

ADAPTIVE WINDOWING FOR OPTIMAL VISUALIZATION OF MEDICAL IMAGES BASED ON NORMALIZED INFORMATION DISTANCE

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ABSTRACT

There has been a growing recent interest of applying Kolmogorov complexity and its related normalized information distance (NID) measures in real-world problems, but their application in the field of medical image processing remains limited. In this work we attempt to incorporate NID in the design of windowing operators for optimal visualization of high dynamic range (HDR) medical images, where predefined intensity interval of interest needs to be mapped to match the low dynamic range (LDR) of standard displays. By approximating NID using a Shannon entropy based method, we are able to optimize parametric windowing operators to maximize the information similarity between the HDR image and the LDR image after mapping. Experimental results demonstrate promising performance of the proposed approach.

Index Terms— Kolmogorov complexity, normalized information distance (NID), entropy, high dynamic range (HDR) imaging, windowing, tone-mapping

1. INTRODUCTION

Recently the theory of Kolmogorov complexity and its associated normalized information distance (NID) metrics have attracted an increasing amount of attention and found a variety of successful applications in bioinformatics, pattern recognition, and natural language processing [1–3]. A popular approach is to approximate NID using a normalized compression distance (NCD) measure, which overcomes the non-computability problem of Kolmogorov complexity and NID, and thus provides practical solutions to many real-world problems [2]. The application of these methodologies in the field of image processing is still at a premature stage [4–10]. In [9], a normalized conditional compression distance (NCCD) method was introduced, which supplies a practical framework to approximate conditional Kolmogorov complexity using an image compressor and a list of image transformations. In [10], a normalized perceptual information similarity (NPIS) method was proposed that incorporates image statistics and perceptual models and employs Shannon entropy to approximate Kolmogorov complexity. Nevertheless, to the best of our knowledge, little progress has been

made in the application of Kolmogorov complexity and related methods in the field of medical image processing.

Medical images are typically captured with higher precisions or higher dynamic ranges of intensity values than what can be directly shown on standard displays with 8-bit depth. Standard medical image formats such as DICOM allow to store such high dynamic range (HDR) images with more bit depths, but to visualize them on regular displays becomes a challenge. In practice, a so-called “windowing” approach is often employed, which linearly maps an intensity interval of interest to the dynamic range of the display. These intervals are defined using two parameters: (i) window width, or the range of the interval, W (which is typically larger than 255); and (ii) window center, or the center of this interval, C . Thus a windowing operator maps the range of intensity values $[C - \frac{1}{2}W, C + \frac{1}{2}W]$ to a low dynamic range (LDR) $[0, 255]$. The default values for W and C may be embedded in the headers of DICOM image files, or determined manually by the end users (radiologists) so that the structural details for specific body region become more visible.

In this work, we aim to develop new windowing operators for optimal visualization of HDR medical images, where the optimality is defined as maximization of the information similarity between the HDR image and the mapped LDR image. A key step in our approach is to approximate NID based information similarity using a Shannon entropy approach. Our experiments show that when the new similarity measure is employed in optimizing two types parametric windowing operators, perceptually appealing images with higher contrast and more visible structural details are obtained.

2. BACKGROUND: KOLMOGOROV COMPLEXITY AND NORMALIZED INFORMATION DISTANCE

The Kolmogorov complexity of an object is defined to be the length of the shortest program that can produce that object on a universal Turing machine and halt:

$$K(x) = \min_{p:U(p)=x} l(p). \quad (1)$$

The conditional Kolmogorov complexity of x relative to y is denoted by $K(x|y)$, and the information distance between the

two objects is defined as the maximum of the length of the shortest program that computes x from y and y from x , i.e. $\max\{K(x|y), K(y|x)\}$. A better way to compare objects of different lengths is to normalize the information distance [2]:

$$\text{NID}(x, y) = \frac{\max\{K(x|y), K(y|x)\}}{\max\{K(x), K(y)\}}. \quad (2)$$

It is shown that NID is a distance metric that satisfies the identity and symmetry axioms and the triangular inequality [2].

