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SSIM-BASED NON-LOCAL MEANS IMAGE DENOISING

Abdul Rehman and Zhou Wang

Department of Electrical & Computer Engineering, University of Waterloo, Waterloo, ON, Canada
Email: abdul.rehman@uwaterloo.ca, zhouwang@ieee.org

ABSTRACT

Perceptually inspired image processing has been an emerging field of study in recent years. Here we make one of the first efforts to incorporate the structural similarity (SSIM) index, a successful perceptual image quality assessment measure, into the framework of non-local means (NLM) image denoising, which is a state-of-the-art method that delivers superior desnoising performance. Specifically, a denoised image patch is obtained by weighted averaging of neighboring patches, where the similarity between patches as well as the weights assigned to the patches are determined based on an estimation of SSIM. A two-stage approach is proposed for robust SSIM estimation in the presence of noise. Moreover, motivated by the ideas behind SSIM, we adjust the contrast and mean of each patch before feeding it into the weighted averaging process. Our experimental results show that the proposed SSIM-based NLM algorithm achieves better SSIM and PSNR performance and provides better visual quality than least square based NLM method.

***Index Terms*—** image denoising, structural similarity, non-local means, perceptual image processing

1. INTRODUCTION

Despite the ubiquitous usage in a wide variety of signal processing applications, the mean squared error (MSE) appears to be a poor measure when perceived image quality is of our major concern [1]. The structural similarity (SSIM) index [2] is a recently proposed image similarity measure that has shown superior performance over MSE in predicting visual quality of images [1]. SSIM and its derivations have been applied to a broad range of applications, ranging from image restoration and compression, to visual communication and pattern recognition [1]. The usage of SSIM should not be restricted to perceptual quality evaluation and algorithm comparison purposes only. Perhaps the more interesting and promising application area is to use it in the design and optimization of image processing algorithms and systems.

Recently there has been a great deal of attention paid to the problem of image denoising, which is not only a practically useful application, but also an ideal test bed for image representation, modeling and estimation theories. One of the most successful image denoising algorithms is the non-local means (NLM) method [3], which has achieved state-of-the-art performance. NLM denoising is a nonlocal filtering (or weighted averaging) technique where the weights are decided based on similarity between the current image patch being denoised and the other patches in the image within a neighborhood. Since MSE is employed for calculating the weights, the resulting denoised image might not have the best perceptual quality. This motivates us to replace the role of MSE with SSIM in the framework. There are two issues that need to be resolved before effective SSIM-based approach can be developed. First, we would need to reliably estimate the SSIM value between two original image patches in the presence of noise. Directly using the SSIM value between two noisy patches to define the weight would not lead to good results. This is because SSIM attempts to match the structures of two patches, but when the signal-to-noise ratio is low, the noise submerges the actual structure of the image, and thus SSIM evaluation would favor those patches with the noise pattern best matched. Second, once weights are calculated based on SSIM, it is important to adjust the contrast and mean values of the patches before weighted averaging. This is because SSIM may pick those patches that are structurally similar but with different contrast and mean values, and thus direct averaging these patches (that have different contrast and mean variations) would provoke further undesired distortions. These issues are tackled with the help of proposed two stage denoising algorithm on similar lines to BM3D [4], a state-of-the-art denoising method which also uses two stages to perform image denoising.

2. PROBLEM FORMULATION

NLM algorithm [5] replaces the intensity of each pixel in the noisy image by a weighted average of all the pixel intensities in the image. More generally, the nonlocal filter (NLF) in the continuous space can be represented as follows [6]

$$P_{NLF}(f(x,y)) = \frac{\int_\Omega w(x,y;x',y')f(x',y')dx'dy'}{\int_\Omega w(x,y;x',y')dx'dy'} , \quad (1)$$

where $w(x,y;x'y')$ is the weighting function related to the similarity between two patches at (x,y) and (x',y'). The

Table 1. Comparisons of NLM denoising using $\mathcal{L}_2$ and SSIM of original image patches for weight calculation

Test image	Barbara			
Noise std (σ)	15	25	30	50
	PSNR comparison (in dB)			
Noisy image	24.61	20.29	18.81	14.74
$\mathcal{L}_2^*$-NLM	31.42	27.33	25.97	22.49
SSIM*-NLM	32.21	29.59	28.65	25.61
	SSIM comparison			
Noisy image	0.729	0.543	0.474	0.289
$\mathcal{L}_2^*$-NLM	0.925	0.818	0.759	0.543
SSIM*-NLM	0.947	0.902	0.879	0.779

weight in NLM denoising is specified by borrowing ideas from the work of nonparametric sampling-based texture synthesis [7]. It is calculated based on $\mathcal{L}_2$ distance between two patches at (x, y) and (x', y').

