursive.

$$dyn :: {}^{s}\mathtt{Type} \times \mathtt{Addr} \times {}^{r}\mathtt{Type} \times {}^{r}\mathtt{Env} \rightarrow {}^{r}\mathtt{Type}$$

$$\text{DYN}\ \frac{\begin{array}{ccc}{}^{r}\mathtt{T} = \iota'\ \_<\_> & \iota \vdash {}^{r}\mathtt{T}\ {}^{r}\!<:\ \iota'\ \mathtt{C}<\overline{{}^{r}\mathtt{T}}> & \iota \vdash {}^{r}\mathtt{T}\ {}^{r}\!<:\ \iota'\ \mathtt{C}<\overline{{}^{r}\mathtt{T}_a}> \Rightarrow \iota \vdash \overline{{}^{r}\mathtt{T}}\ {}^{r}\!<:\ \overline{{}^{r}\mathtt{T}_a} \\ \multicolumn{2}{c}{\mathrm{dom}(\mathtt{C}) = \overline{\mathtt{X}}} & \mathrm{free}({}^{s}\mathtt{T}) \subseteq \overline{\mathtt{X}} \circ \overline{\mathtt{X}'}\end{array}}{dyn({}^{s}\mathtt{T}, \iota, {}^{r}\mathtt{T}, (\overline{\mathtt{X}'\ {}^{r}\mathtt{T}'}; \_)) = {}^{s}\mathtt{T}[\iota'/\mathtt{this}, \iota'/\mathtt{peer}, \iota/\mathtt{rep}, \mathtt{any}_a/\mathtt{any}_u, \overline{{}^{r}\mathtt{T}/\mathtt{X}}, \overline{{}^{r}\mathtt{T}'/\mathtt{X}'}]}$$

Note that the outcome of *dyn* depends on finding $\iota'\ \mathtt{C}<\overline{{}^{r}\mathtt{T}}>$, an appropriate supertype of ${}^{r}\mathtt{T}$, which contains substitutions for all type variables not mapped by the environment (free(${}^{s}\mathtt{T}$) yields the free type variables in ${}^{s}\mathtt{T}$). Thus, one may wonder whether there is more than one such appropriate superclass. However, because type variables are globally unique, if the free variables of ${}^{s}\mathtt{T}$ are in the domain of a class then they are not in the domain of any other class. In addition, we want the most concrete runtime type arguments. To ensure this, we require that all other supertypes of ${}^{r}\mathtt{T}$ that have the same class `C` have equal or less specific type arguments. This needs to be considered for the case that $\iota' = \texttt{any}_a$ when the subtyping rule TA-2 can be used to also change type arguments. Together this ensures that $\iota'\ \texttt{C}\texttt{<}\overline{^{\mathtt{r}}\mathtt{T}}\texttt{>}$ is unique.

To illustrate how the dynamization works, consider an execution of method `put` (Fig. 2), whose `this` object has address ι and runtime type $\iota'\ \texttt{Map}\texttt{<}^{\mathtt{r}}\mathtt{T}_1, {}^{\mathtt{r}}\mathtt{T}_2\texttt{>}$. Now we determine the runtime type of the object created by `new rep Node<K,V>`. The dynamization of the type of the new object w.r.t. `this` is: $\mathit{dyn}(\texttt{rep Node<K,V>}, \iota, \iota'\ \texttt{Map}\texttt{<}^{\mathtt{r}}\mathtt{T}_1, {}^{\mathtt{r}}\mathtt{T}_2\texttt{>}, \_)$, which yields $\iota\ \texttt{Node}\texttt{<}^{\mathtt{r}}\mathtt{T}_1, {}^{\mathtt{r}}\mathtt{T}_2\texttt{>}$. This runtime type correctly reflects that the new object is owned by `this` (owner address ι) and has the same type arguments as the runtime type of `this`.

It is convenient to define the following three overloaded versions of *dyn*:

$$
\begin{array}{lcl}
\mathit{dyn}({}^{\mathtt{s}}\mathtt{T}, \iota, {}^{\mathtt{r}}\mathtt{T}) & = & \mathit{dyn}({}^{\mathtt{s}}\mathtt{T}, \iota, {}^{\mathtt{r}}\mathtt{T}, \epsilon) \\
\mathit{dyn}({}^{\mathtt{s}}\mathtt{T}, \iota, \mathtt{h}, {}^{\mathtt{r}}\Gamma) & = & \mathit{dyn}({}^{\mathtt{s}}\mathtt{T}, \iota, \mathtt{h}(\iota)\downarrow_1, {}^{\mathtt{r}}\Gamma) \\
\mathit{dyn}({}^{\mathtt{s}}\mathtt{T}, \mathtt{h}, {}^{\mathtt{r}}\Gamma) & = & \mathit{dyn}({}^{\mathtt{s}}\mathtt{T}, {}^{\mathtt{r}}\Gamma(\mathtt{this}), \mathtt{h}, {}^{\mathtt{r}}\Gamma)
\end{array}
$$

4.2 Lookup Functions

In this subsection, we define the functions to look up the runtime type of a field or the body of a method.

Field Lookup. The runtime type of a field `f` is essentially the dynamization of its static type. The function ${}^{\mathtt{r}}\mathit{fType}(\iota, {}^{\mathtt{r}}\mathtt{T}, \mathtt{f})$ yields the runtime type of `f` in an object at address ι with runtime type ${}^{\mathtt{r}}\mathtt{T}$. In the following definition, $\mathtt{C}'$ is the superclass of the class `C` of ${}^{\mathtt{r}}\mathtt{T}$, in which `f` is actually defined.

$$
\text{RFT}\ \frac{{}^{\mathtt{r}}\mathtt{T} = \_\ \mathtt{C}\texttt{<}\_\texttt{>} \qquad \mathtt{C}\texttt{<}\_\texttt{>} \sqsubseteq \mathtt{C}'\texttt{<}\_\texttt{>}}{{}^{\mathtt{r}}\mathit{fType}(\iota, {}^{\mathtt{r}}\mathtt{T}, \mathtt{f}) = \mathit{dyn}({}^{\mathtt{s}}\mathit{fType}(\mathtt{C}', \mathtt{f}), \iota, {}^{\mathtt{r}}\mathtt{T})}
$$

Method Lookup. The function $\mathit{mBody}(\mathtt{C}, \mathtt{m})$ yields a tuple consisting of `m`'s body expression as well as the identifiers of its formal parameters and type variables. This is trivial if `m` is declared in `C` (RMT-1). Otherwise, `m` is looked up in `C`'s superclass $\mathtt{C}'$ (RMT-2).

$$
\text{RMT-1}\ \frac{\texttt{class C<\_> extends \_<\_>}\ \{\ \_\ \_;\ \ldots \texttt{<}\overline{\mathtt{X}\ \_}\texttt{>}\ \_\ \_\ \mathtt{m}(\overline{\mathtt{x}\ \_})\ \{\ \texttt{return e}\ \}\ldots\ \}}{\mathit{mBody}(\mathtt{C}, \mathtt{m}) = (\mathtt{e}, \overline{\mathtt{x}}, \overline{\mathtt{X}})}
$$

$$
\text{RMT-2}\ \frac{\texttt{class C<\_> extends}\ \mathtt{C}'\texttt{<\_>}\ \{\ \text{no method}\ \mathtt{m}\ \}}{\mathit{mBody}(\mathtt{C}, \mathtt{m}) = \mathit{mBody}(\mathtt{C}', \mathtt{m})}
$$

4.3 Well-Formedness

In this subsection, we define well-formedness of runtime types, heaps, and runtime environments.

Well-Formed Runtime Types. The judgement $\iota \vdash \iota'\ \mathtt{C}\texttt{<}\overline{^{\mathtt{r}}\mathtt{T}}\texttt{>}\ \mathtt{ok}$ expresses that runtime type $\iota'\ \mathtt{C}\texttt{<}\overline{^{\mathtt{r}}\mathtt{T}}\texttt{>}$ is well-formed for viewpoint address ι. According to rule WFRT, a runtime type must have a type argument for each type variables of its class. Each runtime type argument must be a subtype of the dynamization of the type variable's upper bound.

Since upper bounds of well-formed classes do not contain rep (see rule WFC in Fig. 10), we can pass an arbitrary address to *dyn*.

$$\text{WFRT}\;\frac{\begin{array}{c}\texttt{class C<}\overline{\texttt{X}\ {}^{\texttt{s}}\texttt{N}}\texttt{> ...}\\ \iota \vdash \overline{{}^{\texttt{r}}\texttt{T}}\ {}^{\texttt{r}}\texttt{<:}\ \mathit{dyn}(\overline{{}^{\texttt{s}}\texttt{N}}, \_\,, \iota'\ \texttt{C<}\overline{{}^{\texttt{r}}\texttt{T}}\texttt{>})\end{array}}{\iota \vdash \iota'\ \texttt{C<}\overline{{}^{\texttt{r}}\texttt{T}}\texttt{> ok}}$$

Well-Formed Heaps. A heap h is well-formed, denoted by h ok, if and only if the runtime types of all objects are well-formed, the $\texttt{null}_a$ address is not mapped to an object, and all addresses stored in fields are well-typed.

$$\text{WFH}\;\frac{\begin{array}{c}(\forall\iota:\quad \iota \vdash \texttt{h}(\iota)\downarrow_1\ \ \texttt{ok}) \qquad\qquad \texttt{null}_a \notin \text{dom}(\texttt{h})\\ (\forall\iota,\texttt{f}:\quad \texttt{h}(\iota) = {}^{\texttt{r}}\texttt{T},\texttt{Fs}\ \wedge\ {}^{\texttt{r}}\!\mathit{fType}(\iota, {}^{\texttt{r}}\texttt{T}, \texttt{f}) = {}^{\texttt{r}}\texttt{T}' \implies \texttt{h} \vdash \texttt{Fs}(\texttt{f}) : {}^{\texttt{r}}\texttt{T}')\end{array}}{\texttt{h ok}}$$

Well-Formed Runtime Environments. The judgement $\texttt{h} \vdash {}^{\texttt{r}}\Gamma : {}^{\texttt{s}}\Gamma$ expresses that runtime environment ${}^{\texttt{r}}\Gamma$ is well-formed w.r.t. a well-formed heap h and a well-formed static type environment ${}^{\texttt{s}}\Gamma$. This is the case if and only if: (1) ${}^{\texttt{r}}\Gamma$ maps all method type variables $\overline{\texttt{X}}$ that are contained in ${}^{\texttt{s}}\Gamma$ to well-formed runtime types $\overline{{}^{\texttt{r}}\texttt{T}}$, which are subtypes of the dynamizations of the corresponding upper bounds $\overline{{}^{\texttt{s}}\texttt{N}}$; (2) ${}^{\texttt{r}}\Gamma$ maps this to an address ι, which is not allowed to be $\texttt{null}_a$. The runtime type of the this object at address ι is exactly the dynamization of the static type of this, $\texttt{this}_u\ \texttt{C<}\overline{\texttt{X}'}\texttt{>}$. (3) ${}^{\texttt{r}}\Gamma$ maps the formal parameters $\overline{\texttt{x}}$ that are contained in ${}^{\texttt{s}}\Gamma$ to addresses $\overline{\iota'}$. The objects at addresses $\overline{\iota'}$ are well-typed with the dynamization of the static types of $\overline{\texttt{x}}$, $\overline{{}^{\texttt{s}}\texttt{T}'}$.

$$\text{WFRE}\;\frac{\begin{array}{c}{}^{\texttt{r}}\Gamma = \overline{\texttt{X}\ {}^{\texttt{r}}\texttt{T}}\ ;\ \texttt{this}\ \iota, \overline{\texttt{x}\ \iota'}\\ {}^{\texttt{s}}\Gamma = \overline{\texttt{X}\ {}^{\texttt{s}}\texttt{N}},\ \overline{\texttt{X}'\ \_}\ ;\ \texttt{this}\ (\texttt{this}_u\ \texttt{C<}\overline{\texttt{X}'}\texttt{>}), \overline{\texttt{x}\ {}^{\texttt{s}}\texttt{T}'}\\ \texttt{h ok} \qquad\qquad {}^{\texttt{s}}\Gamma\ \texttt{ok} \qquad\qquad \iota \neq \texttt{null}_a\\ \iota \vdash \overline{{}^{\texttt{r}}\texttt{T}}\ \texttt{ok} \qquad\qquad \iota \vdash \overline{{}^{\texttt{r}}\texttt{T}}\ {}^{\texttt{r}}\texttt{<:}\ \overline{\mathit{dyn}({}^{\texttt{s}}\texttt{N}, \texttt{h}, {}^{\texttt{r}}\Gamma)}\\ \texttt{h} \vdash \iota :\ \mathit{dyn}(\texttt{this}_u\ \texttt{C<}\overline{\texttt{X}'}\texttt{>}, \texttt{h}, {}^{\texttt{r}}\Gamma) \qquad \texttt{h} \vdash \overline{\iota'} :\ \mathit{dyn}(\overline{{}^{\texttt{s}}\texttt{T}'}, \texttt{h}, {}^{\texttt{r}}\Gamma)\end{array}}{\texttt{h} \vdash {}^{\texttt{r}}\Gamma : {}^{\texttt{s}}\Gamma}$$

4.4 Operational Semantics

We describe program execution by a big-step operational semantics. The transition $\texttt{h}, {}^{\texttt{r}}\Gamma, \texttt{e} \rightsquigarrow \texttt{h}', \iota$ expresses that the evaluation of an expression e in heap h and runtime environment ${}^{\texttt{r}}\Gamma$ results in address ι and successor heap $\texttt{h}'$. A program with main class C is executed by evaluating the main expression in a heap $\texttt{h}_0$ that contains exactly one C instance in the root context where all fields $\overline{\texttt{f}} = \mathit{fields}(\texttt{C})$ are initialized to $\texttt{null}_a$ ($\texttt{h}_0 = \{\iota \mapsto (\texttt{null}_a\ \texttt{C<>},\ \overline{\texttt{f}\ \texttt{null}_a})\}$) and a runtime environment ${}^{\texttt{r}}\Gamma_0$ that maps this to this C instance (${}^{\texttt{r}}\Gamma_0 = \epsilon\texttt{;this}\ \iota$). The rules for evaluating expressions are presented in Fig. 14 and explained in the following.

Parameters, including this, are evaluated by looking up the stored address in the stack, which is part of the runtime environment ${}^{\texttt{r}}\Gamma$ (OS-Var). The null expression always evaluates to the $\texttt{null}_a$ address (OS-Null). Object creation picks a fresh address, allocates an object of the appropriate type, and initializes its fields to $\texttt{null}_a$ (OS-New). For cast

$$\text{OS-Var}\ \frac{}{\mathtt{h}, {}^{\mathtt{r}}\Gamma, \mathtt{x} \rightsquigarrow \mathtt{h}, {}^{\mathtt{r}}\Gamma(\mathtt{x})} \qquad \text{OS-Null}\ \frac{}{\mathtt{h}, {}^{\mathtt{r}}\Gamma, \mathtt{null} \rightsquigarrow \mathtt{h}, \mathtt{null}_a}$$

$$\text{OS-New}\ \frac{\begin{array}{c}\iota \notin \mathrm{dom}(h) \quad \iota \neq \mathtt{null}_a \\ {}^{\mathtt{r}}\mathtt{T} = dyn({}^{\mathtt{s}}\mathtt{T}, \mathtt{h}, {}^{\mathtt{r}}\Gamma) \\ {}^{\mathtt{r}}\mathtt{T} = \_\ \mathtt{C}\texttt{<}\_\texttt{>} \\ \mathtt{Fs}(fields(\mathtt{C})) = \mathtt{null}_a \\ \mathtt{h}' = \mathtt{h}[\iota \mapsto ({}^{\mathtt{r}}\mathtt{T}, \mathtt{Fs})]\end{array}}{\mathtt{h}, {}^{\mathtt{r}}\Gamma, \mathtt{new}\ {}^{\mathtt{s}}\mathtt{T} \rightsquigarrow \mathtt{h}', \iota} \qquad \text{OS-Cast}\ \frac{\begin{array}{c}\mathtt{h}, {}^{\mathtt{r}}\Gamma, \mathtt{e}_0 \rightsquigarrow \mathtt{h}', \iota \\ \mathtt{h}' \vdash \iota : dyn({}^{\mathtt{s}}\mathtt{T}, \mathtt{h}, {}^{\mathtt{r}}\Gamma)\end{array}}{\mathtt{h}, {}^{\mathtt{r}}\Gamma, ({}^{\mathtt{s}}\mathtt{T})\ \mathtt{e}_0 \rightsquigarrow \mathtt{h}', \iota}$$

$$\text{OS-Read}\ \frac{\begin{array}{c}\mathtt{h}, {}^{\mathtt{r}}\Gamma, \mathtt{e}_0 \rightsquigarrow \mathtt{h}', \iota_0 \\ \iota_0 \neq \mathtt{null}_a \\ \iota = \mathtt{h}'(\iota_0)\downarrow_2 (\mathtt{f})\end{array}}{\mathtt{h}, {}^{\mathtt{r}}\Gamma, \mathtt{e}_0.\mathtt{f} \rightsquigarrow \mathtt{h}', \iota} \qquad \text{OS-Write}\ \frac{\begin{array}{c}\mathtt{h}, {}^{\mathtt{r}}\Gamma, \mathtt{e}_0 \rightsquigarrow \mathtt{h}_0, \iota_0 \\ \iota_0 \neq \mathtt{null}_a \\ \mathtt{h}_0, {}^{\mathtt{r}}\Gamma, \mathtt{e}_2 \rightsquigarrow \mathtt{h}_2, \iota \\ \mathtt{h}' = \mathtt{h}_2[\iota_0.\mathtt{f} := \iota]\end{array}}{\mathtt{h}, {}^{\mathtt{r}}\Gamma, \mathtt{e}_0.\mathtt{f}\texttt{=}\mathtt{e}_2 \rightsquigarrow \mathtt{h}', \iota}$$

$$\text{OS-Invk}\ \frac{\begin{array}{ccc}\mathtt{h}, {}^{\mathtt{r}}\Gamma, \mathtt{e}_0 \rightsquigarrow \mathtt{h}_0, \iota_0 & \iota_0 \neq \mathtt{null}_a & \mathtt{h}_0, {}^{\mathtt{r}}\Gamma, \overline{\mathtt{e}_2} \rightsquigarrow \mathtt{h}_2, \overline{\iota_2} \\ \mathtt{h}_0(\iota_0)\downarrow_1 = \_\ \mathtt{C}_0\texttt{<}\_\texttt{>} & & mBody(\mathtt{C}_0, \mathtt{m}) = (\mathtt{e}_1, \overline{\mathtt{x}}, \overline{\mathtt{X}}) \\ \overline{{}^{\mathtt{r}}\mathtt{T}} = dyn(\overline{{}^{\mathtt{s}}\mathtt{T}}, \mathtt{h}, {}^{\mathtt{r}}\Gamma) & {}^{\mathtt{r}}\Gamma' = \overline{\mathtt{X}\ {}^{\mathtt{r}}\mathtt{T}}\ ;\ \mathtt{this}\ \iota_0, \overline{\mathtt{x}\ \iota_2} & \mathtt{h}_2, {}^{\mathtt{r}}\Gamma', \mathtt{e}_1 \rightsquigarrow \mathtt{h}', \iota\end{array}}{\mathtt{h}, {}^{\mathtt{r}}\Gamma, \mathtt{e}_0.\mathtt{m}\texttt{<}\overline{{}^{\mathtt{s}}\mathtt{T}}\texttt{>}(\overline{\mathtt{e}_2}) \rightsquigarrow \mathtt{h}', \iota}$$

Figure 14: Operational semantics.

expressions, we evaluate the expression $\mathtt{e}_0$ and check that the resulting address has the runtime type that is the dynamization of the static type given in the cast expression w.r.t. the current `this` address (OS-Cast). Runtime information about type arguments and ownership is mainly required to check casts. However, we also use this information to evaluate `new` expressions, where the type is a type variable.

For field read, we evaluate the receiver expression and then look up the field in the heap, provided that the receiver is non-null (OS-Read). For field updates, we evaluate the receiver expression and the right-hand side expression, and update the heap (OS-Write). Note that the limited covariance of Generic Universe Types does not require a runtime ownership check for field updates.

Rule OS-Invk describes how to evaluate a call of a method `m`. We evaluate the receiver expression and the actual method arguments in the usual order. The class of the receiver object is used to look up the method body. Its expression is then evaluated in the runtime environment that maps `m`'s type variables to actual type arguments as well as `m`'s formal method parameters (including `this`) to the actual method arguments. The resulting heap and address are the result of the call. Note that method invocations do not need any runtime type checks or purity checks.