Due to the non-computability of Kolmogorov complexity, direct computation of NID is impossible, and in all cases, NID has to be approximated by employing either data compression techniques or other computable quantities such as the Shannon entropy. In particular, it has been shown that Kolmogorov complexity and Shannon entropy are equivalent for a wide class of information sources. For any computable probability mass function $f(x) = P(X = x)$ on sample space $\chi = \{0, 1\}^*$ with entropy $H(X) = -\sum_x f(x) \log_2 f(x)$ the following inequality holds true [11]:

$$0 \leq \left(\sum_x f(x) K(x) - H(X) \right) \leq K(f) + O(1) \quad (3)$$

which states that the expected Kolmogorov complexity of the source is close to its entropy.

3. METHOD: ADAPTIVE WINDOWING FOR MAXIMAL INFORMATION SIMILARITY

The windowing process in medical imaging may be understood as a special case of the tone-mapping operation (TMO) that converts HDR images to LDR images [12]. TMO has been an active research topic in the past decades that has resulted in a number of successful techniques [12–15]. Existing TMO methods may be categorized into four groups, namely global operators, local operators, frequency operators, and gradient operators [13]. In the context of medical imaging, global operators implemented using monotonic intensity transformations are preferred because it is the only category that maintains one-to-one mapping of intensity values and preserves the ranks of pixel intensity values. By contrast, other TMOs may map the same intensity value in the HDR image to different values in the LDR image, which may confuse the understanding of the physical meanings behind the intensity values.

Standard windowing operation in medical imaging linearly maps the intensity interval of interest $[l_l, l_u]$ to the dynamic range of the LDR image, typically $[0, 255]$. This has often been shown to be far from optimal in terms of perceived image quality [16]. To develop a better windowing method, we relax the mapping operation to be a continuous and monotonically increasing function f lives in the function space of

$$\mathcal{F}_{[l_l, l_u]} = \{f : [l_l, l_u] \rightarrow [0, 255] | f \text{ monotonically increasing}\} \quad (4)$$

For any given f , we can then define a windowing operator T_f over an input HDR image x by

$$y = T_f(x) = \text{round}\{f(x)\}, \quad (5)$$

where since both images can take only integer intensity values, a rounding operator is necessary. The key question now is to obtain an LDR image y that is optimal in certain criterion. Motivated by the ideas behind NID, we would want to find an image y such that the normalized information similarity between x and y is maximized. Therefore, the problem of finding the optimal windowing operator can be expressed as

$$f_{\text{opt-NID}} = \arg \min_{f \in \mathcal{F}_{[l_l, l_u]}} \text{NID}(x, T_f(x)). \quad (6)$$

To provide a practical algorithm to compute NID, we resort to a Shannon entropy approximation of the Kolmogorov complexity, leading to a normalized Shannon information distance

$$\text{NID}(x, y) \approx \frac{\max\{H(x|y), H(y|x)\}}{\max\{H(x), H(y)\}} \quad (7)$$

Since the conversion from x to y is unique, there is no uncertainty in y given x , thus $H(y|x) = 0$. To compute $H(x|y)$, we first need to apply a reconstruction operator that “invert” the windowing function f :

$$\hat{x} = R_{f^{-1}}(y) = \text{round}\{f^{-1}(y)\}. \quad (8)$$

Note that such an “inversion” will not fully reconstruct x because there is information loss in the forward conversion and all values are integers that create rounding errors. Therefore, the actual uncertainty of $H(x|y)$ roughly lies in the prediction residual between x and \hat{x} . Also note that x as an HDR image contains more information (and uncertainty) than the LDR image y , thus $H(x) > H(y)$. Considering all the above factors, the actually computation simplifies to

$$\text{NID}(x, y) \approx \frac{H(x - R_{f^{-1}}(y))}{H(x)}. \quad (9)$$

Combining this with Eq. (6), the actual optimization problem we would need to solve reduces to

$$f_{\text{opt-NID}} = \arg \min_{f \in \mathcal{F}_{[l_l, l_u]}} \frac{H(x - R_{f^{-1}}(T_f(x)))}{H(x)}. \quad (10)$$

To fully solve Eq. (10) requires finding the best function in the function space $\mathcal{F}_{[l_l, l_u]}$, and is in general a difficult problem. Here we constrain the solutions to live in two families of parametric functions. In both cases, we express f as a linear combination of basis functions by

$$f(l) = \sum_{k=0}^{n-1} c_k \phi_k(l) = \phi_0(l) + \sum_{k=1}^{n-1} c_k \phi_k(l), \quad (11)$$

where $c_0 = 1$ and $\phi_0(l)$ is a ‘‘ramp’’ function that corresponds to direct linear mapping given by

$$\phi_0(l) = \begin{cases} (l - l_l)/(l_u - l_l), & l_0 \leq l \leq l_u \\ 0, & \text{otherwise} \end{cases}. \quad (12)$$

The other basis functions are different for the two cases.