To better reflect the perceptual similarity between two patches and also to give favor to the patches that are structurally similar, we opt to replace the role of $\mathcal{L}_2$ by SSIM in computing the weight function. Let X_1 and X_2 be two image patches extracted from the original noise-free image. The SSIM index between them is defined as

$$S(X_1, X_2) = \frac{(2\mu_{X_1}\mu_{X_2} + C_1)(2\sigma_{X_1X_2} + C_2)}{(\mu_{X_1}^2 + \mu_{X_2}^2 + C_1)(\sigma_{X_1}^2 + \sigma_{X_2}^2 + C_2)}, \quad (2)$$

where μ_X, σ_X, and $\sigma_{X_1X_2}$ are the mean, standard deviation, and cross correlation between the two patches, respectively, and C_1 and C_2 are positive stabilizing constants.

To understand the impact of replacing $\mathcal{L}_2$ with SSIM, we carried out an empirical study where all weights were calculated using patches extracted from the original image but computed using $\mathcal{L}_2$ and SSIM, respectively. With these weights, the NLM denoising results of "Babara" image at different noise levels are shown in Table 1, where we observe large gains in both PSNR and SSIM values of the denoised image when SSIM is employed for weight computation.

The above empirical study, though very instructive, does not provide a working denoising algorithm, because the original image patches are not accessible. Therefore, the critical problem here is how to predict the SSIM value between X_1 and X_2 from their noisy observations.

3. PROPOSED SCHEME

Let Y_1 and Y_2 be two observed noisy patches that are created from two clean original patches X_1 and X_2 by

$$Y_1 = X_1 + N_1, \quad (3)$$

$$Y_2 = X_2 + N_2, \quad (4)$$

where N_1 and N_2 are the corresponding *i.i.d* Gaussian noise patches with standard deviation σ_n. The purpose here is to estimate $S(X_1, X_2)$ using Y_1, Y_2. A simple approximation would be

$$S(X_1, X_2) \approx \frac{(2\mu_{Y_1}\mu_{Y_2} + C_1)(2\sigma_{Y_1Y_2} + C_2)}{(\mu_{Y_1}^2 + \mu_{Y_2}^2 + C_1)(\sigma_{Y_1}^2 + \sigma_{Y_2}^2 - 2\sigma_n^2 + C_2)} \quad (5)$$

Here we have made use of the assumptions that the noise N_1 and N_2 are zero-mean, the signal X_1 and X_2 are uncorrelated with noise, and the noise N_1 and N_2 added at different locations are also uncorrelated. Our studies suggest that the approximation in Eq. ((5)) does not achieve desired accuracy in estimating $S(X_1, X_2)$ because the assumptions does not hold for small patches. Also, when the variance of noise is significant as compared to that of the image patch, SSIM is in favor of similar noise patterns rather than image structures.

To overcome the problem above, we propose a two-stage method. In the first stage, we compute a local estimate of the noise using the method proposed in [5]. As mentioned in [3], NLM denoising is based on the "method noise" and the residual image obtained after subtracting the denoised image from the noise-free image looks like random noise and does not contain structures similar to those contained in the original image. We believe that the noise estimated by NLM denoising can be used to provide a better estimate of $S(X_1, X_2)$ because more accurate information about the noise pattern at the local patch is available. Suppose the estimated noise is given by $\hat{N}_1$ and $\hat{N}_2$, respectively. It enables us to estimate X_1 and X_2 by

$$\hat{X}_1 = Y_1 - \hat{N}_1, \quad (6)$$

$$\hat{X}_2 = Y_2 - \hat{N}_2. \quad (7)$$

We can then use $\hat{X}_1$ and $\hat{X}_2$ in the second step to estimate $S(X_1, X_2)$ and define our SSIM-based weight as

$$w_{\text{SSIM}} = S(\hat{X}_1, \hat{X}_2). \quad (8)$$

Before computing the weighted averaging for each patch, we perform further adjustment on the mean and contrast of each patch Y by

$$Y' = \frac{\sigma_{\hat{X}_c} + c}{\sigma_Y + c}(Y - \mu_Y) + \mu_{\hat{X}_c} \quad (9)$$

where μ_Y, σ_Y and $\mu_{\hat{X}_c}$, $\sigma_{\hat{X}_c}$ are the mean and contrast values of the current patch and the patch to be denoised (estimated using Eq. (6)), respectively and c is the stabilizing constant. This adjustment is motivated by the ideas behind SSIM, which separates the measurement of mean, contrast and structure. Indeed, SSIM-based weight calculation may help collect those image patches that are structurally similar to the patch being denoised but with different contrast and mean values. To avoid creating bias in mean or contrast, it is useful to normalize the patch first, such that only the structural part of the patch contributes to the denoising task.