5 Properties and Proofs

5.1 Properties

5.1.1 Adaptation from a Viewpoint

The following lemma expresses that viewpoint adaptation from a viewpoint to `this` is correct. Consider the `this` object of a well-formed runtime environment as well as two objects $\mathtt{o}_1$ and $\mathtt{o}_2$. If from the viewpoint `this`, $\mathtt{o}_1$ has the type that is the dynamization of a static type ${}^{\mathtt{s}}\mathtt{N}$, and from viewpoint $\mathtt{o}_1$, $\mathtt{o}_2$ has the type that is the dynamization of a static type ${}^{\mathtt{s}}\mathtt{T}$, then from the viewpoint `this`, $\mathtt{o}_2$ has the type that is the dynamization of ${}^{\mathtt{s}}\mathtt{T}$ adapted from ${}^{\mathtt{s}}\mathtt{N}$, ${}^{\mathtt{s}}\mathtt{N}\rhd{}^{\mathtt{s}}\mathtt{T}$. The following lemma expresses this property using the addresses ι_1 and ι_2 of the objects $\mathtt{o}_1$ and $\mathtt{o}_2$, respectively.

Lemma 5.1 (Adaptation from a Viewpoint).

$$\left.\begin{array}{l}
\mathtt{h}\vdash{}^{\mathtt{r}}\Gamma:{}^{\mathtt{s}}\Gamma \qquad \mathtt{h}\vdash{}^{\mathtt{r}}\Gamma':{}^{\mathtt{s}}\Gamma' \\
\mathtt{h}\vdash\iota_1:dyn({}^{\mathtt{s}}\mathtt{N},\mathtt{h},{}^{\mathtt{r}}\Gamma) \\
\mathtt{h}\vdash\iota_2:dyn({}^{\mathtt{s}}\mathtt{T},\iota_1,{}^{\mathtt{r}}\mathtt{T},{}^{\mathtt{r}}\Gamma') \\
\mathtt{h}(\iota_1)\downarrow_1={}^{\mathtt{r}}\mathtt{T} \\
{}^{\mathtt{s}}\mathtt{N}=\mathtt{u}_N\ \mathtt{C}_N\mathtt{<_>} \\
\mathtt{u}_N=\mathtt{this}_u\Rightarrow{}^{\mathtt{r}}\Gamma(\mathtt{this})=\iota_1 \\
free({}^{\mathtt{s}}\mathtt{T})\subseteq dom(\mathtt{C}_N)\circ\overline{\mathtt{X}_m} \\
\overline{{}^{\mathtt{r}}\mathtt{T}_m}=dyn(\overline{{}^{\mathtt{s}}\mathtt{T}_m},\mathtt{h},{}^{\mathtt{r}}\Gamma) \\
{}^{\mathtt{r}}\Gamma'=\overline{\mathtt{X}_m\ {}^{\mathtt{r}}\mathtt{T}_m};_
\end{array}\right\}\Longrightarrow\ \mathtt{h}\vdash\iota_2:dyn(({}^{\mathtt{s}}\mathtt{N}\rhd{}^{\mathtt{s}}\mathtt{T})[\overline{{}^{\mathtt{s}}\mathtt{T}_m/\mathtt{X}_m}],\mathtt{h},{}^{\mathtt{r}}\Gamma)$$

This lemma justifies the type rule GT-Read. The last antecedent ensures that, if the main modifier is $\mathtt{this}_u$ then ι_1 corresponds to the current object in the environment. The proof of this lemma runs by induction on the shape of static type ${}^{\mathtt{s}}\mathtt{T}$. The base case deals with type variables and non-generic types. The induction step considers generic types, assuming that the lemma holds for the actual type arguments. Each of the cases is done by a case distinction on the main modifiers of ${}^{\mathtt{s}}\mathtt{N}$ and ${}^{\mathtt{s}}\mathtt{T}$.

5.1.2 Adaptation to a Viewpoint

The following lemma in the converse of Lemma 5.1. It expresses that viewpoint adaptation from `this` to an object $\mathtt{o}_1$ is correct. If from the viewpoint `this`, $\mathtt{o}_1$ has the type that is the dynamization of a static type ${}^{\mathtt{s}}\mathtt{N}$ and $\mathtt{o}_2$ has the type that is the dynamization of ${}^{\mathtt{s}}\mathtt{N}\rhd{}^{\mathtt{s}}\mathtt{T}$, then from viewpoint $\mathtt{o}_1$, $\mathtt{o}_2$ has the type that is the dynamization of a static type ${}^{\mathtt{s}}\mathtt{T}$. The lemma requires that the adaptation of ${}^{\mathtt{s}}\mathtt{T}$ does not change ownership modifiers in ${}^{\mathtt{s}}\mathtt{T}$ from non-`any` to `any`, because the lost ownership information cannot be recovered. Such a change occurs if ${}^{\mathtt{s}}\mathtt{N}$'s main modifier is `any` or if ${}^{\mathtt{s}}\mathtt{T}$ contains `rep` and is not accessed through `this` (see definition of *rp*, Sec. 3.6).

Lemma 5.2 (Adaptation to a Viewpoint).

$$\left.\begin{array}{l}
\mathtt{h} \vdash {}^{r}\Gamma : {}^{s}\Gamma \qquad \mathtt{h} \vdash {}^{r}\Gamma' : {}^{s}\Gamma' \\
\mathtt{h} \vdash \iota_1 : dyn({}^{s}\mathtt{N}, \mathtt{h}, {}^{r}\Gamma) \\
\mathtt{h} \vdash \iota_2 : dyn(({}^{s}\mathtt{N} \triangleright {}^{s}\mathtt{T})[\overline{{}^{s}\mathtt{T}_m / \mathtt{X}_m}], \mathtt{h}, {}^{r}\Gamma) \\
\mathtt{h}(\iota_1)\downarrow_1 = {}^{r}\mathtt{T} \\
{}^{s}\mathtt{N} = \mathtt{u}_N\ \mathtt{C}_N\mathtt{<_>}, \quad \mathtt{u}_N \neq \mathtt{any}, \quad rp(\mathtt{u}_N, {}^{s}\mathtt{T}) \\
\mathtt{u}_N = \mathtt{this}_u \Rightarrow {}^{r}\Gamma(\mathtt{this}) = \iota_1 \\
free({}^{s}\mathtt{T}) \subseteq dom(\mathtt{C}_N) \circ \overline{\mathtt{X}_m} \\
\overline{{}^{r}\mathtt{T}_m} = dyn(\overline{{}^{s}\mathtt{T}_m}, \mathtt{h}, {}^{r}\Gamma) \\
{}^{r}\Gamma' = \overline{\mathtt{X}_m\ {}^{r}\mathtt{T}_m}; _
\end{array}\right\} \implies \mathtt{h} \vdash \iota_2 : dyn({}^{s}\mathtt{T}, \iota_1, {}^{r}\mathtt{T}, {}^{r}\Gamma')$$

This lemma justifies the type rule GT-Write and the requirements for the types of the parameters in GT-Invk. The proof of this lemma is analogous to the proof for Lemma 5.1.

5.1.3 Soundness

Type safety of Generic Universe Types is expressed by the following theorem. If a well-typed expression e is evaluated in a well-formed environment (including a well-formed heap), then the resulting environment is well-formed and the result of e's evaluation has the type that is the dynamization of e's static type. Moreover, our theorem expresses that the main modifier $\mathtt{this}_u$ is used solely for the type of this.

Theorem 5.3 (Soundness).

$$\left.\begin{array}{l}
\mathtt{h} \vdash {}^{r}\Gamma : {}^{s}\Gamma \\
{}^{s}\Gamma \vdash \mathtt{e} : {}^{s}\mathtt{T} \\
\mathtt{h}, {}^{r}\Gamma, \mathtt{e} \rightsquigarrow \mathtt{h}', \iota
\end{array}\right\} \implies \left\{\begin{array}{l}
\mathtt{h}' \vdash {}^{r}\Gamma : {}^{s}\Gamma \\
\mathtt{h}' \vdash \iota : dyn({}^{s}\mathtt{T}, \mathtt{h}, {}^{r}\Gamma) \\
{}^{s}\mathtt{T} = \mathtt{this}_u\ _\mathtt{<_>} \Rightarrow \iota \in \{{}^{r}\Gamma(\mathtt{this}), \mathtt{null}_a\}
\end{array}\right.$$

The proof of Theorem 5.3 runs by rule induction on the operational semantics. Lemma 5.1 is used to prove field read and method results, whereas Lemma 5.2 is used to prove field updates and method parameter passing.

We omit a proof of progress since this property is not affected by adding ownership to a Java-like language. The basic proof can easily be adapted from FGJ [16]. Extensions to include field updates and casts have also been done before [15, 4]. Only the additional check of the ownership information in a cast is different from these previous approaches; its treatment is analogous to a standard Java cast.

5.1.4 Owner-as-Modifier

A property that is relevant for Generic Universe Types is the enforcement of the owner-as-modifier discipline, which is expressed by the following theorem. The evaluation of a well-typed expression e in a well-formed environment modifies only those objects that are (transitively) owned by the owner of this.

Theorem 5.4 (Owner-as-Modifier).

$$\left.\begin{array}{l} \mathtt{h} \vdash {}^{\mathtt{r}}\Gamma : {}^{\mathtt{s}}\Gamma \\ {}^{\mathtt{s}}\Gamma \vdash \mathtt{e} : {}^{\mathtt{s}}\mathtt{T} \\ \iota_T = {}^{\mathtt{r}}\Gamma(\mathtt{this}) \\ \mathtt{h}, {}^{\mathtt{r}}\Gamma, \mathtt{e} \rightsquigarrow \mathtt{h}', \_ \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \forall \iota \in \mathit{dom}(\mathtt{h}), \mathtt{f} : \\ \quad \mathtt{h}(\iota) \downarrow_2 (\mathtt{f}) = \mathtt{h}' \downarrow_2 (\mathtt{f}) \vee \\ \quad \mathit{owner}(\mathtt{h}, \iota_T) \in \mathit{owners}(\mathtt{h}, \iota) \end{array}\right.$$

where $\mathit{owner}(\mathtt{h}, \iota)$ denotes the direct owner of the object at address ι in heap $\mathtt{h}$, and $\mathit{owners}(\mathtt{h}, \iota)$ denotes the set of all (transitive) owners of this object.

The proof of Theorem 5.4 runs by rule induction on the operational semantics. The interesting cases are field update and calls of non-pure methods. In both cases, the type rules (Fig. 11) enforce that the receiver expression does not have the main modifier `any`. That is, the receiver object is owned by `this` or the owner of `this`.

For the proof of Theorem 5.4 we assume that pure methods do not modify objects that exist in the prestate of the call:

Lemma 5.5 (Pure Methods).

$$\left.\begin{array}{l} \mathtt{h} \vdash {}^{\mathtt{r}}\Gamma : {}^{\mathtt{s}}\Gamma \\ {}^{\mathtt{s}}\Gamma \vdash \mathtt{e}_0.\mathtt{m}\texttt{<}\overline{{}^{\mathtt{s}}\mathtt{T}}\texttt{>}(\mathtt{e}_2) : {}^{\mathtt{s}}\mathtt{T} \\ \mathtt{h}, {}^{\mathtt{r}}\Gamma, \mathtt{e}_0.\mathtt{m}\texttt{<}\overline{{}^{\mathtt{s}}\mathtt{T}}\texttt{>}(\mathtt{e}_2) \rightsquigarrow \mathtt{h}', \_ \\ {}^{\mathtt{s}}\Gamma \vdash \mathtt{e}_0 : \_\ \mathtt{C}_0\texttt{<}\_\texttt{>} \\ \mathit{mType}(\mathtt{C}_0, \mathtt{m}) = \texttt{<}\_\texttt{>}\ \mathtt{pure}\ \_\ \mathtt{m}(\_) \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \forall \iota \in \mathit{dom}(\mathtt{h}), \mathtt{f} : \\ \quad \mathtt{h}(\iota) \downarrow_2 (\mathtt{f}) = \mathtt{h}' \downarrow_2 (\mathtt{f}) \end{array}\right.$$

In this paper we do not describe how this is enforced in the program. A simple but conservative approach forbids all object creations, field updates, and calls of methods that are not pure [22]. The above definition also allows weaker forms of purity that allow object creations [13] and also approaches that allow the modification of newly created objects [27].

5.1.5 Adaptation from a Viewpoint Helper Lemma

The following lemma is used in the proof of the two viewpoint adaptation lemmas. This lemma is basically the same as Lemma 5.1, but is more suitable for a proof by induction.

Lemma 5.6 (Adaptation from a Viewpoint Helper Lemma).

$$\left.\begin{array}{l} \mathtt{h} \vdash {}^{\mathtt{r}}\Gamma : {}^{\mathtt{s}}\Gamma \qquad \mathtt{h} \vdash {}^{\mathtt{r}}\Gamma' : {}^{\mathtt{s}}\Gamma' \\ \mathtt{h} \vdash \iota_1 : \mathit{dyn}({}^{\mathtt{s}}\mathtt{N}, \mathtt{h}, {}^{\mathtt{r}}\Gamma) \\ \mathtt{h}(\iota_1) \downarrow_1 = {}^{\mathtt{r}}\mathtt{T} \\ {}^{\mathtt{s}}\mathtt{N} = \mathtt{u}_N\ \mathtt{C}_N\texttt{<}\_\texttt{>} \\ \mathtt{u}_N = \mathtt{this}_u \Rightarrow {}^{\mathtt{r}}\Gamma(\mathtt{this}) = \iota_1 \\ \mathit{free}({}^{\mathtt{s}}\mathtt{T}) \subseteq \mathit{dom}(\mathtt{C}_N) \circ \overline{\mathtt{X}_m} \\ \overline{{}^{\mathtt{r}}\mathtt{T}_m} = \mathit{dyn}(\overline{{}^{\mathtt{s}}\mathtt{T}_m}, \mathtt{h}, {}^{\mathtt{r}}\Gamma) \\ {}^{\mathtt{r}}\Gamma' = \overline{\mathtt{X}_m\ {}^{\mathtt{r}}\mathtt{T}_m}; \_ \\ {}^{\mathtt{r}}\mathtt{T}_2 = \mathit{dyn}({}^{\mathtt{s}}\mathtt{T}, \iota_1, {}^{\mathtt{r}}\mathtt{T}, {}^{\mathtt{r}}\Gamma') \end{array}\right\} \Longrightarrow \ {}^{\mathtt{r}}\mathtt{T}_2\ {}^{\mathtt{r}}\texttt{<:}_{\mathtt{a}}\ \mathit{dyn}(({}^{\mathtt{s}}\mathtt{N} \rhd {}^{\mathtt{s}}\mathtt{T})[\overline{{}^{\mathtt{s}}\mathtt{T}_m / \mathtt{X}_m}], \mathtt{h}, {}^{\mathtt{r}}\Gamma)$$

The proof runs by induction on the shape of ${}^{\mathtt{s}}\mathtt{T}$.

5.1.6 Adaptation to a Viewpoint Helper Lemma

The following lemma is used in the proof of the two viewpoint adaptation lemmas. This lemma is basically the same as Lemma 5.2, but is more suitable for a proof by induction.

Lemma 5.7 (Adaptation to a Viewpoint Helper Lemma).

$$\left.\begin{array}{l}
\mathtt{h} \vdash {}^{\mathtt{r}}\Gamma : {}^{\mathtt{s}}\Gamma \qquad \mathtt{h} \vdash {}^{\mathtt{r}}\Gamma' : {}^{\mathtt{s}}\Gamma' \\
\mathtt{h} \vdash \iota_1 : dyn({}^{\mathtt{s}}\mathtt{N}, \mathtt{h}, {}^{\mathtt{r}}\Gamma) \\
{}^{\mathtt{r}}\mathtt{T}_2 = dyn(({}^{\mathtt{s}}\mathtt{N} \rhd {}^{\mathtt{s}}\mathtt{T})[\overline{{}^{\mathtt{s}}\mathtt{T}_m / \mathtt{X}_m}], \mathtt{h}, {}^{\mathtt{r}}\Gamma) \\
\mathtt{h}(\iota_1)\downarrow_1 = {}^{\mathtt{r}}\mathtt{T} \\
{}^{\mathtt{s}}\mathtt{N} = \mathtt{u}_N\ \mathtt{C}_N\mathtt{<_>}, \quad \mathtt{u}_N \neq \mathtt{any}, \quad rp(\mathtt{u}_N, {}^{\mathtt{s}}\mathtt{T}) \\
\mathtt{u}_N = \mathtt{this}_u \Rightarrow {}^{\mathtt{r}}\Gamma(\mathtt{this}) = \iota_1 \\
free({}^{\mathtt{s}}\mathtt{T}) \subseteq dom(\mathtt{C}_N) \circ \overline{\mathtt{X}_m} \\
\overline{{}^{\mathtt{r}}\mathtt{T}_m} = dyn(\overline{{}^{\mathtt{s}}\mathtt{T}_m}, \mathtt{h}, {}^{\mathtt{r}}\Gamma) \\
{}^{\mathtt{r}}\Gamma' = \overline{\mathtt{X}_m\ {}^{\mathtt{r}}\mathtt{T}_m}; _
\end{array}\right\} \implies {}^{\mathtt{r}}\mathtt{T}_2 = dyn({}^{\mathtt{s}}\mathtt{T}, \iota_1, {}^{\mathtt{r}}\mathtt{T}, {}^{\mathtt{r}}\Gamma')$$

The proof runs by induction on the shape of ${}^{\mathtt{s}}\mathtt{T}$.

5.1.7 Properties of Runtime Types

If we have two possible runtime supertypes of ${}^{\mathtt{r}}\mathtt{T}$, both having the same owner address and class, and know that all type arguments of one supertype are subtypes of the other type arguments, then we know that the type arguments are also in the $<:_{\mathtt{a}}$ relation, *i.e.,* all the classes in the two types are equal and only the owner addresses vary.

Lemma 5.8 (Covariance of Runtime Subtyping 1).

$$\left.\begin{array}{l}
\iota_0 \vdash {}^{\mathtt{r}}\mathtt{T}\ {}^{\mathtt{r}}\!\!<:\ \iota'\ \mathtt{C}\mathtt{<}\overline{{}^{\mathtt{r}}\mathtt{T}}\mathtt{>} \\
\iota_0 \vdash {}^{\mathtt{r}}\mathtt{T}\ {}^{\mathtt{r}}\!\!<:\ \iota'\ \mathtt{C}\mathtt{<}\overline{{}^{\mathtt{r}}\mathtt{T}'}\mathtt{>} \\
\iota_0 \vdash \overline{{}^{\mathtt{r}}\mathtt{T}}\ {}^{\mathtt{r}}\!\!<:\ \overline{{}^{\mathtt{r}}\mathtt{T}'}
\end{array}\right\} \Rightarrow \overline{{}^{\mathtt{r}}\mathtt{T}}\ {}^{\mathtt{r}}\!\!<:_{\mathtt{a}}\ \overline{{}^{\mathtt{r}}\mathtt{T}'}$$

The proof runs by induction on the runtime subtyping rules.

If we have two possible runtime supertypes of ${}^{\mathtt{r}}\mathtt{T}$, both having the same owner address and class, and know that the owner of the supertypes is not $\mathtt{any}_a$, then we know that the type arguments are equal.

Lemma 5.9 (Covariance of Runtime Subtyping 2).

$$\left.\begin{array}{l}
\iota_0 \vdash {}^{\mathtt{r}}\mathtt{T}\ {}^{\mathtt{r}}\!\!<:\ \iota'\ \mathtt{C}\mathtt{<}\overline{{}^{\mathtt{r}}\mathtt{T}}\mathtt{>} \\
\iota_0 \vdash {}^{\mathtt{r}}\mathtt{T}\ {}^{\mathtt{r}}\!\!<:\ \iota'\ \mathtt{C}\mathtt{<}\overline{{}^{\mathtt{r}}\mathtt{T}'}\mathtt{>} \\
\iota' \neq \mathtt{any}_a
\end{array}\right\} \Rightarrow \overline{{}^{\mathtt{r}}\mathtt{T}} = \overline{{}^{\mathtt{r}}\mathtt{T}'}$$

The proof runs by induction on the runtime subtyping rules.

A static subtype relation will result in a runtime subtype relation, if they are converted to runtime types in the same context.