In the first case, we consider equipartition piecewise linear approximation, where we divide the full intensity interval $[l_l, l_u]$ into n subintervals $I_k = [l_{k-1}, l_k]$ for $1 \leq k \leq K$ of length $\Delta l = (l_u - l_l)/K$. The partition points are given by $l_k = l_l + k\Delta l$, $0 \leq k \leq n$, as such $l_l = l_0$ and $l_u = l_n$. The basis functions for piecewise linear approximation are ‘‘hat’’ function given by

$$\phi_k(l) = t \left(\frac{l - l_k}{\Delta l} \right), \text{ for } k = 1, \dots, n - 1, \quad (13)$$

where

$$t(l) = \begin{cases} 1 - |l|, & -1 \leq l \leq 1 \\ 0, & \text{otherwise} \end{cases}. \quad (14)$$

For the function $f(l)$ to be monotonically increasing, we need $0 \leq \dots \leq f(l_{k-1}) \leq f(l_k) \leq \dots \leq 1$, which yields

$$0 \leq \dots \leq c_{k-1} + \frac{k-1}{n} \leq c_k + \frac{k}{n} \leq \dots \leq 1. \quad (15)$$

For example, in the case that $n = 3$, we can derive the following constraints on the solutions of the coefficients:

$$\begin{cases} c_1 \geq -\frac{1}{3}; \\ c_2 - c_1 \geq -\frac{1}{3}; \\ c_2 \leq \frac{1}{3}. \end{cases} \quad (16)$$

In the second case, we approximate the mapping function using the family of sine functions by

$$\phi_k(l) = \sin \left(\frac{k\pi(l - l_l)}{l_u - l_l} \right) \text{ for } l_l \leq l \leq l_u \text{ and } k = 1, 2, \dots, n \quad (17)$$

To ensure that the mapping function $f(l)$ to be monotonically increasing, we would need $f'(l) \geq 0$. Plug Eq. (17) into Eq. (11) and take derivatives with respect to l and let it be no less than 0, we can obtain a set of constraints on the solutions of the coefficients. For example, in the case of $n = 3$, the constraints are given by

$$\begin{cases} c_1 + 2c_2 \geq -\frac{1}{\pi} \\ -c_1 + 2c_2 \geq -\frac{1}{\pi} \\ \frac{c_1^2}{16c_2} + 2c_2 \leq \frac{1}{\pi} \end{cases} \quad (18)$$

Having the aforementioned two types of parametric windowing functions, we can then search in the coefficient space (c_1, c_2, \dots, c_n) to solve for the optimization problem defined in Eq. (10 under the constraints on the coefficients (e.g., for the case of $n = 3$, the constraints are (16) for piecewise linear functions or (18) for sine basis functions). The search space is typically complex and to solve the problem, we would need to employ numerical optimization methods or resort to software optimization tools (e.g., Matlab *fmincon* function). Examples and detailed experimental results are presented in Section 4.

4. EXPERIMENT AND COMPARISON

We use real-world medical images in DICOM format to test the proposed method. In addition, we compare it with the most widely used image similarity/distortion measures in the literature, i.e., mean squared error (MSE) and structural similarity index (SSIM) [17]. Note that the images before and after windowing have different dynamic ranges, and thus direct computation of MSE and SSIM is not feasible. Therefore, we search for the best windowing methods by optimizing MSE or SSIM between the original and reconstructed HDR images. These can be expressed as

$$f_{opt-MSE} = \arg \min_{f \in \mathcal{F}_{[l_l, l_u]}} \text{MSE}(x, R_{f^{-1}}(T_f(x))), \quad (19)$$

$$f_{opt-SSIM} = \arg \max_{f \in \mathcal{F}_{[l_l, l_u]}} \text{SSIM}(x, R_{f^{-1}}(T_f(x))). \quad (20)$$

In DICOM images, the window width and window center parameters are embedded in the image header, and thus the values of l_l and l_u are fixed. All windowing methods under test do not change these values, but attempt to find the best mapping functions with different optimization criteria.