Finally, we create our final denoised patch at location i by

$$\tilde{X}(i) = \frac{\sum_{j \in \mathcal{N}_i} w_{\text{SSIM}}(i, j) Y'(j)}{\sum_{j \in \mathcal{N}_i} w_{\text{SSIM}}(i, j)}, \quad (10)$$

Table 2. SSIM and PSNR comparisons of image denoising results

Test image	Barbara				Lena				Boat			
Noise std (σ)	15	25	30	50	15	25	30	50	15	25	30	50
	PSNR comparison (in dB)											
Noisy image	24.61	20.29	18.81	14.74	24.62	20.27	18.78	14.71	24.65	20.27	18.76	14.61
$\mathcal{L}_2$-NLM	31.44	28.69	27.55	23.85	32.71	29.94	28.57	25.52	30.87	28.09	27.05	23.91
SSIM-NLM	32.10	29.28	28.21	24.85	33.11	30.52	29.42	25.92	31.26	28.54	27.58	24.39
	SSIM comparison											
Noisy image	0.729	0.543	0.474	0.289	0.489	0.402	0.338	0.192	0.676	0.475	0.406	0.239
$\mathcal{L}_2$-NLM	0.934	0.871	0.832	0.651	0.869	0.832	0.781	0.619	0.889	0.796	0.748	0.565
SSIM-NLM	0.944	0.889	0.858	0.721	0.893	0.858	0.818	0.645	0.900	0.815	0.779	0.599

where $\mathcal{N}_i$ denotes the union of the neighbors around i and $w_{\mathrm{SSIM}}(i,j)$ is the SSIM weight computed between the patches located at i and j.

4. SIMULATION RESULTS

We test image denoising algorithms on various images with noise standard deviation σ ranging from 15 to 50. The $\mathcal{L}_2$ and SSIM based NLM methods are denoted as $\mathcal{L}_2$-NLM [3] and SSIM-NLM, respectively. All $\mathcal{L}_2$-NLM results are obtained using the code provided by Buades *et. al.* at [8]. The search ranges for both algorithms are fixed at 7×7 in order to limit the complexity of the algorithm. The added computational complexity of SSIM-NLM over $\mathcal{L}_2$-NLM mostly lies in estimating the SSIM values between patches. In our experiment, we found it negligible compared with the overall computational cost of the NLM algorithm.

Table 2 shows the results for images "Barbara", "Lena" and "Boat". It can be observed that the proposed SSIM-NLM method achieves better performance than $\mathcal{L}_2$-NLM in terms of not only SSIM, but also PSNR. This may be due to SSIM's capability of collecting those image patches that have similar structure but with different mean and/or contrast. We also observe in our experiment that the performance gap between the two methods increases further when the search range is increased.

It is interesting to compare the denoising results of "Babara" image in Table 2 with those in Table 1. It can be observed that when similarity values are calculated by using the original noise-free image, SSIM-NLM performs significantly better than $\mathcal{L}_2$-NLM in terms of both SSIM and PSNR. Another observation is that the denoising performance of $\mathcal{L}_2$-NLM degrades when the original image is used to compute the weights. This is likely because of the weight mapping function and thresholds used in the implementation in [5,8]. When the original image is used, many more patches with lower $\mathcal{L}_2$ distances also make significant impact on denoising. This often results in blur of the denoised image. By contrast, the SSIM-NLM method does not suffer from such a problem, implying that SSIM is probably a better measure to select similar patches.

To provide visual comparisons of the denoising algorithms, Fig. 1 shows two image areas cropped from the "Babara" image denoised by $\mathcal{L}_2$-NLM [3] and SSIM-NLM, respectively. It can be seen that the proposed SSIM-NLM scheme preserves many local structures better and therefore has better perceptual image quality. The visual quality improvement is also reflected in the corresponding SSIM maps, which provide useful guidance on how local image quality is improved over space. It can be observed from the SSIM maps that the areas which are relatively more structured benefit more from the proposed denoising algorithm as the quality measure used is better at calculating the similarity of structures as compared to MSE.

5. CONCLUSIONS

We proposed an SSIM-based NLM method for image denoising. The key of our approach is to replace the role of MSE with SSIM in measuring patch similarities and in calculating weights. We propose a robust method to estimate SSIM in the presence of noise and adjust the mean and contrast of image patches before using them for weighted averaging. Our simulation results demonstrate the promises of the proposed approach and also indicate the potentials of replacing the ubiquitous PSNR/MSE with SSIM as the optimization criterion in image processing applications.