Lemma 5.10 (Subtyping preserved by *dyn* 1).

$$\left.\begin{array}{l}\mathtt{h} \vdash {}^{\mathtt{r}}\Gamma : {}^{\mathtt{s}}\Gamma \\ {}^{\mathtt{s}}\Gamma \vdash {}^{\mathtt{s}}\mathtt{T} <: {}^{\mathtt{s}}\mathtt{T}'\end{array}\right\} \Rightarrow \iota \vdash dyn({}^{\mathtt{s}}\mathtt{T}, \iota, {}^{\mathtt{r}}\mathtt{T}, {}^{\mathtt{r}}\Gamma)\ {}^{\mathtt{r}}\!\!<: dyn({}^{\mathtt{s}}\mathtt{T}', \iota, {}^{\mathtt{r}}\mathtt{T}, {}^{\mathtt{r}}\Gamma)$$

The proof runs by induction on the shape of ${}^{\mathtt{s}}\mathtt{T}$ and the static subtyping rules.

If an address ι can be typed with the dynamization of a subtype ${}^{\mathtt{s}}\mathtt{T}$, it can also be typed with the dynamization of a supertype ${}^{\mathtt{s}}\mathtt{T}'$.

Lemma 5.11 (Subtyping preserved by *dyn* 2).

$$\left.\begin{array}{l}\mathtt{h} \vdash {}^{\mathtt{r}}\Gamma : {}^{\mathtt{s}}\Gamma \\ {}^{\mathtt{s}}\Gamma \vdash {}^{\mathtt{s}}\mathtt{T} <: {}^{\mathtt{s}}\mathtt{T}' \\ \mathtt{h} \vdash \iota : dyn({}^{\mathtt{s}}\mathtt{T}, \mathtt{h}, {}^{\mathtt{r}}\Gamma)\end{array}\right\} \Rightarrow \mathtt{h} \vdash \iota : dyn({}^{\mathtt{s}}\mathtt{T}', \mathtt{h}, {}^{\mathtt{r}}\Gamma)$$

The proof runs by induction on the shape of ${}^{\mathtt{s}}\mathtt{T}$ and the static subtyping rules.

5.1.8 Evaluation Preserves Types

The evaluation of any expression does not change the runtime types of existing objects.

Lemma 5.12 (Evaluation Preserves Types).
If $\mathtt{h}, {}^{\mathtt{r}}\Gamma, \mathtt{e} \rightsquigarrow \mathtt{h}', \iota'$, *then*

- $\iota \in dom(\mathtt{h}) \;\Rightarrow\; \mathtt{h}(\iota)\downarrow_1 = \mathtt{h}'(\iota)\downarrow_1$.
- $dyn({}^{\mathtt{s}}\mathtt{T}, \mathtt{h}, {}^{\mathtt{r}}\Gamma)$ *is defined* $\Rightarrow\; dyn({}^{\mathtt{s}}\mathtt{T}, \mathtt{h}, {}^{\mathtt{r}}\Gamma) = dyn({}^{\mathtt{s}}\mathtt{T}, \mathtt{h}', {}^{\mathtt{r}}\Gamma)$.

The proof is a case analysis of all expressions.

5.1.9 Runtime Meaning of Ownership Modifiers

The following lemma connects the meaning of the static ownership modifiers and the runtime owner. For $\mathtt{this}_u$ and `peer` references, the owner of the referenced object is the owner of the current object. For `rep` references, the owner of the referenced object is the current object. From $\mathtt{any}_u$ references, we do not gain any information about the runtime ownership.

Lemma 5.13 (Runtime Meaning of Ownership Modifiers).
If $h \vdash \iota : dyn({}^{\mathtt{s}}\mathtt{T}, \mathtt{h}, {}^{\mathtt{r}}\Gamma)$ *and* $\iota \neq \mathtt{null}_a$, *then*

1. ${}^{\mathtt{s}}\mathtt{T} = \mathtt{this}_u\ \_\!<\!\_\!> \quad \Rightarrow owner(\mathtt{h}, \iota) = owner(\mathtt{h}, {}^{\mathtt{r}}\Gamma(\mathtt{this}))$
2. ${}^{\mathtt{s}}\mathtt{T} = \mathtt{peer}\ \_\!<\!\_\!> \quad \Rightarrow owner(\mathtt{h}, \iota) = owner(\mathtt{h}, {}^{\mathtt{r}}\Gamma(\mathtt{this}))$
3. ${}^{\mathtt{s}}\mathtt{T} = \mathtt{rep}\ \_\!<\!\_\!> \quad \Rightarrow owner(\mathtt{h}, \iota) = {}^{\mathtt{r}}\Gamma(\mathtt{this})$

The proof is a case analysis and the application of the definition of *dyn*.

5.1.10 Generation Lemma

The following generation lemma allows us to draw conclusions on the possible derivation of the typing. We know that some expression $\mathtt{e}$ has a type ${}^s\mathtt{T}$ in an environment ${}^s\Gamma$. Then there is a unique shape of the expression by which we can determine which type rule has been used to derive the type ${}^s\mathtt{T}$. This gives us information about all the conditions that must hold for this expression.

Lemma 5.14 (Generation Lemma).

$$
\begin{array}{ll}
{}^s\Gamma \vdash \mathtt{e} : {}^s\mathtt{T} & \\
\mathtt{e} \equiv \mathtt{x} : & \mathtt{x} \in \mathit{dom}(\Gamma) \ \wedge \ \Gamma \vdash \mathtt{x} : \Gamma(\mathtt{x}) \ \wedge \ \Gamma \vdash \Gamma(\mathtt{x}) <: {}^s\mathtt{T} \\
\mathtt{e} \equiv \mathtt{null} : & \Gamma \vdash \mathtt{null} : {}^s\mathtt{T}' \ \wedge \ \Gamma \vdash {}^s\mathtt{T}' <: {}^s\mathtt{T} \\
\mathtt{e} \equiv \mathtt{new}\ {}^s\mathtt{T}' : & \Gamma \vdash \mathtt{new}\ {}^s\mathtt{T}' : {}^s\mathtt{T}' \ \wedge \ \Gamma \vdash {}^s\mathtt{T}' <: {}^s\mathtt{T} \\
\mathtt{e} \equiv ({}^s\mathtt{T}')\ \mathtt{e}_0 : & \Gamma \vdash \mathtt{e}_0 : {}^s\mathtt{N}_0 \ \wedge \ \Gamma \vdash ({}^s\mathtt{T}')\ \mathtt{e}_0 : {}^s\mathtt{T}' \ \wedge \ \Gamma \vdash {}^s\mathtt{T}' <: {}^s\mathtt{T} \\
\mathtt{e} \equiv \mathtt{e}_0.\mathtt{f} : & {}^s\Gamma \vdash \mathtt{e}_0 : {}^s\mathtt{N}_0 \ \wedge \ {}^s\mathtt{N}_0 = \_\ \mathtt{C}_0\texttt{<\_>} \ \wedge \\
 & {}^s\Gamma \vdash \mathtt{e}_0.\mathtt{f} : {}^s\mathtt{N}_0 \triangleright \mathit{fType}(\mathtt{C}_0, \mathtt{f}) \ \wedge \\
 & {}^s\Gamma \vdash {}^s\mathtt{N}_0 \triangleright \mathit{fType}(\mathtt{C}_0, \mathtt{f}) <: {}^s\mathtt{T} \\
\mathtt{e} \equiv \mathtt{e}_0.\mathtt{f}\!=\!\mathtt{e}_2 : & {}^s\Gamma \vdash \mathtt{e}_0 : {}^s\mathtt{N}_0 \ \wedge \ {}^s\mathtt{N}_0 = \mathtt{u}_0\ \mathtt{C}_0\texttt{<\_>} \ \wedge \\
 & {}^s\mathtt{T}_1 = \mathit{fType}(\mathtt{C}_0, \mathtt{f}) \ \wedge \ {}^s\Gamma \vdash \mathtt{e}_2 : {}^s\mathtt{N}_0 \triangleright {}^s\mathtt{T}_1 \ \wedge \\
 & \mathtt{u}_0 \neq \mathtt{any} \ \wedge \ \mathit{rp}(\mathtt{u}_0, {}^s\mathtt{T}_1) \ \wedge \ {}^s\Gamma \vdash \mathtt{e}_0.\mathtt{f}\!=\!\mathtt{e}_2 : {}^s\mathtt{N}_0 \triangleright {}^s\mathtt{T}_1 \ \wedge \\
 & {}^s\Gamma \vdash {}^s\mathtt{N}_0 \triangleright {}^s\mathtt{T}_1 <: {}^s\mathtt{T} \\
\mathtt{e} \equiv \mathtt{e}_0.\mathtt{m}\texttt{<}\overline{{}^s\mathtt{T}}\texttt{>}(\overline{\mathtt{e}_2}) : & {}^s\Gamma \vdash \mathtt{e}_0 : {}^s\mathtt{N}_0 \ \wedge \ {}^s\mathtt{N}_0 = \mathtt{u}_0\ \mathtt{C}_0\texttt{<\_>} \ \wedge \\
 & \mathit{mType}(\mathtt{C}_0, \mathtt{m}) = \texttt{<}\overline{\mathtt{X}_m\ {}^sN_b}\texttt{>}\ \mathtt{w}\ {}^s\mathtt{T}_r\ \mathtt{m}(\overline{\mathtt{x}\ {}^s\mathtt{T}_p}) \ \wedge \\
 & {}^s\Gamma \vdash \overline{{}^s\mathtt{T}} <: ({}^s\mathtt{N}_0 \triangleright \overline{{}^sN_b})[\overline{{}^s\mathtt{T}/\mathtt{X}_m}] \ \wedge \ {}^s\Gamma \vdash \overline{\mathtt{e}_2} : ({}^s\mathtt{N}_0 \triangleright \overline{{}^s\mathtt{T}_p})[\overline{{}^s\mathtt{T}/\mathtt{X}_m}] \ \wedge \\
 & (\mathtt{u}_0\!=\!\mathtt{any} \Rightarrow \mathtt{w}\!=\!\mathtt{pure}) \ \wedge \ \mathit{rp}(\mathtt{u}_0, \overline{{}^s\mathtt{T}_p} \circ \overline{{}^sN_b}) \ \wedge \\
 & {}^s\Gamma \vdash \mathtt{e}_0.\mathtt{m}\texttt{<}\overline{\mathtt{T}}\texttt{>}(\overline{\mathtt{e}_2}) : ({}^s\mathtt{N}_0 \triangleright {}^s\mathtt{T}_r)[\overline{{}^s\mathtt{T}/\mathtt{X}_m}] \ \wedge \\
 & {}^s\Gamma \vdash ({}^s\mathtt{N}_0 \triangleright {}^s\mathtt{T}_r)[\overline{{}^s\mathtt{T}/\mathtt{X}_m}] <: {}^s\mathtt{T}
\end{array}
$$

The proof of Lemma 5.14 runs by rule induction on the shape of the expression $\mathtt{e}$. There are always two type rules that could apply to an expression: the rule for the particular kind of expression and the subsumption rule. From the particular rule we get all the conditions that are checked for this kind of expression; subsumption allows one to go to an arbitrary supertype of this type.

5.2 Proofs

5.2.1 Proof of Theorem 5.3 — Soundness

Our soundness theorem is:

$$
\left.\begin{array}{ll}
1. & \mathtt{h}\ \mathtt{ok} \\
2. & \mathtt{h} \vdash {}^r\Gamma : {}^s\Gamma \\
3. & {}^s\Gamma \vdash \mathtt{e} : {}^s\mathtt{T} \\
4. & \mathtt{h}, {}^r\Gamma, \mathtt{e} \rightsquigarrow \mathtt{h}', \iota
\end{array}\right\} \Longrightarrow \left\{\begin{array}{ll}
I. & \mathtt{h}'\ \mathtt{ok} \\
II. & \mathtt{h}' \vdash {}^r\Gamma : {}^s\Gamma \\
III. & \mathtt{h}' \vdash \iota : \mathit{dyn}({}^s\mathtt{T}, \mathtt{h}, {}^r\Gamma) \\
IV. & {}^s\mathtt{T} = \mathtt{this}_u\ \_\texttt{<\_>} \Rightarrow \iota \in \{{}^r\Gamma(\mathtt{this}), \mathtt{null}_a\}
\end{array}\right.
$$

We prove this by induction on the shape of the derivation tree of 4.

Case 1: $\mathtt{e} \equiv \mathtt{x}$

We have the assumptions of the theorem:

1. $\mathtt{h\ ok}$	2. $\mathtt{h} \vdash {}^{\mathtt{r}}\Gamma : {}^{\mathtt{s}}\Gamma$
3. ${}^{\mathtt{s}}\Gamma \vdash \mathtt{x} : {}^{\mathtt{s}}\mathtt{T}$	4. $\mathtt{h}, {}^{\mathtt{r}}\Gamma, \mathtt{x} \rightsquigarrow \mathtt{h}', \iota$

From 3. and Lemma 5.14 we get:

$$\mathtt{x} \in \mathrm{dom}({}^{\mathtt{s}}\Gamma) \qquad {}^{\mathtt{s}}\Gamma \vdash \mathtt{x} : {}^{\mathtt{s}}\Gamma(\mathtt{x}) \qquad {}^{\mathtt{s}}\Gamma \vdash {}^{\mathtt{s}}\Gamma(\mathtt{x}) <: {}^{\mathtt{s}}\mathtt{T}$$

From the operational semantics we know:

$$\mathtt{h}, {}^{\mathtt{r}}\Gamma, \mathtt{x} \rightsquigarrow \mathtt{h}, {}^{\mathtt{r}}\Gamma(\mathtt{x})$$

Part I: $\mathtt{h}'\ \mathtt{ok}$
We have 1. and $\mathtt{h} = \mathtt{h}'$.

Part II: $\mathtt{h}' \vdash {}^{\mathtt{r}}\Gamma : {}^{\mathtt{s}}\Gamma$
We have 2. and $\mathtt{h} = \mathtt{h}'$.

Part III: $\mathtt{h}' \vdash \iota : dyn({}^{\mathtt{s}}\mathtt{T}, \mathtt{h}, {}^{\mathtt{r}}\Gamma)$
From Part II we know that the environments conform. We know from the operational semantics that the ι is from the runtime environment. Therefore we know from the definition of well-formed runtime environment (WFRE, Page 24), that $\mathtt{h}' \vdash \iota : dyn({}^{\mathtt{s}}\Gamma(\mathtt{x}), \mathtt{h}, {}^{\mathtt{r}}\Gamma)$ holds. We also know that ${}^{\mathtt{s}}\Gamma \vdash {}^{\mathtt{s}}\Gamma(\mathtt{x}) <: {}^{\mathtt{s}}\mathtt{T}$ and can therefore apply Lemma 5.11 to arrive at III.

Part IV: ${}^{\mathtt{s}}\mathtt{T}{=}\mathtt{this}_u\ \_{<}\_{>} \Rightarrow \iota \in \{{}^{\mathtt{r}}\Gamma(\mathtt{this}), \mathtt{null}_a\}$
The static type only has the `this` ownership modifier, if we read the `this` variable. Then we know from the operational semantics and Part II that the variable we read is ${}^{\mathtt{r}}\Gamma(\mathtt{this})$. □

Case 2: $\mathtt{e} \equiv \mathtt{null}$

We have the assumptions of the theorem:

1. $\mathtt{h\ ok}$	2. $\mathtt{h} \vdash {}^{\mathtt{r}}\Gamma : {}^{\mathtt{s}}\Gamma$
3. ${}^{\mathtt{s}}\Gamma \vdash \mathtt{null} : {}^{\mathtt{s}}\mathtt{T}$	4. $\mathtt{h}, {}^{\mathtt{r}}\Gamma, \mathtt{null} \rightsquigarrow \mathtt{h}', \iota$

From 3. and Lemma 5.14 we get:

$${}^{\mathtt{s}}\Gamma \vdash \mathtt{null} : {}^{\mathtt{s}}\mathtt{T}' \qquad {}^{\mathtt{s}}\Gamma \vdash {}^{\mathtt{s}}\mathtt{T}' <: {}^{\mathtt{s}}\mathtt{T}$$

From the operational semantics we know:

$$\mathtt{h}, {}^{\mathtt{r}}\Gamma, \mathtt{null} \rightsquigarrow \mathtt{h}, \mathtt{null}_a$$

Part I: $\mathtt{h}'\ \mathtt{ok}$
The heap does not change.

Part II: $\mathtt{h}' \vdash {}^{\mathtt{r}}\Gamma : {}^{\mathtt{s}}\Gamma$
The heap does not change.

Part III: $\mathtt{h}' \vdash \iota : dyn({}^{\mathtt{s}}\mathtt{T}, \mathtt{h}, {}^{\mathtt{r}}\Gamma)$
We know from the operational semantics that $\iota = \mathtt{null}_a$. Rule RT-5 allows us to assign any runtime type to $\mathtt{null}_a$.

Part IV: ${}^{\mathtt{s}}\mathtt{T}{=}\mathtt{this}_u\ \_{<}\_{>} \Rightarrow \iota \in \{{}^{\mathtt{r}}\Gamma(\mathtt{this}), \mathtt{null}_a\}$
We know from the operational semantics that $\iota = \mathtt{null}_a$. □

Case 3: $\mathtt{e} \equiv \mathtt{e}_0.\mathtt{f}$

We have the assumptions of the theorem:

$$\begin{array}{llll} 1. & \mathtt{h\ ok} & 2. & \mathtt{h} \vdash {}^{\mathtt{r}}\Gamma : {}^{\mathtt{s}}\Gamma \\ 3. & {}^{\mathtt{s}}\Gamma \vdash \mathtt{e}_0.\mathtt{f} : {}^{\mathtt{s}}\mathtt{T} & 4. & \mathtt{h}, {}^{\mathtt{r}}\Gamma, \mathtt{e}_0.\mathtt{f} \rightsquigarrow \mathtt{h}', \iota \end{array}$$

From 3. and Lemma 5.14 we get:

$$\begin{array}{ll} {}^{\mathtt{s}}\Gamma \vdash \mathtt{e}_0 : {}^{\mathtt{s}}\mathtt{N}_0 & {}^{\mathtt{s}}\mathtt{N}_0 = \_\ \mathtt{C}_0{<}\_{>} \\ {}^{\mathtt{s}}\Gamma \vdash \mathtt{e}_0.\mathtt{f} : {}^{\mathtt{s}}\mathtt{N}_0 \triangleright fType(\mathtt{C}_0, \mathtt{f}) & \Gamma \vdash {}^{\mathtt{s}}\mathtt{N}_0 \triangleright fType(\mathtt{C}_0, \mathtt{f}) <: {}^{\mathtt{s}}\mathtt{T} \end{array}$$

From the operational semantics we know:

$$\begin{array}{ll} \mathtt{h}, {}^{\mathtt{r}}\Gamma, \mathtt{e}_0 \rightsquigarrow \mathtt{h}', \iota_0 & \iota_0 \neq \mathtt{null}_a \\ \iota = \mathtt{h}'(\iota_0)\downarrow_2(\mathtt{f}) & \mathtt{h}, {}^{\mathtt{r}}\Gamma, \mathtt{e}_0.\mathtt{f} \rightsquigarrow \mathtt{h}', \iota \end{array}$$

We apply the induction hypothesis to $\mathtt{e}_0$:

$$\left.\begin{array}{ll} 1_0. & \mathtt{h\ ok} \\ 2_0. & \mathtt{h} \vdash {}^{\mathtt{r}}\Gamma : {}^{\mathtt{s}}\Gamma \\ 3_0. & {}^{\mathtt{s}}\Gamma \vdash \mathtt{e}_0 : {}^{\mathtt{s}}\mathtt{N}_0 \\ 4_0. & \mathtt{h}, {}^{\mathtt{r}}\Gamma, \mathtt{e}_0 \rightsquigarrow \mathtt{h}', \iota_0 \end{array}\right\} \Longrightarrow \left\{\begin{array}{ll} I_0. & \mathtt{h}'\ \mathtt{ok} \\ II_0. & \mathtt{h}' \vdash {}^{\mathtt{r}}\Gamma : {}^{\mathtt{s}}\Gamma \\ III_0. & \mathtt{h}' \vdash \iota_0 : dyn({}^{\mathtt{s}}\mathtt{N}_0, \mathtt{h}, {}^{\mathtt{r}}\Gamma) \\ IV_0. & {}^{\mathtt{s}}\mathtt{N}_0 = \mathtt{this}_u\ \_{<}\_{>} \Rightarrow \iota_0 \in \{{}^{\mathtt{r}}\Gamma(\mathtt{this}), \mathtt{null}_a\} \end{array}\right.$$

1_0. and 2_0. correspond to 1. and 2. 3_0. is from the type rules and 4_0. is from the operational semantics.

Part I: $\mathtt{h}'\ \mathtt{ok}$
From I_0.

Part II: $\mathtt{h}' \vdash {}^{\mathtt{r}}\Gamma : {}^{\mathtt{s}}\Gamma$
From II_0.

Part III: $\mathtt{h}' \vdash \iota : dyn({}^{\mathtt{s}}\mathtt{T}, \mathtt{h}, {}^{\mathtt{r}}\Gamma)$
We know from III_0 that $\mathtt{h}' \vdash \iota_0 : dyn({}^{\mathtt{s}}\mathtt{N}_0, \mathtt{h}, {}^{\mathtt{r}}\Gamma)$.