Figure 1 compares the images created using default DICOM direct linear windowing, and optimal MSE windowing, optimal SSIM windowing, and optimal NID windowing, all using sine basis. Their corresponding windowing functions are also given. It can be observed that the structural details are best preserved in NID optimal windowing image, which also appears to have higher contrast and better perceptual quality. To better illustrate how NID behaves in the parameter space, Fig. 2 shows NID as a function of the c_1 and c_2 parameters in piecewise linear windowing, where brighter pixels indicate larger NID values. Sample images corresponding to different choices of c_1 and c_2 values are also given. It can be seen that the quality of the windowing results is quite sensitive to the selection of the parameters, and NID provides a useful tool to automatically select the best parameters that produces the best quality image.

The major computational cost of the proposed method lies in the search procedure in the parameter space. In our experiment using a computer with a Core-i5 CPU running at 2.27Ghz, it takes about 250 seconds for our unoptimized program to find the optimal NID windowing operator for an 512×512 image using an exhaustive search method on a grid of 0.02×0.02 precision. The time can be largely shortened by using advanced optimization method. For example, an MATLAB *fmincon* function that employs gradient optimization and trust-region-reflective algorithm reduces the search time to about 13 seconds.

5. CONCLUSION

In this work, we make one of the first attempts to apply the theory of Kolmogorov complexity and NID to the field of

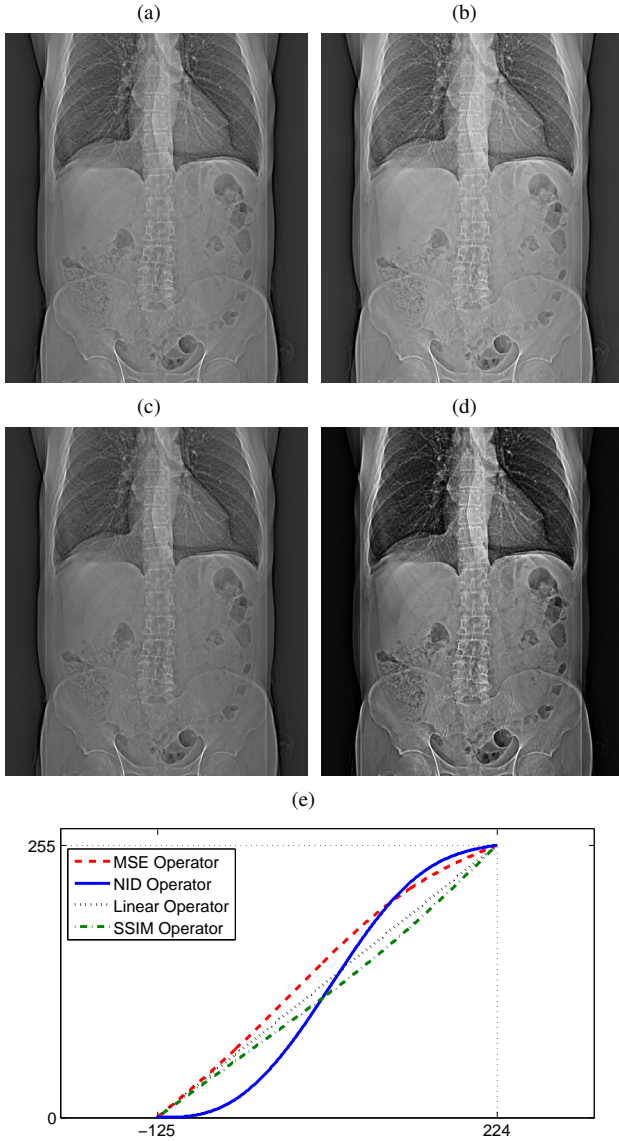


Fig. 1: Adaptive windowing using (a) Linear, (b) MSE, (c) SSIM, and (d) NID optimization of sine basis operators. (e) Corresponding optimal windowing function.