From the well-formed heap judgement from Part I, we can deduce that $\mathtt{h}' \vdash \iota : {}^{\mathtt{r}}fType(\iota_0, \mathtt{h}'(\iota_0)\downarrow_1, \mathtt{f})$. The definition of ${}^{\mathtt{r}}fType$ and the type rules give us $\mathtt{h}' \vdash \iota : dyn(fType(\mathtt{C}_0, \mathtt{f}), \iota_0, \mathtt{h}'(\iota_0)\downarrow_1)$.

Now we can apply Lemma 5.1 to arrive at $\mathtt{h}' \vdash \iota : dyn({}^{\mathtt{s}}\mathtt{N}_0 \triangleright fType(\mathtt{C}_0, \mathtt{f}), \mathtt{h}, {}^{\mathtt{r}}\Gamma)$. Finally, with Lemma 5.11 we arrive at the conclusion.

Part IV: $^{s}\mathtt{T}{=}\mathtt{this}_u\ \_\texttt{<}\_\texttt{>} \Rightarrow \iota \in \{^{r}\Gamma(\mathtt{this}), \mathtt{null}_a\}$
The declared type of a field can never have $\mathtt{this}_u$ as main modifier. The type $^{s}\mathtt{T}$ is a supertype of the result of applying $\triangleright$ to this field type and can therefore also never have $\mathtt{this}_u$ as main modifier. □

Case 4: $\mathtt{e} \equiv \mathtt{e}_0.\mathtt{f}{=}\mathtt{e}_2$

We have the assumptions of the theorem:

1. $\mathtt{h\ ok}$
2. $\mathtt{h} \vdash {}^{r}\Gamma : {}^{s}\Gamma$
3. $^{s}\Gamma \vdash \mathtt{e}_0.\mathtt{f}{=}\mathtt{e}_2 : {}^{s}\mathtt{T}$
4. $\mathtt{h}, {}^{r}\Gamma, \mathtt{e}_0.\mathtt{f}{=}\mathtt{e}_2 \rightsquigarrow \mathtt{h}', \iota$

From 3. and Lemma 5.14 we get:

$$
\begin{array}{ll}
^{s}\Gamma \vdash \mathtt{e}_0 : {}^{s}\mathtt{N}_0 & ^{s}\mathtt{N}_0 = \mathtt{u}_0\ \mathtt{C}_0\texttt{<}\_\texttt{>} \\
^{s}\mathtt{T}_1 = {}^{s}\!fType(\mathtt{C}_0, \mathtt{f}) & ^{s}\Gamma \vdash \mathtt{e}_2 : {}^{s}\mathtt{N}_0 \triangleright {}^{s}\mathtt{T}_1 \\
\mathtt{u}_0 \neq \mathtt{any} & rp(\mathtt{u}_0, {}^{s}\mathtt{T}_1) \\
^{s}\Gamma \vdash \mathtt{e}_0.\mathtt{f}{=}\mathtt{e}_2 : {}^{s}\mathtt{N}_0 \triangleright {}^{s}\mathtt{T}_1 & ^{s}\Gamma \vdash {}^{s}\mathtt{N}_0 \triangleright {}^{s}\mathtt{T}_1 <: {}^{s}\mathtt{T}
\end{array}
$$

From the operational semantics we know:

$$
\begin{array}{ll}
\mathtt{h}, {}^{r}\Gamma, \mathtt{e}_0 \rightsquigarrow \mathtt{h}_0, \iota_0 & \iota_0 \neq \mathtt{null}_a \\
\mathtt{h}_0, {}^{r}\Gamma, \mathtt{e}_2 \rightsquigarrow \mathtt{h}_2, \iota & \mathtt{h}' = \mathtt{h}_2[\iota_0.\mathtt{f} := \iota] \\
\mathtt{h}, {}^{r}\Gamma, \mathtt{e}_0.\mathtt{f}{=}\mathtt{e}_2 \rightsquigarrow \mathtt{h}', \iota &
\end{array}
$$

We apply the induction hypothesis to $\mathtt{e}_0$:

$$
\left.\begin{array}{ll}
1_0. & \mathtt{h\ ok} \\
2_0. & \mathtt{h} \vdash {}^{r}\Gamma : {}^{s}\Gamma \\
3_0. & ^{s}\Gamma \vdash \mathtt{e}_0 : {}^{s}\mathtt{N}_0 \\
4_0. & \mathtt{h}, {}^{r}\Gamma, \mathtt{e}_0 \rightsquigarrow \mathtt{h}_0, \iota_0
\end{array}\right\} \Longrightarrow \left\{\begin{array}{ll}
I_0. & \mathtt{h}_0\ \mathtt{ok} \\
II_0. & \mathtt{h}_0 \vdash {}^{r}\Gamma : {}^{s}\Gamma \\
III_0. & \mathtt{h}_0 \vdash \iota_0 : dyn({}^{s}\mathtt{N}_0, \mathtt{h}, {}^{r}\Gamma) \\
IV_0. & ^{s}\mathtt{N}_0 = \mathtt{this}_u\ \_\texttt{<}\_\texttt{>} \Rightarrow \iota_0 \in \{^{r}\Gamma(\mathtt{this}), \mathtt{null}_a\}
\end{array}\right.
$$

1_0. and 2_0. correspond to 1. and 2. 3_0. is from the type rules and 4_0. is from the operational semantics.

We apply the induction hypothesis to $\mathtt{e}_2$:

$$
\left.\begin{array}{ll}
1_2. & \mathtt{h}_0\ \mathtt{ok} \\
2_2. & \mathtt{h}_0 \vdash {}^{r}\Gamma : {}^{s}\Gamma \\
3_2. & ^{s}\Gamma \vdash \mathtt{e}_2 : {}^{s}\mathtt{N}_0 \triangleright {}^{s}\mathtt{T}_1 \\
4_2. & \mathtt{h}_0, {}^{r}\Gamma, \mathtt{e}_2 \rightsquigarrow \mathtt{h}_2, \iota
\end{array}\right\} \Longrightarrow \left\{\begin{array}{ll}
I_2. & \mathtt{h}_2\ \mathtt{ok} \\
II_2. & \mathtt{h}_2 \vdash {}^{r}\Gamma : {}^{s}\Gamma \\
III_2. & \mathtt{h}_2 \vdash \iota : dyn({}^{s}\mathtt{N}_0 \triangleright {}^{s}\mathtt{T}_1, \mathtt{h}_0, {}^{r}\Gamma) \\
IV_2. & ^{s}\mathtt{N}_0 \triangleright {}^{s}\mathtt{T}_1 = \mathtt{this}_u\ \_\texttt{<}\_\texttt{>} \Rightarrow \iota \in \{^{r}\Gamma(\mathtt{this}), \mathtt{null}_a\}
\end{array}\right.
$$

1_2. is I_0. and 2_2. is II_0. 3_2. is from the type rules and 4_2. is from the operational semantics.

Part I: $\mathtt{h}'\ \mathtt{ok}$
From I_2. we have $\mathtt{h}_2\ \mathtt{ok}$. We have $\mathtt{h}' = \mathtt{h}_2[\iota_0.\mathtt{f} := \iota]$.

From the definition of well-formed heap, we see that it remains to show that $\mathtt{h}' \vdash \iota : {}^{r}\!fType(\iota_0, \mathtt{h}'(\iota_0)\downarrow_1, \mathtt{f})$. Let us give a name to the runtime type of the target and to the class of that object: ${}^{r}\mathtt{T}_0 = \mathtt{h}'(\iota_0)\downarrow_1 = \_\ \mathtt{C}_R\mathtt{<\_>}$. From the definition of ${}^{r}\mathit{fType}(\iota_0, {}^{r}\mathtt{T}_0, \mathtt{f})$ and from the type rules we get:

$$\begin{array}{ll}
{}^{s}\mathtt{T}_1 = {}^{s}\mathit{fType}(\mathtt{C}_0, \mathtt{f}) & \text{type rules} \\
\mathtt{C}_R \sqsubseteq \mathtt{C}_0 & \text{from III}_0\text{., RT-4, RT-1, dyn.} \\
{}^{r}\mathit{fType}(\iota_0, {}^{r}\mathtt{T}_0, \mathtt{f}) = \mathit{dyn}({}^{s}\mathtt{T}_1, \iota_0, {}^{r}\mathtt{T}_0) & \text{definition } {}^{r}\mathit{fType}
\end{array}$$

It remains to show that $\mathtt{h}' \vdash \iota : \mathit{dyn}({}^{s}\mathtt{T}_1, \iota_0, {}^{r}\mathtt{T}_0)$.

We have: III_0. $\mathtt{h}_0 \vdash \iota_0 : \mathit{dyn}({}^{s}\mathtt{N}_0, \mathtt{h}, {}^{r}\Gamma)$ and III_2. $\mathtt{h}_2 \vdash \iota : \mathit{dyn}({}^{s}\mathtt{N}_0 \rhd {}^{s}\mathtt{T}_1, \mathtt{h}_0, {}^{r}\Gamma)$. Because of Lemma 5.12 both of these also apply to $\mathtt{h}'$ instead of $\mathtt{h}_0$, $\mathtt{h}$, or $\mathtt{h}_2$. Together with 1. and what we have from the type rules, this allows us to apply Lemma 5.2 to arrive at our conclusion.

Part II: $\mathtt{h}' \vdash {}^{r}\Gamma : {}^{s}\Gamma$
From II_2. we have $\mathtt{h}_2 \vdash {}^{r}\Gamma : {}^{s}\Gamma$. We have $\mathtt{h}' = \mathtt{h}_2[\iota_0.\mathtt{f} := \iota]$.

Because only the field value is changed (and Lemma 5.12 in general) we get $\mathtt{h}' \vdash {}^{r}\Gamma : {}^{s}\Gamma$.

Part III: $\mathtt{h}' \vdash \iota : \mathit{dyn}({}^{s}\mathtt{T}, \mathtt{h}, {}^{r}\Gamma)$
We have ${}^{s}\Gamma \vdash {}^{s}\mathtt{N}_0 \rhd {}^{s}\mathtt{T}_1 <: {}^{s}\mathtt{T}$. We first show that $\mathtt{h}' \vdash \iota : \mathit{dyn}({}^{s}\mathtt{N}_0 \rhd {}^{s}\mathtt{T}_1, \mathtt{h}, {}^{r}\Gamma)$ and then use Lemma 5.11 to arrive at the conclusion.

Because of Lemma 5.12 the result from III_2., which uses $\mathtt{h}_2$, also applies to $\mathtt{h}'$. Thus, we have: $\mathtt{h}' \vdash \iota : \mathit{dyn}({}^{s}\mathtt{N}_0 \rhd {}^{s}\mathtt{T}_1, \mathtt{h}_0, {}^{r}\Gamma)$ We already have 2., the well-formedness of the runtime environment. We know that evaluation preserves types and know that *dyn* only uses the runtime type of the current object in the environment. Therefore, the judgement still holds if the second argument of *dyn* is $\mathtt{h}$ instead of $\mathtt{h}_0$. We thus arrive at $\mathtt{h}' \vdash \iota : \mathit{dyn}({}^{s}\mathtt{N}_0 \rhd {}^{s}\mathtt{T}_1, \mathtt{h}, {}^{r}\Gamma)$.

Part IV: ${}^{s}\mathtt{T}{=}\mathtt{this}_u\ \_\mathtt{<\_>} \Rightarrow \iota \in \{{}^{r}\Gamma(\mathtt{this}), \mathtt{null}_a\}$
We have ${}^{s}\Gamma \vdash {}^{s}\mathtt{N}_0 \rhd {}^{s}\mathtt{T}_1 <: {}^{s}\mathtt{T}$; ${}^{s}\mathtt{T}_1$ is the declared field type and can not have $\mathtt{this}_u$ as main modifier. Therefore, also the result of the combination can not have the $\mathtt{this}_u$ main modifier. Finally, also no supertype thereof can have the $\mathtt{this}_u$ main modifier. □

Case 5: $\mathtt{e} \equiv \mathtt{e}_0.\mathtt{m}\mathtt{<}\overline{{}^{s}\mathtt{T}}\mathtt{>}(\mathtt{e}_2)$

For simplicity, we assume there is only one method argument.
We have the assumptions of the theorem:

1. $\mathtt{h\ ok}$	2. $\mathtt{h} \vdash {}^{r}\Gamma : {}^{s}\Gamma$
3. ${}^{s}\Gamma \vdash \mathtt{e}_0.\mathtt{m}\mathtt{<}\overline{{}^{s}\mathtt{T}}\mathtt{>}(\mathtt{e}_2) : {}^{s}\mathtt{T}$	4. $\mathtt{h}, {}^{r}\Gamma, \mathtt{e}_0.\mathtt{m}\mathtt{<}\overline{{}^{s}\mathtt{T}}\mathtt{>}(\mathtt{e}_2) \rightsquigarrow \mathtt{h}', \iota$

From 3. and Lemma 5.14 we get:

$$\begin{array}{ll}
{}^{s}\Gamma \vdash \mathtt{e}_0 : {}^{s}\mathtt{N}_0 & {}^{s}\mathtt{N}_0 = \mathtt{u}_0\ \mathtt{C}_{0S}\mathtt{<_>} \\
\mathit{mType}(\mathtt{C}_{0S}, \mathtt{m}) = \mathtt{<}\overline{\mathtt{X}_m\ {}^{s}\mathtt{N}_{bS}}\mathtt{>}\ \mathtt{w}\ {}^{s}\mathtt{T}_{rS}\ \mathtt{m}(\mathtt{x}\ {}^{s}\mathtt{T}_{pS}) & \\
{}^{s}\Gamma \vdash \overline{{}^{s}\mathtt{T}} <: ({}^{s}\mathtt{N}_0 \rhd \overline{{}^{s}\mathtt{N}_{bS}})[\overline{{}^{s}\mathtt{T}/\mathtt{X}_m}] & {}^{s}\Gamma \vdash \mathtt{e}_2 : ({}^{s}\mathtt{N}_0 \rhd {}^{s}\mathtt{T}_{pS})[\overline{{}^{s}\mathtt{T}/\mathtt{X}_m}] \\
\mathtt{u}_0{=}\mathtt{any} \Rightarrow \mathtt{w}{=}\mathtt{pure} & \mathit{rp}(\mathtt{u}_0, \overline{{}^{s}\mathtt{T}_{pS} \circ {}^{s}\mathtt{N}_{bS}}) \\
{}^{s}\Gamma \vdash \mathtt{e}_0.\mathtt{m}\mathtt{<}\overline{{}^{s}\mathtt{T}}\mathtt{>}(\mathtt{e}_2) : ({}^{s}\mathtt{N}_0 \rhd {}^{s}\mathtt{T}_{rS})[\overline{{}^{s}\mathtt{T}/\mathtt{X}_m}] & {}^{s}\Gamma \vdash ({}^{s}\mathtt{N}_0 \rhd {}^{s}\mathtt{T}_{rS})[\overline{{}^{s}\mathtt{T}/\mathtt{X}_m}] <: {}^{s}\mathtt{T}
\end{array}$$

From the operational semantics we know:

$$\begin{array}{lll}
\mathtt{h}, {}^{\mathtt{r}}\Gamma, \mathtt{e}_0 \rightsquigarrow \mathtt{h}_0, \iota_0 & \iota_0 \neq \mathtt{null}_a & \mathtt{h}_0, {}^{\mathtt{r}}\Gamma, \mathtt{e}_2 \rightsquigarrow \mathtt{h}_2, \iota_2 \\
\mathtt{h}_0(\iota_0)\downarrow_1 = _\ \mathtt{C}_{0R}\texttt{<_>} & mBody(\mathtt{C}_{0R}, \mathtt{m}) = (\mathtt{e}_1, \mathtt{x}, \overline{\mathtt{X}_m}) & \\
\overline{{}^{\mathtt{r}}\mathtt{T}} = dyn(\overline{{}^{\mathtt{s}}\mathtt{T}}, \mathtt{h}, {}^{\mathtt{r}}\Gamma) & {}^{\mathtt{r}}\Gamma' = \overline{\mathtt{X}_m\ {}^{\mathtt{r}}\mathtt{T}}\ ;\ \mathtt{this}\ \iota_0, \mathtt{x}\ \iota_2 & \\
\mathtt{h}_2, {}^{\mathtt{r}}\Gamma', \mathtt{e}_1 \rightsquigarrow \mathtt{h}', \iota & \mathtt{h}, {}^{\mathtt{r}}\Gamma, \mathtt{e}_0.\mathtt{m}\texttt{<}\overline{{}^{\mathtt{s}}\mathtt{T}}\texttt{>}(\mathtt{e}_2) \rightsquigarrow \mathtt{h}', \iota &
\end{array}$$

For a method call, we have to distinguish three different classes for the receiver type.

Statically, we know that the receiver has class $\mathtt{C}_{0\mathtt{S}}\texttt{<}\overline{\mathtt{X}_\mathtt{S}\ {}^{\mathtt{s}}\mathtt{N}_\mathtt{S}}\texttt{>}$ and we have the signature $mType(\mathtt{C}_{0S}, \mathtt{m}) = \texttt{<}\overline{\mathtt{X}_m\ {}^{\mathtt{s}}\mathtt{N}_{bS}}\texttt{>}\ \mathtt{w}\ {}^{\mathtt{s}}\mathtt{T}_{rS}\ \mathtt{m}(\mathtt{x}\ {}^{\mathtt{s}}\mathtt{T}_{pS})$.

At runtime, the receiver object ι_0 has class $\mathtt{C}_{0\mathtt{R}}\texttt{<}\overline{\mathtt{X}_\mathtt{R}\ {}^{\mathtt{s}}\mathtt{N}_\mathtt{R}}\texttt{>}$ and we have the signature $mType(\mathtt{C}_{0R}, \mathtt{m}) = \texttt{<}\overline{\mathtt{X}_m\ {}^{\mathtt{s}}\mathtt{N}_{bR}}\texttt{>}\ \mathtt{w}\ {}^{\mathtt{s}}\mathtt{T}_{rR}\ \mathtt{m}(\mathtt{x}\ {}^{\mathtt{s}}\mathtt{T}_{pR})$ and the method body $mBody(\mathtt{C}_{0R}, \mathtt{m}) = (\mathtt{e}_1, \mathtt{x}, \overline{\mathtt{X}_m})$.

The third class to consider is the class $\mathtt{C}_{0\mathtt{C}}\texttt{<}\overline{\mathtt{X}_\mathtt{C}\ {}^{\mathtt{s}}\mathtt{N}_\mathtt{C}}\texttt{>}$ which contains the most concrete implementation of the method. Here we have the signature
$mType(\mathtt{C}_{0C}, \mathtt{m}) = \texttt{<}\overline{\mathtt{X}_m\ {}^{\mathtt{s}}\mathtt{N}_{bC}}\texttt{>}\ \mathtt{w}\ {}^{\mathtt{s}}\mathtt{T}_{rC}\ \mathtt{m}(\mathtt{x}\ {}^{\mathtt{s}}\mathtt{T}_{pC})$ and the method body $mBody(\mathtt{C}_{0C}, \mathtt{m}) = (\mathtt{e}_1, \mathtt{x}, \overline{\mathtt{X}_m})$. This class is important, because we take the method body from this class and execute it. Note that the method body returned for class $\mathtt{C}_{0R}$ is the same as the one for $\mathtt{C}_{0C}$.

The three classes are in the following subclass relationship, disregarding the concrete type arguments: $\mathtt{C}_{0R} \sqsubseteq \mathtt{C}_{0C} \sqsubseteq \mathtt{C}_{0S}$.

We use the subscripts R, C, and S to distinguish the level we are talking about. Note that the name of the method type variables and method parameters are the same on all three levels.

We apply the induction hypothesis to $\mathtt{e}_0$:

$$\left.\begin{array}{ll}
1_0. & \mathtt{h}\ \mathtt{ok} \\
2_0. & \mathtt{h} \vdash {}^{\mathtt{r}}\Gamma : {}^{\mathtt{s}}\Gamma \\
3_0. & {}^{\mathtt{s}}\Gamma \vdash \mathtt{e}_0 : {}^{\mathtt{s}}\mathtt{N}_0 \\
4_0. & \mathtt{h}, {}^{\mathtt{r}}\Gamma, \mathtt{e}_0 \rightsquigarrow \mathtt{h}_0, \iota_0
\end{array}\right\} \Longrightarrow \left\{\begin{array}{ll}
I_0. & \mathtt{h}_0\ \mathtt{ok} \\
II_0. & \mathtt{h}_0 \vdash {}^{\mathtt{r}}\Gamma : {}^{\mathtt{s}}\Gamma \\
III_0. & \mathtt{h}_0 \vdash \iota_0 : dyn({}^{\mathtt{s}}\mathtt{N}_0, \mathtt{h}, {}^{\mathtt{r}}\Gamma) \\
IV_0. & {}^{\mathtt{s}}\mathtt{N}_0 = \mathtt{this}_u\ _\texttt{<_>} \Rightarrow \iota_0 \in \{{}^{\mathtt{r}}\Gamma(\mathtt{this}), \mathtt{null}_a\}
\end{array}\right.$$

1_0. and 2_0. correspond to 1. and 2. 3_0. is from the type rules and 4_0. is from the operational semantics.