medical image processing. Specifically, we use an NID-motivated criterion in the optimal design of windowing operators for the visualization of HDR medical images on standard displays. A Shannon entropy based approximation was made that converts the uncomputable NID minimization problem into a practical algorithm that optimizes parametric windowing operators. Experiments using medical images demonstrate that the proposed method provides a powerful tool in finding the best parametric windowing functions, which create images with higher contrast and more visible structural details. In the future, the proposed method may be extended to higher order parametric windowing functions. The promising results obtained in this work also inspires us

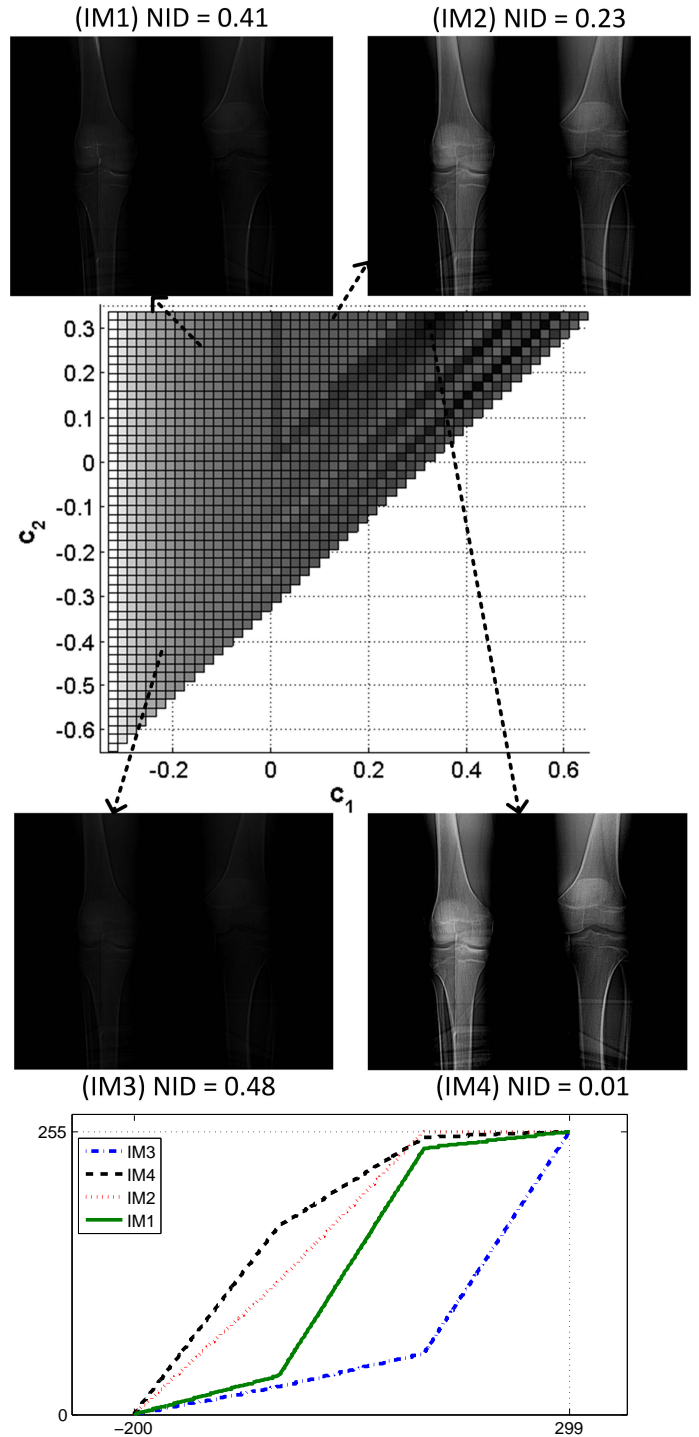


Fig. 2: NID as a function of the parameters in piecewise linear windowing operator. (IM1)-(IM4): images correspond to 4 different options of c_1 and c_2 parameters, which result in different image quality and NID values.

to explore more applications of Kolmogorov complexity in the field of medical image processing.

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