We apply the induction hypothesis to $\mathtt{e}_2$ (we call ${}^{\mathtt{s}}\mathtt{T}_a = ({}^{\mathtt{s}}\mathtt{N}_0 \rhd {}^{\mathtt{s}}\mathtt{T}_{pS})[\overline{{}^{\mathtt{s}}\mathtt{T}/\mathtt{X}_m}]$):

$$\left.\begin{array}{ll}
1_2. & \mathtt{h}_0\ \mathtt{ok} \\
2_2. & \mathtt{h}_0 \vdash {}^{\mathtt{r}}\Gamma : {}^{\mathtt{s}}\Gamma \\
3_2. & {}^{\mathtt{s}}\Gamma \vdash \mathtt{e}_2 : {}^{\mathtt{s}}\mathtt{T}_a \\
4_2. & \mathtt{h}_0, {}^{\mathtt{r}}\Gamma, \mathtt{e}_2 \rightsquigarrow \mathtt{h}_2, \iota_2
\end{array}\right\} \Longrightarrow \left\{\begin{array}{ll}
I_2. & \mathtt{h}_2\ \mathtt{ok} \\
II_2. & \mathtt{h}_2 \vdash {}^{\mathtt{r}}\Gamma : {}^{\mathtt{s}}\Gamma \\
III_2. & \mathtt{h}_2 \vdash \iota_2 : dyn({}^{\mathtt{s}}\mathtt{T}_a, \mathtt{h}_0, {}^{\mathtt{r}}\Gamma) \\
IV_2. & {}^{\mathtt{s}}\mathtt{T}_a = \mathtt{this}_u\ _\texttt{<_>} \Rightarrow \iota_2 \in \{{}^{\mathtt{r}}\Gamma(\mathtt{this}), \mathtt{null}_a\}
\end{array}\right.$$

1_2. is I_0. and 2_2. is II_0. 3_2. is from the type rules and 4_2. is from the operational semantics.

Finally, we apply the induction hypothesis to $\mathtt{e}_1$:

$$\left.\begin{array}{ll}
1_1. & \mathtt{h}_2\ \mathtt{ok} \\
2_1. & \mathtt{h}_2 \vdash {}^{\mathtt{r}}\Gamma' : {}^{\mathtt{s}}\Gamma_C \\
3_1. & {}^{\mathtt{s}}\Gamma_C \vdash \mathtt{e}_1 : {}^{\mathtt{s}}\mathtt{T}_{rC} \\
4_1. & \mathtt{h}_2, {}^{\mathtt{r}}\Gamma', \mathtt{e}_1 \rightsquigarrow \mathtt{h}', \iota
\end{array}\right\} \Longrightarrow \left\{\begin{array}{ll}
I_1. & \mathtt{h}'\ \mathtt{ok} \\
II_1. & \mathtt{h}' \vdash {}^{\mathtt{r}}\Gamma' : {}^{\mathtt{s}}\Gamma_C \\
III_1. & \mathtt{h}' \vdash \iota : dyn({}^{\mathtt{s}}\mathtt{T}_{rC}, \mathtt{h}_2, {}^{\mathtt{r}}\Gamma') \\
IV_1. & {}^{\mathtt{s}}\mathtt{T}_{rC} = \mathtt{this}_u\ _\texttt{<_>} \Rightarrow \iota \in \{{}^{\mathtt{r}}\Gamma'(\mathtt{this}), \mathtt{null}_a\}
\end{array}\right.$$

We get 1_1. from I_2. and 4_1. is from the operational semantics. The other two requirements need more work and are developed next.

As static environment we use ${}^s\Gamma_C$ the environment that was used for checking the the well-formedness of method `m` (see rule WFM-1).

Requirement 3_1. ${}^s\Gamma_C \vdash \mathtt{e}_1 : {}^s\mathtt{T}_{rC}$ We know that the program was type checked; especially, WFM-1 was used to check well-formedness of the method in class $\mathtt{C}_{0C}$. The environment that was used is ${}^s\Gamma_C = \overline{\mathtt{X}_m\ {}^s\mathtt{N}_{bC}}, \overline{\mathtt{X}_C\ {}^s\mathtt{N}_C}; \mathtt{this}\ (\mathtt{this}_u\ \mathtt{C}_{0C}\texttt{<}\overline{\mathtt{X}_C}\texttt{>}), \mathtt{x}\ {}^s\mathtt{T}_{pC}$. This environment was used to check that the method body can be typed with the declared return type ${}^s\Gamma_C \vdash \mathtt{e}_1 : {}^s\mathtt{T}_{rC}$.

Requirement 2_1. $\mathtt{h}_2 \vdash {}^r\Gamma' : {}^s\Gamma_C$ From the semantics we have ${}^r\Gamma' = \overline{\mathtt{X}_m\ {}^r\mathtt{T}}\ ;\ \mathtt{this}\ \iota_0, \mathtt{x}\ \iota_2$.

For WFRE we have to show:

$$\begin{array}{lll} \mathtt{h}_2\ \mathtt{ok} & {}^s\Gamma_C\ \mathtt{ok} & \iota_0 \vdash \overline{{}^r\mathtt{T}}\ \mathtt{ok} \\ \iota_0 \vdash \overline{{}^r\mathtt{T}\ {}^r\texttt{<:}\ dyn({}^s\mathtt{N}_{bC}, \mathtt{h}_2, {}^r\Gamma')} & & \mathtt{h}_2 \vdash \iota_0 :\ dyn(\mathtt{this}_u\ \mathtt{C}_{0C}\texttt{<}\overline{\mathtt{X}_C}\texttt{>}, \mathtt{h}_2, {}^r\Gamma') \\ \mathtt{h}_2 \vdash \iota_2 :\ dyn({}^s\mathtt{T}_{pC}, \mathtt{h}_2, {}^r\Gamma') & & \end{array}$$

These follow from I_2., III_0., III_2., the knowledge we have from the type checks and the operational semantics, and corresponding applications of the viewpoint adaptation lemma.

Part I: $\mathtt{h}'\ \mathtt{ok}$
We get this from I_1.

Part II: $\mathtt{h}' \vdash {}^r\Gamma : {}^s\Gamma$
We had II_2. $\mathtt{h}_2 \vdash {}^r\Gamma : {}^s\Gamma$. The evaluation of $\mathtt{e}_1$ does not change the runtime types in the heap, see Lemma 5.12. So the environments are still well formed.

Part III: $\mathtt{h}' \vdash \iota : dyn({}^s\mathtt{T}, \mathtt{h}, {}^r\Gamma)$
We assume show that $\mathtt{h}' \vdash \iota : dyn(({}^s\mathtt{N}_0 \triangleright {}^s\mathtt{T}_{rS})[\overline{{}^s\mathtt{T}/\mathtt{X}_m}], \mathtt{h}, {}^r\Gamma)$ and then apply Lemma 5.11 to arrive at our conclusion.

Above we have shown III_1. $\mathtt{h}' \vdash \iota : dyn({}^s\mathtt{T}_{rC}, \mathtt{h}_2, {}^r\Gamma')$ in which we can expand the usage of dyn to $\mathtt{h}' \vdash \iota : dyn({}^s\mathtt{T}_{rC}, \iota_0, \mathtt{h}_2, {}^r\Gamma')$.

We also have shown III_0. $\mathtt{h}_0 \vdash \iota_0 : dyn({}^s\mathtt{N}_0, \mathtt{h}, {}^r\Gamma)$.

Now we are ready to use Lemma 5.1 to arrive at $\mathtt{h}' \vdash \iota : dyn(({}^s\mathtt{N}_0 \triangleright {}^s\mathtt{T}_{rS})[\overline{{}^s\mathtt{T}/\mathtt{X}_m}], \mathtt{h}, {}^r\Gamma)$. The assumptions of this lemma correspond exactly to the information we just derived.

Part IV: ${}^s\mathtt{T}{=}\mathtt{this}\ \_\texttt{<}\_\texttt{>} \Rightarrow \iota \in \{{}^r\Gamma(\mathtt{this}), \mathtt{null}_a\}$
We have ${}^s\mathtt{T} = ({}^s\mathtt{N}_0 \triangleright {}^s\mathtt{T}_{rS})[\overline{{}^s\mathtt{T}/\mathtt{X}_m}]$. ${}^s\mathtt{T}_{rS}$ is the declared return type and can not have $\mathtt{this}_u$ as main modifier. Therefore, also the result of the combination can not have the $\mathtt{this}_u$ main modifier. □

Case 6: $\mathtt{e} \equiv \mathtt{new}\ {}^s\mathtt{T}'$

We have the assumptions of the theorem:

1. $\mathtt{h}\ \mathtt{ok}$
2. $\mathtt{h} \vdash {}^r\Gamma : {}^s\Gamma$
3. ${}^s\Gamma \vdash \mathtt{new}\ {}^s\mathtt{T}' : {}^s\mathtt{T}$
4. $\mathtt{h}, {}^r\Gamma, \mathtt{new}\ {}^s\mathtt{T}' \rightsquigarrow \mathtt{h}', \iota$

From 3. and Lemma 5.14 we get:

$$ {}^{s}\Gamma \vdash \texttt{new}\ {}^{s}\texttt{T}' : {}^{s}\texttt{T}' \qquad {}^{s}\Gamma \vdash {}^{s}\texttt{T}' <: {}^{s}\texttt{T} $$

From the operational semantics we know:

$$ \begin{array}{lll} \iota \notin \mathrm{dom}(h) & \iota \neq \texttt{null}_a & {}^{r}\texttt{T} = \mathit{dyn}({}^{s}\texttt{T}', \texttt{h}, {}^{r}\Gamma) \\ {}^{r}\texttt{T} = \_\ \texttt{C}\texttt{<}\_\texttt{>} & \texttt{Fs}(\mathit{fields}(\texttt{C})) = \texttt{null}_a & \texttt{h}' = \texttt{h}[\iota \mapsto ({}^{r}\texttt{T}, \texttt{Fs})] \\ \texttt{h}, {}^{r}\Gamma, \texttt{new}\ {}^{s}\texttt{T}' \rightsquigarrow \texttt{h}', \iota & & \end{array} $$

Part I: $\texttt{h}'\ \texttt{ok}$
We extend the heap with an additional object; we therefore have to show that the runtime type of the new object is well-formed, that the newly added address is not $\texttt{null}_a$, and that all field values are well-typed (see WFH, Page 24).

We have to ensure that for the newly added address ι the runtime type is well-formed: $\iota \vdash \texttt{h}'(\iota)\downarrow_1\ \ \texttt{ok}$. From the operational semantics and the definition of *dyn* we know that $\texttt{h}'(\iota)\downarrow_1 = {}^{r}\texttt{T} = \mathit{dyn}({}^{s}\texttt{T}', \texttt{h}, {}^{r}\Gamma) = \mathit{dyn}({}^{s}\texttt{T}', {}^{r}\Gamma(\texttt{this}), {}^{r}\Gamma(\texttt{this})\downarrow_1, {}^{r}\Gamma)$. From the type rules we know that ${}^{s}\texttt{T}'$ is a well-formed type and therefore that all type arguments are subtypes of their upper bounds. We know from 2. that the types in the static and runtime environments are all well-formed. Together with Lemma 5.10 this allows us to show the well-formedness of ${}^{r}\texttt{T}$ (see WFRT, Page 23).

From the operational semantics we know that $\iota \neq \texttt{null}_a$.

From the definition of well-formed heap and RT-5 we see, that an object whose fields are all initialized to $\texttt{null}_a$ is valid in a heap.

From 1. and these observation we can conclude that I. holds.

Part II: $\texttt{h}' \vdash {}^{r}\Gamma : {}^{s}\Gamma$
We have $\texttt{h}' = \texttt{h}[\iota \mapsto ({}^{r}\texttt{T}, \texttt{Fs})]$, where ι is a fresh address. This update can not influence the existing environments, which are well-formed according to 2.

Part III: $\texttt{h}' \vdash \iota : \mathit{dyn}({}^{s}\texttt{T}, \texttt{h}, {}^{r}\Gamma)$
From the operational semantics we know that $\texttt{h}'(\iota) = {}^{r}\texttt{T} = \mathit{dyn}({}^{s}\texttt{T}', \texttt{h}, {}^{r}\Gamma)$. We also have ${}^{s}\Gamma \vdash {}^{s}\texttt{T}' <: {}^{s}\texttt{T}$ and can therefore use Lemma 5.11 to arrive at our conclusion.

Part IV: ${}^{s}\texttt{T}{=}\texttt{this}\ \_\texttt{<}\_\texttt{>} \Rightarrow \iota \in \{{}^{r}\Gamma(\texttt{this}), \texttt{null}_a\}$
The syntax of the language forbids that the main modifier of the static type in a new expression can be $\texttt{this}_u$. □

Case 7: $\texttt{e} \equiv ({}^{s}\texttt{T}')\ \texttt{e}_0$

We have the assumptions of the theorem:

$$ \begin{array}{llll} 1. & \texttt{h ok} & 2. & \texttt{h} \vdash {}^{r}\Gamma : {}^{s}\Gamma \\ 3. & {}^{s}\Gamma \vdash ({}^{s}\texttt{T}')\ \texttt{e}_0 : {}^{s}\texttt{T} & 4. & \texttt{h}, {}^{r}\Gamma, ({}^{s}\texttt{T}')\ \texttt{e}_0 \rightsquigarrow \texttt{h}', \iota \end{array} $$

From 3. and Lemma 5.14 we get:

$$ {}^{s}\Gamma \vdash \texttt{e}_0 : {}^{s}\texttt{N}_0 \qquad {}^{s}\Gamma \vdash ({}^{s}\texttt{T}')\ \texttt{e}_0 : {}^{s}\texttt{T}' \qquad {}^{s}\Gamma \vdash {}^{s}\texttt{T}' <: {}^{s}\texttt{T} $$

From the operational semantics we know:

$$\mathtt{h}, {}^{\mathtt{r}}\Gamma, \mathtt{e}_0 \rightsquigarrow \mathtt{h}', \iota \qquad \mathtt{h}' \vdash \iota : dyn({}^{\mathtt{s}}\mathtt{T}', \mathtt{h}, {}^{\mathtt{r}}\Gamma) \qquad \mathtt{h}, {}^{\mathtt{r}}\Gamma, ({}^{\mathtt{s}}\mathtt{T}')\ \mathtt{e}_0 \rightsquigarrow \mathtt{h}', \iota$$

We apply the induction hypothesis to $\mathtt{e}_0$:

$$\left.\begin{array}{ll} 1_0. & \mathtt{h\ ok} \\ 2_0. & \mathtt{h} \vdash {}^{\mathtt{r}}\Gamma : {}^{\mathtt{s}}\Gamma \\ 3_0. & {}^{\mathtt{s}}\Gamma \vdash \mathtt{e}_0 : {}^{\mathtt{s}}\mathtt{N}_0 \\ 4_0. & \mathtt{h}, {}^{\mathtt{r}}\Gamma, \mathtt{e}_0 \rightsquigarrow \mathtt{h}_0, \iota \end{array}\right\} \Longrightarrow \left\{\begin{array}{ll} I_0. & \mathtt{h}'\ \mathtt{ok} \\ II_0. & \mathtt{h}' \vdash {}^{\mathtt{r}}\Gamma : {}^{\mathtt{s}}\Gamma \\ III_0. & \mathtt{h}' \vdash \iota : dyn({}^{\mathtt{s}}\mathtt{N}_0, \mathtt{h}, {}^{\mathtt{r}}\Gamma) \\ IV_0. & {}^{\mathtt{s}}\mathtt{N}_0 = \mathtt{this}_u\ \_\texttt{<}\_\texttt{>} \Rightarrow \iota_0 \in \{{}^{\mathtt{r}}\Gamma(\mathtt{this}), \mathtt{null}_a\} \end{array}\right.$$

1_0. and 2_0. correspond to 1. and 2. 3_0. is from the type rules and 4_0. is from the operational semantics.

Part I: $\mathtt{h}'\ \mathtt{ok}$
From I_0.

Part II: $\mathtt{h}' \vdash {}^{\mathtt{r}}\Gamma : {}^{\mathtt{s}}\Gamma$
From II_0.

Part III: $\mathtt{h}' \vdash \iota : dyn({}^{\mathtt{s}}\mathtt{T}, \mathtt{h}, {}^{\mathtt{r}}\Gamma)$
From the operational semantic we have: $\mathtt{h}' \vdash \iota : dyn({}^{\mathtt{s}}\mathtt{T}', \mathtt{h}, {}^{\mathtt{r}}\Gamma)$.

We also have $\mathtt{h} \vdash {}^{\mathtt{r}}\Gamma : {}^{\mathtt{s}}\Gamma$ and ${}^{\mathtt{s}}\Gamma \vdash {}^{\mathtt{s}}\mathtt{T}' <: {}^{\mathtt{s}}\mathtt{T}$. These allow us to use Lemma 5.11 to arrive at $\mathtt{h}' \vdash \iota : dyn({}^{\mathtt{s}}\mathtt{T}, \mathtt{h}, {}^{\mathtt{r}}\Gamma)$.

Part IV: ${}^{\mathtt{s}}\mathtt{T}{=}\mathtt{this}\ \_\texttt{<}\_\texttt{>} \Rightarrow \iota \in \{{}^{\mathtt{r}}\Gamma(\mathtt{this}), \mathtt{null}_a\}$
The syntax of the language forbids that the main modifier of the static type in a cast expression can be $\mathtt{this}_u$. □

5.2.2 Proof of Theorem 5.4 — Owner-as-Modifier

The Owner-as-Modifier theorem is:

$$\left.\begin{array}{ll} 1. & \mathtt{h} \vdash {}^{\mathtt{r}}\Gamma : {}^{\mathtt{s}}\Gamma \\ 2. & {}^{\mathtt{s}}\Gamma \vdash \mathtt{e} : {}^{\mathtt{s}}\mathtt{T} \\ 3. & \iota_T = {}^{\mathtt{r}}\Gamma(\mathtt{this}) \\ 4. & \mathtt{h}, {}^{\mathtt{r}}\Gamma, \mathtt{e} \rightsquigarrow \mathtt{h}', \_ \end{array}\right\} \Rightarrow \left\{\begin{array}{ll} \multicolumn{2}{l}{\forall \iota \in \mathrm{dom}(\mathtt{h}), \mathtt{f} :} \\ I. & \mathtt{h}(\iota)\downarrow_2(\mathtt{f}) = \mathtt{h}'\downarrow_2(\mathtt{f}) \vee \\ II. & owner(\mathtt{h}, \iota_T) \in owners(\mathtt{h}, \iota) \end{array}\right.$$

We prove this by induction on the shape of the derivation tree of 4.

Case 1: $\mathtt{e} \equiv \mathtt{x}$

We have the assumptions of the theorem:

$$\begin{array}{llll} 1. & \mathtt{h} \vdash {}^{\mathtt{r}}\Gamma : {}^{\mathtt{s}}\Gamma & 2. & {}^{\mathtt{s}}\Gamma \vdash \mathtt{x} : {}^{\mathtt{s}}\mathtt{T} \\ 3. & \iota_T = {}^{\mathtt{r}}\Gamma(\mathtt{this}) & 4. & \mathtt{h}, {}^{\mathtt{r}}\Gamma, \mathtt{x} \rightsquigarrow \mathtt{h}', \_ \end{array}$$

From 2. and Lemma 5.14 we get:

$$\mathtt{x} \in \mathrm{dom}({}^{\mathtt{s}}\Gamma) \qquad {}^{\mathtt{s}}\Gamma \vdash \mathtt{x} : {}^{\mathtt{s}}\Gamma(\mathtt{x}) \qquad {}^{\mathtt{s}}\Gamma \vdash {}^{\mathtt{s}}\Gamma(\mathtt{x}) <: {}^{\mathtt{s}}\mathtt{T}$$

From the operational semantics we know:

$$\mathtt{h}, {}^{\mathtt{r}}\Gamma, \mathtt{x} \rightsquigarrow \mathtt{h}, {}^{\mathtt{r}}\Gamma(\mathtt{x})$$

The heap does not change, so I. holds. □

Case 2: $\mathtt{e} \equiv \mathtt{null}$

We have the assumptions of the theorem:

1. $\mathtt{h} \vdash {}^{r}\Gamma : {}^{s}\Gamma$	2. ${}^{s}\Gamma \vdash \mathtt{null} : {}^{s}\mathtt{T}$
3. $\iota_T = {}^{r}\Gamma(\mathtt{this})$	4. $\mathtt{h}, {}^{r}\Gamma, \mathtt{null} \rightsquigarrow \mathtt{h}', \_$

From 2. and Lemma 5.14 we get:

$$ {}^{s}\Gamma \vdash \mathtt{null} : {}^{s}\mathtt{T}' \qquad {}^{s}\Gamma \vdash {}^{s}\mathtt{T}' <: {}^{s}\mathtt{T} $$

From the operational semantics we know:

$$ \mathtt{h}, {}^{r}\Gamma, \mathtt{null} \rightsquigarrow \mathtt{h}, \mathtt{null}_a $$

The heap does not change, so I. holds. □

Case 3: $\mathtt{e} \equiv \mathtt{e}_0.\mathtt{f}$

We have the assumptions of the theorem:

1. $\mathtt{h} \vdash {}^{r}\Gamma : {}^{s}\Gamma$	2. ${}^{s}\Gamma \vdash \mathtt{e}_0.\mathtt{f} : {}^{s}\mathtt{T}$
3. $\iota_T = {}^{r}\Gamma(\mathtt{this})$	4. $\mathtt{h}, {}^{r}\Gamma, \mathtt{e}_0.\mathtt{f} \rightsquigarrow \mathtt{h}', \_$

From 2. and Lemma 5.14 we get:

$$ {}^{s}\Gamma \vdash \mathtt{e}_0 : {}^{s}\mathtt{N}_0 \qquad {}^{s}\mathtt{N}_0 = \_\ \mathtt{C}_0\mathtt{<\_>} $$
$$ {}^{s}\Gamma \vdash \mathtt{e}_0.\mathtt{f} : {}^{s}\mathtt{N}_0 \triangleright \mathit{fType}(\mathtt{C}_0, \mathtt{f}) \qquad {}^{s}\Gamma \vdash {}^{s}\mathtt{N}_0 \triangleright \mathit{fType}(\mathtt{C}_0, \mathtt{f}) <: {}^{s}\mathtt{T} $$

From the operational semantics we know:

$$ \mathtt{h}, {}^{r}\Gamma, \mathtt{e}_0 \rightsquigarrow \mathtt{h}', \iota_0 \qquad \iota_0 \neq \mathtt{null}_a $$
$$ \iota = \mathtt{h}'(\iota_0)\downarrow_2(\mathtt{f}) \qquad \mathtt{h}, {}^{r}\Gamma, \mathtt{e}_0.\mathtt{f} \rightsquigarrow \mathtt{h}', \iota $$

We apply the induction hypothesis to $\mathtt{e}_0$:

$$ \left.\begin{array}{ll} 1_0. & \mathtt{h} \vdash {}^{r}\Gamma : {}^{s}\Gamma \\ 2_0. & {}^{s}\Gamma \vdash \mathtt{e}_0 : {}^{s}\mathtt{N}_0 \\ 3_0. & \iota_T = {}^{r}\Gamma(\mathtt{this}) \\ 4_0. & \mathtt{h}, {}^{r}\Gamma, \mathtt{e}_0 \rightsquigarrow \mathtt{h}', \_ \end{array}\right\} \Longrightarrow \left\{\begin{array}{l} \forall \iota \in \mathrm{dom}(\mathtt{h}), \mathtt{f} : \\ \quad \mathtt{h}(\iota)\downarrow_2(\mathtt{f}) = \mathtt{h}'\downarrow_2(\mathtt{f}) \lor \\ \quad \mathit{owner}(\mathtt{h}, \iota_T) \in \mathit{owners}(\mathtt{h}, \iota) \end{array}\right. $$

1_0. and 3_0. correspond to 1. and 3. 2_0. is from the type rules and 4_0. is from the operational semantics.

This is already the final heap. □

Case 4: $\texttt{e} \equiv \texttt{e}_0\texttt{.f=e}_2$

We have the assumptions of the theorem:

1. $\texttt{h} \vdash {}^{\texttt{r}}\Gamma : {}^{\texttt{s}}\Gamma$	2. ${}^{\texttt{s}}\Gamma \vdash \texttt{e}_0\texttt{.f=e}_2 : {}^{\texttt{s}}\texttt{T}$
3. $\iota_T = {}^{\texttt{r}}\Gamma(\texttt{this})$	4. $\texttt{h}, {}^{\texttt{r}}\Gamma, \texttt{e}_0\texttt{.f=e}_2 \rightsquigarrow \texttt{h}', \_$

From 2. and Lemma 5.14 we get:

$$\begin{array}{ll} {}^{\texttt{s}}\Gamma \vdash \texttt{e}_0 : {}^{\texttt{s}}\texttt{N}_0 & {}^{\texttt{s}}\texttt{N}_0 = \texttt{u}_0\ \texttt{C}_0\texttt{<\_>} \\ {}^{\texttt{s}}\texttt{T}_1 = {}^{s}\!fType(\texttt{C}_0, \texttt{f}) & {}^{\texttt{s}}\Gamma \vdash \texttt{e}_2 : {}^{\texttt{s}}\texttt{N}_0 \triangleright {}^{\texttt{s}}\texttt{T}_1 \\ \texttt{u}_0 \neq \texttt{any} & rp(\texttt{u}_0, {}^{\texttt{s}}\texttt{T}_1) \\ {}^{\texttt{s}}\Gamma \vdash \texttt{e}_0\texttt{.f=e}_2 : {}^{\texttt{s}}\texttt{N}_0 \triangleright {}^{\texttt{s}}\texttt{T}_1 & {}^{\texttt{s}}\Gamma \vdash {}^{\texttt{s}}\texttt{N}_0 \triangleright {}^{\texttt{s}}\texttt{T}_1 <: {}^{\texttt{s}}\texttt{T} \end{array}$$

From the operational semantics we know:

$$\begin{array}{ll} \texttt{h}, {}^{\texttt{r}}\Gamma, \texttt{e}_0 \rightsquigarrow \texttt{h}_0, \iota_0 & \iota_0 \neq \texttt{null}_a \\ \texttt{h}_0, {}^{\texttt{r}}\Gamma, \texttt{e}_2 \rightsquigarrow \texttt{h}_2, \iota & \texttt{h}' = \texttt{h}_2[\iota_0.\texttt{f} := \iota] \\ \texttt{h}, {}^{\texttt{r}}\Gamma, \texttt{e}_0\texttt{.f=e}_2 \rightsquigarrow \texttt{h}', \iota & \end{array}$$

We apply the induction hypothesis to $\texttt{e}_0$:

$$\left.\begin{array}{ll} 1_0. & \texttt{h} \vdash {}^{\texttt{r}}\Gamma : {}^{\texttt{s}}\Gamma \\ 2_0. & {}^{\texttt{s}}\Gamma \vdash \texttt{e}_0 : {}^{\texttt{s}}\texttt{N}_0 \\ 3_0. & \iota_T = {}^{\texttt{r}}\Gamma(\texttt{this}) \\ 4_0. & \texttt{h}, {}^{\texttt{r}}\Gamma, \texttt{e}_0 \rightsquigarrow \texttt{h}_0, \_ \end{array}\right\} \Longrightarrow \left\{\begin{array}{l} \forall \iota \in \mathrm{dom}(\texttt{h}), \texttt{f} : \\ \quad \texttt{h}(\iota)\downarrow_2(\texttt{f}) = \texttt{h}_0\downarrow_2(\texttt{f}) \vee \\ \quad owner(\texttt{h}, \iota_T) \in owners(\texttt{h}, \iota) \end{array}\right.$$

1_0. and 3_0. correspond to 1. and 3. 2_0. is from the type rules and 4_0. is from the operational semantics.

We apply the induction hypothesis to $\texttt{e}_2$:

$$\left.\begin{array}{ll} 1_0. & \texttt{h}_0 \vdash {}^{\texttt{r}}\Gamma : {}^{\texttt{s}}\Gamma \\ 2_0. & {}^{\texttt{s}}\Gamma \vdash \texttt{e}_2 : {}^{\texttt{s}}\texttt{N}_0 \triangleright {}^{\texttt{s}}\texttt{T}_1 \\ 3_0. & \iota_T = {}^{\texttt{r}}\Gamma(\texttt{this}) \\ 4_0. & \texttt{h}_0, {}^{\texttt{r}}\Gamma, \texttt{e}_2 \rightsquigarrow \texttt{h}_2, \_ \end{array}\right\} \Longrightarrow \left\{\begin{array}{l} \forall \iota \in \mathrm{dom}(\texttt{h}_0), \texttt{f} : \\ \quad \texttt{h}_0(\iota)\downarrow_2(\texttt{f}) = \texttt{h}_2\downarrow_2(\texttt{f}) \vee \\ \quad owner(\texttt{h}_0, \iota_T) \in owners(\texttt{h}_0, \iota) \end{array}\right.$$

1_0. and 3_0. correspond to 1. and 3. 2_0. is from the type rules and 4_0. is from the operational semantics.

To receive at the final heap we have to consider $\texttt{h}' = \texttt{h}_2[\iota_0.\texttt{f} := \iota]$. In general, this will change the value of the field $\texttt{f}$ of object ι_0. Therefore, we need to show that $owner(\texttt{h}, \iota_T) \in owners(\texttt{h}_2, \iota_0)$.

From the type rules we know that $\texttt{u}_0 \neq \texttt{any}$, i.e. the ownership modifier is either `this`, `peer`, or `rep`. From the proof of Theorem 5.3 we know that $\texttt{h}_0 \vdash \iota_0 : dyn({}^{\texttt{s}}\texttt{N}_0, \texttt{h}, {}^{\texttt{r}}\Gamma)$. From the operational semantics we know that $\iota_0 \neq \texttt{null}_a$. Therefore we can apply Lemma 5.13 to gain knowledge of the runtime owners:

1. ${}^{\texttt{s}}\texttt{N}_0 = \texttt{this}_u\ \_\texttt{<\_>} \Rightarrow owner(\texttt{h}_0, \iota_0) = owner(\texttt{h}_0, \iota_T)$

2. ${}^{s}\mathtt{N}_0 = \mathtt{peer}\ \_\texttt{<}\_\texttt{>} \Rightarrow \mathit{owner}(\mathtt{h}_0, \iota_0) = \mathit{owner}(\mathtt{h}_0, \iota_T)$

3. ${}^{s}\mathtt{N}_0 = \mathtt{rep}\ \_\texttt{<}\_\texttt{>} \Rightarrow \mathit{owner}(\mathtt{h}_0, \iota_0) = \iota_T$

From the type rules we know that the main modifier is not $\mathtt{any}_u$. That is, the owner of ι_0 is either ι_T, or the owner of ι_T. Therefore, $\mathit{owner}(\mathtt{h}, \iota_T) \in \mathit{owners}(\mathtt{h}_2, \iota_0)$ holds. □

Case 5: $\mathtt{e} \equiv \mathtt{e}_0.\mathtt{m}\texttt{<}\overline{{}^{s}\mathtt{T}}\texttt{>}(\mathtt{e}_2)$

For simplicity, we assume there is only one method argument and formal parameter.

We have the assumptions of the theorem:

1. $\mathtt{h} \vdash {}^{r}\Gamma : {}^{s}\Gamma$	2. ${}^{s}\Gamma \vdash \mathtt{e}_0.\mathtt{m}\texttt{<}\overline{{}^{s}\mathtt{T}}\texttt{>}(\mathtt{e}_2) : {}^{s}\mathtt{T}$
3. $\iota_T = {}^{r}\Gamma(\mathtt{this})$	4. $\mathtt{h}, {}^{r}\Gamma, \mathtt{e}_0.\mathtt{m}\texttt{<}\overline{{}^{s}\mathtt{T}}\texttt{>}(\mathtt{e}_2) \rightsquigarrow \mathtt{h}', \_$

From 2. and Lemma 5.14 we get:

${}^{s}\Gamma \vdash \mathtt{e}_0 : {}^{s}\mathtt{N}_0$	${}^{s}\mathtt{N}_0 = \mathtt{u}_0\ \mathtt{C}_{0S}\texttt{<}\_\texttt{>}$
$\mathit{mType}(\mathtt{C}_{0S}, \mathtt{m}) = \texttt{<}\overline{\mathtt{X}_m\ {}^{s}\mathtt{N}_{bS}}\texttt{>}\ \mathtt{w}\ {}^{s}\mathtt{T}_{rS}\ \mathtt{m}(\mathtt{x}\ {}^{s}\mathtt{T}_{pS})$	
${}^{s}\Gamma \vdash \overline{{}^{s}\mathtt{T}} <: ({}^{s}\mathtt{N}_0 \triangleright \overline{{}^{s}\mathtt{N}_{bS}})[\overline{{}^{s}\mathtt{T}/\mathtt{X}_m}]$	${}^{s}\Gamma \vdash \mathtt{e}_2 : ({}^{s}\mathtt{N}_0 \triangleright {}^{s}\mathtt{T}_{pS})[\overline{{}^{s}\mathtt{T}/\mathtt{X}_m}]$
$\mathtt{u}_0 = \mathtt{any} \Rightarrow \mathtt{w} = \mathtt{pure}$	$\mathit{rp}(\mathtt{u}_0, \overline{{}^{s}\mathtt{T}_{pS}} \circ \overline{{}^{s}\mathtt{N}_{bS}})$
${}^{s}\Gamma \vdash \mathtt{e}_0.\mathtt{m}\texttt{<}\overline{{}^{s}\mathtt{T}}\texttt{>}(\mathtt{e}_2) : ({}^{s}\mathtt{N}_0 \triangleright {}^{s}\mathtt{T}_{rS})[\overline{{}^{s}\mathtt{T}/\mathtt{X}_m}]$	${}^{s}\Gamma \vdash ({}^{s}\mathtt{N}_0 \triangleright {}^{s}\mathtt{T}_{rS})[\overline{{}^{s}\mathtt{T}/\mathtt{X}_m}] <: {}^{s}\mathtt{T}$

From the operational semantics we know:

$\mathtt{h}, {}^{r}\Gamma, \mathtt{e}_0 \rightsquigarrow \mathtt{h}_0, \iota_0$	$\iota_0 \neq \mathtt{null}_a$	$\mathtt{h}_0, {}^{r}\Gamma, \mathtt{e}_2 \rightsquigarrow \mathtt{h}_2, \iota_2$
$\mathtt{h}_0(\iota_0)\downarrow_1 = \_\ \mathtt{C}_{0R}\texttt{<}\_\texttt{>}$	$\mathit{mBody}(\mathtt{C}_{0R}, \mathtt{m}) = (\mathtt{e}_1, \mathtt{x}, \overline{\mathtt{X}_m})$	
$\overline{{}^{r}\mathtt{T}} = \mathit{dyn}(\overline{{}^{s}\mathtt{T}}, \mathtt{h}, {}^{r}\Gamma)$	${}^{r}\Gamma' = \overline{\mathtt{X}_m\ {}^{r}\mathtt{T}}\ ;\ \mathtt{this}\ \iota_0, \mathtt{x}\ \iota_2$	
$\mathtt{h}_2, {}^{r}\Gamma', \mathtt{e}_1 \rightsquigarrow \mathtt{h}', \iota$	$\mathtt{h}, {}^{r}\Gamma, \mathtt{e}_0.\mathtt{m}\texttt{<}\overline{{}^{s}\mathtt{T}}\texttt{>}(\mathtt{e}_2) \rightsquigarrow \mathtt{h}', \iota$	

We apply the induction hypothesis to $\mathtt{e}_0$:

$$\left.\begin{array}{ll} 1_0. & \mathtt{h} \vdash {}^{r}\Gamma : {}^{s}\Gamma \\ 2_0. & {}^{s}\Gamma \vdash \mathtt{e}_0 : {}^{s}\mathtt{N}_0 \\ 3_0. & \iota_T = {}^{r}\Gamma(\mathtt{this}) \\ 4_0. & \mathtt{h}, {}^{r}\Gamma, \mathtt{e}_0 \rightsquigarrow \mathtt{h}_0, \_ \end{array}\right\} \Longrightarrow \left\{\begin{array}{l} \forall \iota \in \mathrm{dom}(\mathtt{h}), \mathtt{f} : \\ \quad \mathtt{h}(\iota)\downarrow_2(\mathtt{f}) = \mathtt{h}_0\downarrow_2(\mathtt{f}) \lor \\ \quad \mathit{owner}(\mathtt{h}, \iota_T) \in \mathit{owners}(\mathtt{h}, \iota) \end{array}\right.$$

1_0. and 3_0. correspond to 1. and 3. 2_0. is from the type rules and 4_0. is from the operational semantics.

We apply the induction hypothesis to $\mathtt{e}_2$:

$$\left.\begin{array}{ll} 1_0. & \mathtt{h}_0 \vdash {}^{r}\Gamma : {}^{s}\Gamma \\ 2_0. & {}^{s}\Gamma \vdash \mathtt{e}_2 : \_ \\ 3_0. & \iota_T = {}^{r}\Gamma(\mathtt{this}) \\ 4_0. & \mathtt{h}_0, {}^{r}\Gamma, \mathtt{e}_2 \rightsquigarrow \mathtt{h}_2, \_ \end{array}\right\} \Longrightarrow \left\{\begin{array}{l} \forall \iota \in \mathrm{dom}(\mathtt{h}_0), \mathtt{f} : \\ \quad \mathtt{h}_0(\iota)\downarrow_2(\mathtt{f}) = \mathtt{h}_2\downarrow_2(\mathtt{f}) \lor \\ \quad \mathit{owner}(\mathtt{h}_0, \iota_T) \in \mathit{owners}(\mathtt{h}_0, \iota) \end{array}\right.$$

1_0. and 3_0. correspond to 1. and 3. 2_0. is from the type rules and 4_0. is from the operational semantics.

If the method that is called is pure, we know from Lemma 5.5 that the method does not change existing objects in the heap and we are done.

If the method is not pure, we apply the induction hypothesis to $\mathtt{e}_1$:

$$\left.\begin{array}{ll} 1_0. & \mathtt{h}_2 \vdash {}^{\mathtt{r}}\Gamma' : {}^{\mathtt{s}}\Gamma' \\ 2_0. & {}^{\mathtt{s}}\Gamma' \vdash \mathtt{e}_1 : \_ \\ 3_0. & \iota_0 = {}^{\mathtt{r}}\Gamma'(\mathtt{this}) \\ 4_0. & \mathtt{h}_2, {}^{\mathtt{r}}\Gamma', \mathtt{e}_1 \rightsquigarrow \mathtt{h}', \_ \end{array}\right\} \Longrightarrow \left\{\begin{array}{l} \forall \iota \in \mathrm{dom}(\mathtt{h}_2), \mathtt{f} : \\ \quad \mathtt{h}_2(\iota) \downarrow_2 (\mathtt{f}) = \mathtt{h}' \downarrow_2 (\mathtt{f}) \vee \\ \quad \mathit{owner}(\mathtt{h}_2, \iota_0) \in \mathit{owners}(\mathtt{h}_2, \iota) \end{array}\right.$$

We use the runtime environment ${}^{\mathtt{r}}\Gamma'$ constructed in the operational semantics and the static environment ${}^{\mathtt{s}}\Gamma'$ that was used to type-check the body of the method. 1_0. and 2_0. were previously shown in the proof of the soundness Theorem 5.3 (see Page 35 for details). 3_0. and 4_0. are from the operational semantics.

From the type rules we know that if the method is not pure then $\mathtt{u}_0 \neq \mathtt{any}$, i.e. the ownership modifier is either `this`, `peer`, or `rep`. From the proof of Theorem 5.3 we know that $\mathtt{h}_0 \vdash \iota_0 : \mathit{dyn}({}^{\mathtt{s}}\mathtt{N}_0, \mathtt{h}, {}^{\mathtt{r}}\Gamma)$. From the operational semantics we know that $\iota_0 \neq \mathtt{null}_a$. Therefore we can apply Lemma 5.13 to gain knowledge of the runtime owners:

1. ${}^{\mathtt{s}}\mathtt{N}_0 = \mathtt{this}_u \;\_\texttt{<}\_\texttt{>} \Rightarrow \mathit{owner}(\mathtt{h}_0, \iota_0) = \mathit{owner}(\mathtt{h}_0, \iota_T)$
2. ${}^{\mathtt{s}}\mathtt{N}_0 = \mathtt{peer} \;\_\texttt{<}\_\texttt{>} \Rightarrow \mathit{owner}(\mathtt{h}_0, \iota_0) = \mathit{owner}(\mathtt{h}_0, \iota_T)$
3. ${}^{\mathtt{s}}\mathtt{N}_0 = \mathtt{rep} \;\_\texttt{<}\_\texttt{>} \Rightarrow \mathit{owner}(\mathtt{h}_0, \iota_0) = \iota_T$

That is, the owner of ι_0 is either ι_T, or the owner of ι_T. Therefore, $\mathit{owner}(\mathtt{h}, \iota_T) \in \mathit{owners}(\mathtt{h}_2, \iota_0)$ holds. □

Case 6: $\mathtt{e} \equiv \mathtt{new}\ {}^{\mathtt{s}}\mathtt{T}'$

We have the assumptions of the theorem:

$$\begin{array}{llll} 1. & \mathtt{h} \vdash {}^{\mathtt{r}}\Gamma : {}^{\mathtt{s}}\Gamma & 2. & {}^{\mathtt{s}}\Gamma \vdash \mathtt{new}\ {}^{\mathtt{s}}\mathtt{T}' : {}^{\mathtt{s}}\mathtt{T} \\ 3. & \iota_T = {}^{\mathtt{r}}\Gamma(\mathtt{this}) & 4. & \mathtt{h}, {}^{\mathtt{r}}\Gamma, \mathtt{new}\ {}^{\mathtt{s}}\mathtt{T}' \rightsquigarrow \mathtt{h}', \_ \end{array}$$

From 2. and Lemma 5.14 we get:

$${}^{\mathtt{s}}\Gamma \vdash \mathtt{new}\ {}^{\mathtt{s}}\mathtt{T}' : {}^{\mathtt{s}}\mathtt{T}' \qquad {}^{\mathtt{s}}\Gamma \vdash {}^{\mathtt{s}}\mathtt{T}' <: {}^{\mathtt{s}}\mathtt{T}$$

From the operational semantics we know:

$$\begin{array}{lll} \iota \notin \mathrm{dom}(h) & \iota \neq \mathtt{null}_a & {}^{\mathtt{r}}\mathtt{T} = \mathit{dyn}({}^{\mathtt{s}}\mathtt{T}', \mathtt{h}, {}^{\mathtt{r}}\Gamma) \\ {}^{\mathtt{r}}\mathtt{T} = \_\ \mathtt{C}\texttt{<}\_\texttt{>} & \mathtt{Fs}(\mathit{fields}(\mathtt{C})) = \mathtt{null}_a & \mathtt{h}' = \mathtt{h}[\iota \mapsto ({}^{\mathtt{r}}\mathtt{T}, \mathtt{Fs})] \\ \mathtt{h}, {}^{\mathtt{r}}\Gamma, \mathtt{new}\ {}^{\mathtt{s}}\mathtt{T}' \rightsquigarrow \mathtt{h}', \iota & & \end{array}$$

We add an additional object to the heap and do not modify any existing objects. Therefore I. holds. □

Case 7: $\texttt{e} \equiv ({}^{s}\texttt{T}')\ \texttt{e}_0$

We have the assumptions of the theorem:

1. $\texttt{h} \vdash {}^{r}\Gamma : {}^{s}\Gamma$ 2. ${}^{s}\Gamma \vdash ({}^{s}\texttt{T}')\ \texttt{e}_0 : {}^{s}\texttt{T}$
3. $\iota_T = {}^{r}\Gamma(\texttt{this})$ 4. $\texttt{h}, {}^{r}\Gamma, ({}^{s}\texttt{T}')\ \texttt{e}_0 \rightsquigarrow \texttt{h}', \_$

From 2. and Lemma 5.14 we get:

$${}^{s}\Gamma \vdash \texttt{e}_0 : {}^{s}\texttt{N}_0 \qquad {}^{s}\Gamma \vdash ({}^{s}\texttt{T}')\ \texttt{e}_0 : {}^{s}\texttt{T}' \qquad {}^{s}\Gamma \vdash {}^{s}\texttt{T}' <: {}^{s}\texttt{T}$$

From the operational semantics we know:

$$\texttt{h}, {}^{r}\Gamma, \texttt{e}_0 \rightsquigarrow \texttt{h}', \iota \qquad \texttt{h}' \vdash \iota : dyn({}^{s}\texttt{T}', \texttt{h}, {}^{r}\Gamma) \qquad \texttt{h}, {}^{r}\Gamma, ({}^{s}\texttt{T}')\ \texttt{e}_0 \rightsquigarrow \texttt{h}', \iota$$

We apply the induction hypothesis to $\texttt{e}_0$:

$$\left.\begin{array}{ll} 1_0. & \texttt{h} \vdash {}^{r}\Gamma : {}^{s}\Gamma \\ 2_0. & {}^{s}\Gamma \vdash \texttt{e}_0 : {}^{s}\texttt{T} \\ 3_0. & \iota_T = {}^{r}\Gamma(\texttt{this}) \\ 4_0. & \texttt{h}, {}^{r}\Gamma, \texttt{e}_0 \rightsquigarrow \texttt{h}', \_ \end{array}\right\} \Longrightarrow \left\{\begin{array}{l} \forall \iota \in \text{dom}(\texttt{h}), \texttt{f} : \\ \quad \texttt{h}(\iota)\downarrow_2 (\texttt{f}) = \texttt{h}'\downarrow_2 (\texttt{f}) \vee \\ \quad owner(\texttt{h}, \iota_T) \in owners(\texttt{h}, \iota) \end{array}\right.$$

1_0. and 3_0. correspond to 1. and 3. 2_0. is from the type rules and 4_0. is from the operational semantics.

This is already the final heap. □

5.2.3 Proof of Lemma 5.7 — Adaptation to a Viewpoint Helper Lemma

We want to prove:

$$\left.\begin{array}{ll} 1. & \texttt{h} \vdash {}^{r}\Gamma : {}^{s}\Gamma \qquad \texttt{h} \vdash {}^{r}\Gamma' : {}^{s}\Gamma' \\ 2. & \texttt{h} \vdash \iota_1 : dyn({}^{s}\texttt{N}, \texttt{h}, {}^{r}\Gamma) \\ 3. & \texttt{h}(\iota_1)\downarrow_1 = {}^{r}\texttt{T} \\ & {}^{s}\texttt{N} = \texttt{u}_N\ \texttt{C}_N\texttt{<}\overline{{}^{s}\texttt{T}_N}\texttt{>} \\ 4. & \texttt{u}_N = \texttt{this}_u \Rightarrow {}^{r}\Gamma(\texttt{this}) = \iota_1 \\ 5. & \texttt{u}_N \neq \texttt{any} \\ 6. & rp(\texttt{u}_N, {}^{s}\texttt{T}) \\ 7. & \text{free}({}^{s}\texttt{T}) \subseteq \text{dom}(\texttt{C}_N) \circ \overline{\texttt{X}_m} \\ 8. & \overline{{}^{r}\texttt{T}_m} = dyn(\overline{{}^{s}\texttt{T}_m}, \texttt{h}, {}^{r}\Gamma) \\ 9. & {}^{r}\Gamma' = \overline{\texttt{X}_m\ {}^{r}\texttt{T}_m}; \_ \\ 10. & {}^{r}\texttt{T}_2 = dyn(({}^{s}\texttt{N} \rhd {}^{s}\texttt{T})[\overline{{}^{s}\texttt{T}_m/\texttt{X}_m}], \texttt{h}, {}^{r}\Gamma) \end{array}\right\} \Longrightarrow {}^{r}\texttt{T}_2 = dyn({}^{s}\texttt{T}, \iota_1, {}^{r}\texttt{T}, {}^{r}\Gamma')$$

For the proofs of the adaptation lemmas we assume that the type variables in ${}^{r}\Gamma$ and ${}^{r}\Gamma'$ are disjunct, that is, that the type variables that can appear in ${}^{s}N$ are different from the type variables that can appear in ${}^{s}\texttt{T}$. This could be the case in a recursive method call. However, this is not a restriction on the expressive power, as we can apply the following renaming:

$$\overline{\texttt{X}_T} \notin \text{free}({}^{s}\texttt{N})$$
$$dyn((({}^{s}\texttt{N}[\overline{\texttt{X}_T/\texttt{X}_m}]) \rhd {}^{s}\texttt{T})[\overline{{}^{s}\texttt{T}_m/\texttt{X}_m}][\overline{\texttt{X}_m/\texttt{X}_T}], \texttt{h}, {}^{r}\Gamma)$$

We prove this lemma by induction on the shape of ${}^{s}\texttt{T}$:

Base cases:

Case 1: ${}^{s}\mathtt{T} = \mathtt{X}^{j}_{m}$

$({}^{s}N \rhd \mathtt{X}^{j}_{m})[\overline{{}^{s}\mathtt{T}_{m}/\mathtt{X}_{m}}] = {}^{s}\mathtt{T}^{j}_{m}$
$dyn(({}^{s}N \rhd \mathtt{X}^{j}_{m})[\overline{{}^{s}\mathtt{T}_{m}/\mathtt{X}_{m}}], \mathtt{h}, {}^{r}\Gamma) =$
$dyn({}^{s}\mathtt{T}^{j}_{m}, \mathtt{h}, {}^{r}\Gamma) = [$ from 8. $]$
${}^{r}\mathtt{T}^{j}_{m} = [$ from 9. and DYN $]$
$dyn(\mathtt{X}^{j}_{m}, \iota_1, {}^{r}\mathtt{T}, {}^{r}\Gamma') =$
$dyn({}^{s}\mathtt{T}, \iota_1, {}^{r}\mathtt{T}, {}^{r}\Gamma')$

Case 2: ${}^{s}\mathtt{T} = \mathtt{X}^{j} \in \mathbf{dom}(\mathtt{C}_N)$

From 2. and 3. we get:
$\iota_1 \vdash {}^{r}\mathtt{T} \;{}^{r}\!\!<: dyn({}^{s}\mathtt{N}, \mathtt{h}, {}^{r}\Gamma)$
From 2. we know that dyn is defined and produces some substitution that we call [A]:
$\iota_1 \vdash {}^{r}\mathtt{T} \;{}^{r}\!\!<: {}^{s}\mathtt{N}[A]$
We first simplify the combination:
$({}^{s}N \rhd \mathtt{X}^{j})[\overline{{}^{s}\mathtt{T}_{m}/\mathtt{X}_{m}}] = {}^{s}\mathtt{T}^{j}_{N}$
Now we have 10.:
$dyn({}^{s}\mathtt{T}^{j}_{N}, h, {}^{r}\Gamma) =$
We know that 2. is defined, therefore also this subtype is defined:
${}^{s}\mathtt{T}^{j}_{N}[A]$
The requirements of $dyn(\mathtt{X}^{j}, \iota_1, {}^{r}\mathtt{T}, {}^{r}\Gamma')$ are:
${}^{r}\mathtt{T}\!\downarrow_1 = \iota'$
$\iota_1 \vdash {}^{r}\mathtt{T} \;{}^{r}\!\!<: \iota' \; \mathtt{C}_N\texttt{<}\overline{{}^{s}\mathtt{T}_N}[A]\texttt{>}$
$\mathtt{X}^{j} \in \mathrm{dom}(\mathtt{C}_N)$
$\iota_1 \vdash {}^{r}\mathtt{T} \;{}^{r}\!\!<: \iota' \; \mathtt{C}_N\texttt{<}\overline{{}^{r}\mathtt{T}'}\texttt{>} \Rightarrow \iota_1 \vdash \overline{{}^{r}\mathtt{T}'} \;{}^{r}\!\!<: \overline{{}^{s}\mathtt{T}_N[A]}$
We know that $\iota' \neq \mathtt{any}_a$ because $\mathtt{u}_N \neq \mathtt{any}_u$ and 2. Therefore we know from Lemma 5.9 that the two type arguments are the same.
Now dyn gives us:
$dyn(\mathtt{X}^{j}, \iota_1, {}^{r}\mathtt{T}, {}^{r}\Gamma') = {}^{s}\mathtt{T}^{j}_{N}[A]$ which is as expected.

Case 3: ${}^{s}\mathtt{T} = \mathtt{u}\;\mathtt{C}$

Case 3a: ${}^{s}\mathtt{N} \rhd {}^{s}\mathtt{T} = \mathtt{this}_u\;\mathtt{C}$ **or** $\mathtt{peer}\;\mathtt{C}$

From the definition of $\rhd$ we see that each ownership modifier is either $\mathtt{this}_u$ or $\mathtt{peer}$.
${}^{r}\mathtt{T}_2 = dyn(\mathtt{u}\;\mathtt{C}, \iota_1, {}^{r}\mathtt{T}, {}^{r}\Gamma') = {}^{r}\mathtt{T}\!\downarrow_1$
$dyn(({}^{s}\mathtt{N} \rhd {}^{s}\mathtt{T})[\overline{{}^{s}\mathtt{T}_{m}/\mathtt{X}_{m}}], \mathtt{h}, {}^{r}\Gamma) = \iota''\;\mathtt{C}$ where $\iota'' = owner(\mathtt{h}, {}^{r}\Gamma(\mathtt{this}))$
From 2. we have
$\iota_1 \vdash {}^{r}\mathtt{T} \;{}^{r}\!\!<: dyn({}^{s}\mathtt{N}, \mathtt{h}, {}^{r}\Gamma)$.
We know that
$dyn({}^{s}\mathtt{N}, \mathtt{h}, {}^{r}\Gamma) = \iota''\;\mathtt{C}_N\texttt{<...>}$.
We know that the main modifiers are not $\mathtt{any}_u$ and therefore that the addresses ι' and ι'' are equal.

Case 3b: $^{s}\mathtt{N} \triangleright {}^{s}\mathtt{T} = \mathtt{rep\ C}$
By looking at $\triangleright$ we find two possibilities for this result:
$\mathtt{u}_N = \mathtt{this}_u \wedge \mathtt{u} = \mathtt{rep}$ or
$\mathtt{u}_N = \mathtt{rep} \wedge (\mathtt{u} = \mathtt{this}_u \vee \mathtt{u} = \mathtt{peer})$

Case 3bi: $\mathtt{u}_N = \mathtt{this}_u \wedge \mathtt{u} = \mathtt{rep}$
By 4. we have $^{r}\Gamma(\mathtt{this}) = \iota_1$
$dyn(\mathtt{rep\ C}, \iota_1, {}^{r}\mathtt{T}, {}^{r}\Gamma') = \iota_1\ \mathtt{C} = dyn(\mathtt{rep\ C}, \mathtt{h}, {}^{r}\Gamma)$

Case 3bii: $\mathtt{u}_N = \mathtt{rep} \wedge (\mathtt{u} = \mathtt{this}_u \vee \mathtt{u} = \mathtt{peer})$
The substitutions for $\mathtt{this}_u$ and $\mathtt{peer}$ made by *dyn* are the same, therefore we just look at $\mathtt{this}_u$.
$dyn({}^{s}\mathtt{T}, \iota_1, {}^{r}\mathtt{T}, {}^{r}\Gamma') = dyn(\mathtt{this}_u\ \mathtt{C}, \iota_1, {}^{r}\mathtt{T}, {}^{r}\Gamma') = \iota'\ \mathtt{C}$
$dyn(({}^{s}\mathtt{N} \triangleright {}^{s}\mathtt{T})[\overline{{}^{s}\mathtt{T}_m/\mathtt{X}_m}], \mathtt{h}, {}^{r}\Gamma) = \iota''\ \mathtt{C}$ where $\iota'' = {}^{r}\Gamma(\mathtt{this})$, because in this case the result of the combination is $\mathtt{rep}$ and the definition of *dyn*.
From 2. we have
$\iota_1 \vdash {}^{r}\mathtt{T}\ {}^{r}\!<: dyn(\mathtt{rep\ C}_N\mathtt{<...>}, \mathtt{h}, {}^{r}\Gamma) = \iota_1 \vdash {}^{r}\mathtt{T}\ {}^{r}\!<: \iota''\ \mathtt{C}_N\mathtt{<...>}$ where $^{r}\Gamma(\mathtt{this}) = \iota''$. We know that $^{r}\Gamma(\mathtt{this})$ can not be any (from the range of $^{r}\Gamma$) and therefore know that $\iota = \iota''$.

Case 3c: $^{s}\mathtt{N} \triangleright {}^{s}\mathtt{T} = \mathtt{any}_u\ \mathtt{C}$

By looking at $\triangleright$ and $\triangleright_m$ we find four possibilities for this result:
(i) $(\mathtt{u}_N = \mathtt{rep} \vee \mathtt{u}_N = \mathtt{peer}) \wedge \mathtt{u} = \mathtt{peer} \wedge \mathtt{rep} \in {}^{s}\mathtt{T}$
(ii) $\mathtt{u}_N = \mathtt{any}_u$
(iii) $(\mathtt{u}_N = \mathtt{peer} \wedge \mathtt{u} = \mathtt{rep}) \vee (\mathtt{u}_N = \mathtt{rep} \wedge \mathtt{u} = \mathtt{rep})$
(iv) $\mathtt{u} = \mathtt{any}_u$
Case (i) does not hold, because 6. ensures that $\mathtt{rep}$ is never contained in $^{s}\mathtt{T}$.
Case (ii) is ruled out by 5.
Case (iii) is ruled out by 6.
Case (iv): $dyn(\mathtt{any}_u\ \mathtt{C}, \iota_1, {}^{r}\mathtt{T}, {}^{r}\Gamma') = \mathtt{any}_a\ \mathtt{C} = dyn(\mathtt{any}_u\ \mathtt{C}, \mathtt{h}, {}^{r}\Gamma)$

□

Induction Step: $^{s}\mathtt{T} = \mathtt{u\ C}\mathtt{<}\overline{{}^{s}\mathtt{T}}\mathtt{>}$

The main modifier is not affected by the presence of type arguments.
$dyn({}^{s}\mathtt{T}, \iota_1, {}^{r}\mathtt{T}, {}^{r}\Gamma') = \iota''\ \mathtt{C}\mathtt{<}\overline{{}^{r}\mathtt{T}''}\mathtt{>}$
$dyn(({}^{s}\mathtt{N} \triangleright {}^{s}\mathtt{T})[\overline{{}^{s}\mathtt{T}_m/\mathtt{X}_m}], \mathtt{h}, {}^{r}\Gamma) = \iota'''\ \mathtt{C}\mathtt{<}\overline{{}^{r}\mathtt{T}'''}\mathtt{>}$
The case analysis for the base case can still be applied and we again get that $\iota'' = \iota'''$.
Now we apply the induction hypothesis to the type arguments $\overline{{}^{s}\mathtt{T}}$ (we treat the type arguments $\overline{{}^{s}\mathtt{T}}$ like a single type; this is done for each type in the sequence):

$$
\left.\begin{array}{ll}
1'. & \mathtt{h} \vdash {}^{\mathtt{r}}\Gamma : {}^{\mathtt{s}}\Gamma \qquad \mathtt{h} \vdash {}^{\mathtt{r}}\Gamma' : {}^{\mathtt{s}}\Gamma' \\
2'. & \mathtt{h} \vdash \iota_1 : dyn({}^{\mathtt{s}}\mathtt{N}, \mathtt{h}, {}^{\mathtt{r}}\Gamma) \\
3'. & \mathtt{h}(\iota_1)\downarrow_1 = {}^{\mathtt{r}}\mathtt{T} \\
 & {}^{\mathtt{s}}\mathtt{N} = \mathtt{u}_N\ \mathtt{C}_N\mathtt{<}\overline{{}^{\mathtt{s}}\mathtt{T}_N}\mathtt{>} \\
4'. & \mathtt{u}_N = \mathtt{this}_u \Rightarrow {}^{\mathtt{r}}\Gamma(\mathtt{this}) = \iota_1 \\
5'. & \mathtt{u}_N \neq \mathtt{any} \\
6'. & rp(\mathtt{u}_N, {}^{\mathtt{s}}\mathtt{T}) \\
7'. & \mathrm{free}(\overline{{}^{\mathtt{s}}\mathtt{T}}) \subseteq \mathrm{dom}(\mathtt{C}_N) \circ \overline{\mathtt{X}_m} \\
8'. & \overline{{}^{\mathtt{r}}\mathtt{T}_m} = dyn(\overline{{}^{\mathtt{s}}\mathtt{T}_m}, \mathtt{h}, {}^{\mathtt{r}}\Gamma) \\
9'. & {}^{\mathtt{r}}\Gamma' = \overline{\mathtt{X}_m\ {}^{\mathtt{r}}\mathtt{T}_m}; \_ \\
10'. & {}^{\mathtt{r}}\mathtt{T}'_2 = dyn(\overline{{}^{\mathtt{s}}\mathtt{T}}, \iota_1, {}^{\mathtt{r}}\mathtt{T}, {}^{\mathtt{r}}\Gamma')
\end{array}\right\} \Longrightarrow {}^{\mathtt{r}}\mathtt{T}'_2 = dyn(({}^{\mathtt{s}}\mathtt{N} \triangleright \overline{{}^{\mathtt{s}}\mathtt{T}})[\overline{{}^{\mathtt{s}}\mathtt{T}_m/\mathtt{X}_m}], \mathtt{h}, {}^{\mathtt{r}}\Gamma)
$$

7' holds, because from 7 we know that $\mathrm{free}({}^{\mathtt{s}}\mathtt{T}) \subseteq \mathrm{dom}(\mathtt{C}_N) \circ \overline{\mathtt{X}_m}$ and we now only consider a subpart of ${}^{\mathtt{s}}\mathtt{T}$. The application of *dyn* in 10' is still defined, also because it is defined for the larger type. The other conditions directly carry over from 1. – 10.

We now have that the main address and all the type arguments are the same. □

5.2.4 Proof of Lemma 5.6 — Adaptation from a Viewpoint Helper Lemma

The following lemma is used in the proof of the previous two viewpoint adaptation lemmas:

$$
\left.\begin{array}{ll}
1. & \mathtt{h} \vdash {}^{\mathtt{r}}\Gamma : {}^{\mathtt{s}}\Gamma \qquad \mathtt{h} \vdash {}^{\mathtt{r}}\Gamma' : {}^{\mathtt{s}}\Gamma' \\
2. & \mathtt{h} \vdash \iota_1 : dyn({}^{\mathtt{s}}\mathtt{N}, \mathtt{h}, {}^{\mathtt{r}}\Gamma) \\
3. & \mathtt{h}(\iota_1)\downarrow_1 = {}^{\mathtt{r}}\mathtt{T} \\
 & {}^{\mathtt{s}}\mathtt{N} = \mathtt{u}_N\ \mathtt{C}_N\mathtt{<}\overline{{}^{\mathtt{s}}\mathtt{T}_N}\mathtt{>} \\
4. & \mathtt{u}_N = \mathtt{this}_u \Rightarrow {}^{\mathtt{r}}\Gamma(\mathtt{this}) = \iota_1 \\
5. & \mathrm{free}({}^{\mathtt{s}}\mathtt{T}) \subseteq \mathrm{dom}(\mathtt{C}_N) \circ \overline{\mathtt{X}_m} \\
6. & \overline{{}^{\mathtt{r}}\mathtt{T}_m} = dyn(\overline{{}^{\mathtt{s}}\mathtt{T}_m}, \mathtt{h}, {}^{\mathtt{r}}\Gamma) \\
7. & {}^{\mathtt{r}}\Gamma' = \overline{\mathtt{X}_m\ {}^{\mathtt{r}}\mathtt{T}_m}; \_ \\
8. & {}^{\mathtt{r}}\mathtt{T}_2 = dyn({}^{\mathtt{s}}\mathtt{T}, \iota_1, {}^{\mathtt{r}}\mathtt{T}, {}^{\mathtt{r}}\Gamma')
\end{array}\right\} \Longrightarrow {}^{\mathtt{r}}\mathtt{T}_2\ {}^{\mathtt{r}}\mathtt{<:}_{\mathtt{a}}\ dyn(({}^{\mathtt{s}}\mathtt{N} \triangleright {}^{\mathtt{s}}\mathtt{T})[\overline{{}^{\mathtt{s}}\mathtt{T}_m/\mathtt{X}_m}], \mathtt{h}, {}^{\mathtt{r}}\Gamma)
$$

We prove this by induction on the shape of ${}^{\mathtt{s}}\mathtt{T}$:

Base cases:

Case 1: ${}^{\mathtt{s}}\mathtt{T} = \mathtt{X}^j_m$

$$
\begin{array}{l}
({}^{\mathtt{s}}N \triangleright \mathtt{X}^j_m)[\overline{{}^{\mathtt{s}}\mathtt{T}_m/\mathtt{X}_m}] = {}^{\mathtt{s}}\mathtt{T}^j_m \\
dyn(({}^{\mathtt{s}}N \triangleright \mathtt{X}^j_m)[\overline{{}^{\mathtt{s}}\mathtt{T}_m/\mathtt{X}_m}], \mathtt{h}, {}^{\mathtt{r}}\Gamma) = \\
dyn({}^{\mathtt{s}}\mathtt{T}^j_m, \mathtt{h}, {}^{\mathtt{r}}\Gamma) = [\ \text{from 6.}\ ] \\
{}^{\mathtt{r}}\mathtt{T}^j_m = [\ \text{from 7. and DYN}\ ] \\
dyn(\mathtt{X}^j_m, \iota_1, {}^{\mathtt{r}}\mathtt{T}, {}^{\mathtt{r}}\Gamma') = \\
dyn({}^{\mathtt{s}}\mathtt{T}, \iota_1, {}^{\mathtt{r}}\mathtt{T}, {}^{\mathtt{r}}\Gamma')
\end{array}
$$

The two applications are equal; therefore, by rule RTA-1 we also have that they are ${}^{\mathtt{r}}\mathtt{<:}_{\mathtt{a}}$ subtypes.

Case 2: ${}^{s}\mathtt{T} = \mathtt{X}^{j} \in \mathbf{dom}(\mathtt{C}_N)$

From 2. and 3. we get:
$\iota_1 \vdash {}^{r}\mathtt{T}\ {}^{r}{<:}\ dyn({}^{s}\mathtt{N}, \mathtt{h}, {}^{r}\Gamma)$
From 2. we know that dyn is defined and produces some substitution that we call [A]:
$\iota_1 \vdash {}^{r}\mathtt{T}\ {}^{r}{<:}\ {}^{s}\mathtt{N}[A]$
From 8. and the definition of dyn we get
$\iota_1 \vdash {}^{r}\mathtt{T}\ {}^{r}{<:}\ \iota'\ \mathtt{C}_N{<}\overline{{}^{r}\mathtt{T}}{>}$ and
$\iota_1 \vdash {}^{r}\mathtt{T}\ {}^{r}{<:}\ \iota'\ \mathtt{C}_N{<}\overline{{}^{r}\mathtt{T}'}{>} \Rightarrow \iota_1 \vdash \overline{{}^{r}\mathtt{T}}\ {}^{r}{<:}\ \overline{{}^{r}\mathtt{T}'}$.
in particular this gives:
${}^{r}\mathtt{T}_j\ {}^{r}{<:}_{\mathtt{a}}\ {}^{s}\mathtt{T}^{j}_{N}[A]$
$({}^{s}N \triangleright \mathtt{X}^{j})[\overline{{}^{s}\mathtt{T}_m/\mathtt{X}_m}] = {}^{s}\mathtt{T}^{j}$
$dyn({}^{s}\mathtt{T}^{j}_{N}, \mathtt{h}, {}^{r}\Gamma) = {}^{s}\mathtt{T}^{j}_{N}[A]$

Case 3: ${}^{s}\mathtt{T} = \mathtt{u}\ \mathtt{C}$

From Lemma 5.7 we know that the property holds for
$\mathtt{u}_N \neq \mathtt{any}_u \wedge (\mathtt{u}_N = \mathtt{this} \vee \mathtt{u} \neq \mathtt{rep})$
We prove the remaining two cases now.

Case (a): $\mathtt{u}_N = \mathtt{any}_u$
$dyn(\mathtt{u}\ \mathtt{C}, \iota_1, {}^{r}\mathtt{T}, \overline{{}^{r}\Gamma'}) = \iota''\ \mathtt{C}$
$dyn(({}^{s}N \triangleright \mathtt{u}\ \mathtt{C})[\overline{{}^{s}\mathtt{T}_m/\mathtt{X}_m}], \mathtt{h}, {}^{r}\Gamma) = \mathtt{any}_a\ \mathtt{C}$
From RTA-2 we see that they are in the ${}^{r}{<:}_{\mathtt{a}}$ relation.
Case (b): $\mathtt{u}_N \neq \mathtt{this} \wedge \mathtt{u} = \mathtt{rep}$
$dyn(\mathtt{u}\ \mathtt{C}, \iota_1, {}^{r}\mathtt{T}, \overline{{}^{r}\Gamma'}) = \iota''\ \mathtt{C}$
$dyn(({}^{s}N \triangleright \mathtt{u}\ \mathtt{C})[\overline{{}^{s}\mathtt{T}_m/\mathtt{X}_m}], \mathtt{h}, {}^{r}\Gamma) = dyn(\mathtt{any}_u\ \mathtt{C}, \mathtt{h}, {}^{r}\Gamma) = \mathtt{any}_a\ \mathtt{C}$
From RTA-2 we see that they are in the ${}^{r}{<:}_{\mathtt{a}}$ relation.

Induction step: ${}^{s}\mathtt{T} = \mathtt{u}\ \mathtt{C}{<}\overline{{}^{s}\mathtt{T}}{>}$

Case (a): $\mathtt{u}_N \neq \mathtt{any}_u \wedge (\mathtt{u}_N = \mathtt{this} \vee \mathtt{rep} \notin {}^{s}\mathtt{T})$
Follows directly from Lemma 5.7.

Case (b): $\mathtt{u}_N = \mathtt{any}_u \vee (\mathtt{u}_N \neq \mathtt{this} \wedge \mathtt{rep} \in {}^{s}\mathtt{T})$
${}^{r}\mathtt{T}_2 = dyn({}^{s}\mathtt{T}, \iota_1, {}^{r}\mathtt{T}, {}^{r}\Gamma') = \iota''\ \mathtt{C}{<}\overline{dyn(\overline{{}^{s}\mathtt{T}}, \iota_1, {}^{r}\mathtt{T}, {}^{r}\Gamma')}{>}$
$dyn(({}^{s}N \triangleright {}^{s}\mathtt{T})[\overline{{}^{s}\mathtt{T}_m/\mathtt{X}_m}], \mathtt{h}, {}^{r}\Gamma) = \mathtt{any}_a\ \mathtt{C}{<}\overline{dyn(({}^{s}N \triangleright {}^{s}\mathtt{T})[\overline{{}^{s}\mathtt{T}_m/\mathtt{X}_m}], \mathtt{h}, {}^{r}\Gamma)}{>}$

By the induction hypothesis we have $dyn(\overline{{}^{s}\mathtt{T}}, \iota_1, {}^{r}\mathtt{T}, {}^{r}\Gamma')\ {}^{r}{<:}_{\mathtt{a}}\ dyn(\overline{({}^{s}N \triangleright {}^{s}\mathtt{T})[\overline{{}^{s}\mathtt{T}_m/\mathtt{X}_m}]}, \mathtt{h}, {}^{r}\Gamma)$
□

5.2.5 Proof of Lemma 5.2 — Adaptation to a Viewpoint

We want to prove:

$$\left.\begin{array}{l} \mathtt{h} \vdash {}^{\mathtt{r}}\Gamma : {}^{\mathtt{s}}\Gamma \\ \mathtt{h} \vdash \iota_1 : dyn({}^{\mathtt{s}}\mathtt{N}, \mathtt{h}, {}^{\mathtt{r}}\Gamma) \\ \mathtt{h} \vdash \iota_2 : dyn(({}^{\mathtt{s}}\mathtt{N} \rhd {}^{\mathtt{s}}\mathtt{T})[\overline{{}^{\mathtt{s}}\mathtt{T}/\mathtt{X}_m}], \mathtt{h}, {}^{\mathtt{r}}\Gamma) \\ \mathtt{h}(\iota_1)\downarrow_1 = {}^{\mathtt{r}}\mathtt{T} \\ {}^{\mathtt{s}}\mathtt{N} = \mathtt{u}\ \mathtt{C}\text{<\_>}, \quad \mathtt{u} \neq \mathtt{any}, \quad rp(\mathtt{u}, {}^{\mathtt{s}}\mathtt{T}) \\ \mathtt{u} = \mathtt{this}_u \Rightarrow {}^{\mathtt{r}}\Gamma(\mathtt{this}) = \iota_1 \\ \text{free}({}^{\mathtt{s}}\mathtt{T}) \subseteq \text{dom}(\mathtt{C}) \circ \overline{\mathtt{X}_m} \\ {}^{\mathtt{r}}\Gamma' = \overline{\mathtt{X}_m\ {}^{\mathtt{r}}\mathtt{T}}; \_ \\ \overline{{}^{\mathtt{r}}\mathtt{T}} = dyn(\overline{{}^{\mathtt{s}}\mathtt{T}}, \mathtt{h}, {}^{\mathtt{r}}\Gamma) \end{array}\right\} \implies \mathtt{h} \vdash \iota_2 : dyn({}^{\mathtt{s}}\mathtt{T}, \iota_1, {}^{\mathtt{r}}\mathtt{T}, {}^{\mathtt{r}}\Gamma')$$

We can directly apply Lemma 5.7, because we have the same assumptions. Therefore we have

$dyn(({}^{\mathtt{s}}\mathtt{N} \rhd {}^{\mathtt{s}}\mathtt{T})[\overline{{}^{\mathtt{s}}\mathtt{T}/\mathtt{X}_m}], \mathtt{h}, {}^{\mathtt{r}}\Gamma) = dyn({}^{\mathtt{s}}\mathtt{T}, \iota_1, {}^{\mathtt{r}}\mathtt{T}, {}^{\mathtt{r}}\Gamma') = {}^{\mathtt{r}}\mathtt{T}'$

Then the conclusion is directly one of the assumptions. □

5.2.6 Proof of Lemma 5.1 — Adaptation from a Viewpoint

We want to prove:

$$\left.\begin{array}{ll} 1. & \mathtt{h} \vdash {}^{\mathtt{r}}\Gamma : {}^{\mathtt{s}}\Gamma \qquad \mathtt{h} \vdash {}^{\mathtt{r}}\Gamma' : {}^{\mathtt{s}}\Gamma' \\ 2. & \mathtt{h} \vdash \iota_1 : dyn({}^{\mathtt{s}}\mathtt{N}, \mathtt{h}, {}^{\mathtt{r}}\Gamma) \\ 3. & \mathtt{h} \vdash \iota_2 : dyn({}^{\mathtt{s}}\mathtt{T}, \iota_1, {}^{\mathtt{r}}\mathtt{T}, {}^{\mathtt{r}}\Gamma') \\ 4. & \mathtt{h}(\iota_1)\downarrow_1 = {}^{\mathtt{r}}\mathtt{T} \\ & {}^{\mathtt{s}}\mathtt{N} = \mathtt{u}_N\ \mathtt{C}_N\text{<\_>} \\ 5. & \mathtt{u}_N = \mathtt{this}_u \Rightarrow {}^{\mathtt{r}}\Gamma(\mathtt{this}) = \iota_1 \\ 6. & \text{free}({}^{\mathtt{s}}\mathtt{T}) \subseteq \text{dom}(\mathtt{C}_N) \circ \overline{\mathtt{X}_m} \\ 7. & \overline{{}^{\mathtt{r}}\mathtt{T}_m} = dyn(\overline{{}^{\mathtt{s}}\mathtt{T}_m}, \mathtt{h}, {}^{\mathtt{r}}\Gamma) \\ 8. & {}^{\mathtt{r}}\Gamma' = \overline{\mathtt{X}_m\ {}^{\mathtt{r}}\mathtt{T}_m}; \_ \end{array}\right\} \implies \mathtt{h} \vdash \iota_2 : dyn(({}^{\mathtt{s}}\mathtt{N} \rhd {}^{\mathtt{s}}\mathtt{T})[\overline{{}^{\mathtt{s}}\mathtt{T}_m/\mathtt{X}_m}], \mathtt{h}, {}^{\mathtt{r}}\Gamma)$$

We can apply Lemma 5.6 to arrive at $dyn({}^{\mathtt{s}}\mathtt{T}, \iota_1, {}^{\mathtt{r}}\mathtt{T}, {}^{\mathtt{r}}\Gamma')\ {}^{\mathtt{r}}\!\!<:_{\mathtt{a}} dyn(({}^{\mathtt{s}}\mathtt{N} \rhd {}^{\mathtt{s}}\mathtt{T})[\overline{{}^{\mathtt{s}}\mathtt{T}_m/\mathtt{X}_m}], \mathtt{h}, {}^{\mathtt{r}}\Gamma)$

By RT-3 we get $\iota_2 \vdash dyn({}^{\mathtt{s}}\mathtt{T}, \iota_1, {}^{\mathtt{r}}\mathtt{T}, {}^{\mathtt{r}}\Gamma')\ {}^{\mathtt{r}}\!\!<: dyn(({}^{\mathtt{s}}\mathtt{N} \rhd {}^{\mathtt{s}}\mathtt{T})[\overline{{}^{\mathtt{s}}\mathtt{T}_m/\mathtt{X}_m}], \mathtt{h}, {}^{\mathtt{r}}\Gamma)$

From 3. and RT-4 we know that there is a ${}^{\mathtt{r}}\mathtt{T}_2$ the runtime type of ι_2 in the heap.

Together this allows us to arrive at the conclusion. □

5.2.7 Proof of Lemma 5.12 — Evaluation preserves types

Case analysis of all expressions shows that only the fields are updated in a heap. The second part follows directly from the first, because *dyn* only takes the runtime type from the heap and does not depend on the field values. □

5.2.8 Proof of Lemma 5.13 — Runtime meaning of ownership modifiers

If $h \vdash \iota : dyn({}^s\mathtt{T}, \mathtt{h}, {}^r\Gamma)$, then

1. ${}^s\mathtt{T} = \mathtt{this}\ \_<\_> \Rightarrow owner(\mathtt{h}, \iota) = owner(\mathtt{h}, {}^r\Gamma(\mathtt{this}))$
2. ${}^s\mathtt{T} = \mathtt{peer}\ \_<\_> \Rightarrow owner(\mathtt{h}, \iota) = owner(\mathtt{h}, {}^r\Gamma(\mathtt{this}))$
3. ${}^s\mathtt{T} = \mathtt{rep}\ \_<\_> \Rightarrow owner(\mathtt{h}, \iota) = {}^r\Gamma(\mathtt{this})$

$dyn({}^s\mathtt{T}, \mathtt{h}, {}^r\Gamma)$ is short for $dyn({}^s\mathtt{T}, {}^r\Gamma(\mathtt{this}), \mathtt{h}({}^r\Gamma(\mathtt{this})) \downarrow_1, {}^r\Gamma)$.

Case 1: ${}^s\mathtt{T}$=this $\_<\_>$
$dyn({}^s\mathtt{T}, {}^r\Gamma(\mathtt{this}), \mathtt{h}({}^r\Gamma(\mathtt{this})) \downarrow_1, {}^r\Gamma)$ replaces this by $\mathtt{h}({}^r\Gamma(\mathtt{this})) \downarrow_1 \downarrow_1$, that is, the owner of ${}^r\Gamma(\mathtt{this})$. □

Case 2: ${}^s\mathtt{T}$=peer $\_<\_>$
$dyn({}^s\mathtt{T}, {}^r\Gamma(\mathtt{this}), \mathtt{h}({}^r\Gamma(\mathtt{this})) \downarrow_1, {}^r\Gamma)$ replaces peer by $\mathtt{h}({}^r\Gamma(\mathtt{this})) \downarrow_1 \downarrow_1$, that is, the owner of ${}^r\Gamma(\mathtt{this})$. □

Case 3: ${}^s\mathtt{T}$=rep $\_<\_>$
$dyn({}^s\mathtt{T}, {}^r\Gamma(\mathtt{this}), \mathtt{h}({}^r\Gamma(\mathtt{this})) \downarrow_1, {}^r\Gamma)$ replaces rep by ${}^r\Gamma(\mathtt{this})$. □

6 Conclusions

We presented Generic Universe Types, an ownership type system for Java-like languages with generic types. Our type system permits arbitrary references through any types, but controls modifications of objects, that is, enforces the owner-as-modifier discipline. This allows us to handle interesting implementations beyond simple aggregate objects, for instance, iterators and shared buffers [13]. We show how any types and generics can be combined in a type safe way using limited covariance and viewpoint adaptation.

Generic Universe Types require little annotation overhead for programmers. As we have shown for non-generic Universe Types [13], this overhead can be further reduced by appropriate defaults. The default ownership modifier is generally peer, but the modifier of upper bounds, exceptions, and immutable types (such as String) defaults to any. These defaults make the conversion from Java 5 to Generic Universe Types simple.

The type checker and runtime support for non-generic Universe Types are implemented in JML [19]. An adaptation to Generic Universe Types is ongoing.

As future work, we plan to use Generic Universe Types for program verification, extending our earlier work [22, 23]. One of the interesting challenges there is to relax the restrictions on any types, for instance, to allow field updates and calls of non-pure methods on receivers whose type is a type variable with an any upper bound. We are working on Path-dependent Universe Types to support more fine-grained information about object ownership [25], and plan to extend our existing inference tools for non-generic Universe Types to Generic Universe Types.

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