6. ACKNOWLEDGMENT

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7. REFERENCES

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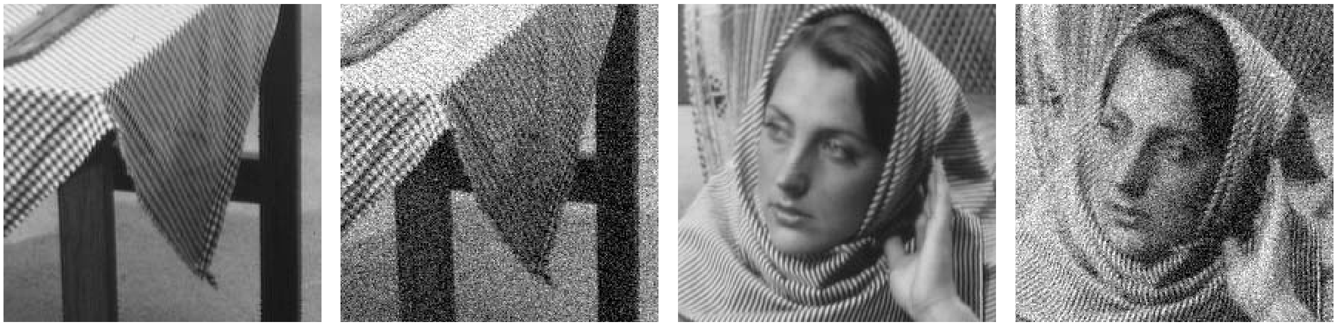
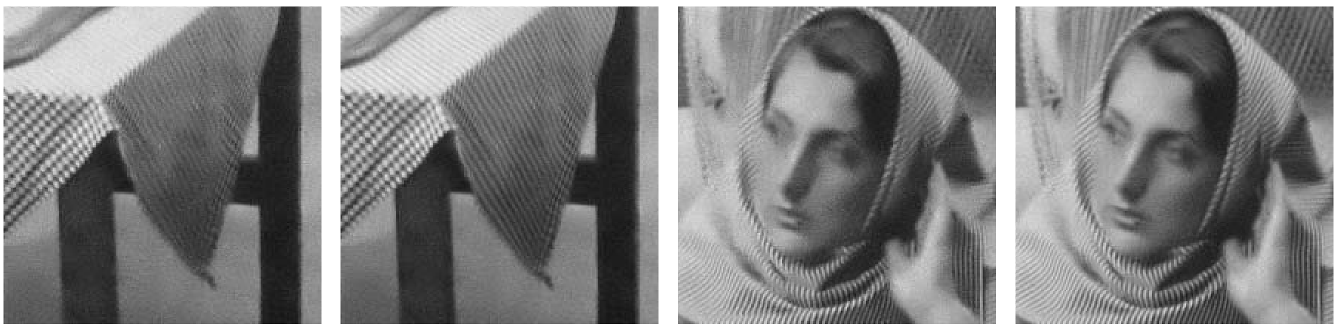
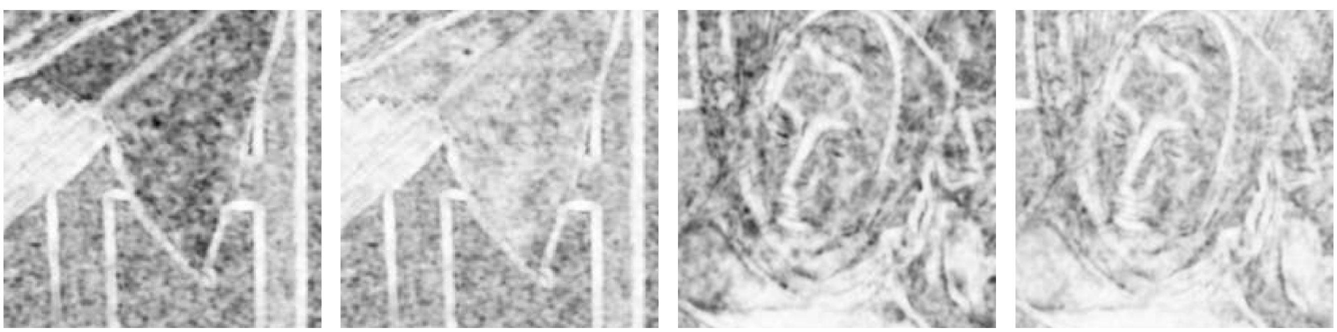


(a) Original image (b) Noisy image ($\sigma = 30$) (c) Original image (d) Noisy image ($\sigma = 30$)



(e) $\mathcal{L}_2$-NLM denoised (f) SSIM-NLM denoised (g) $\mathcal{L}_2$-NLM denoised (h) SSIM-NLM denoised



(i) SSIM map of (e) (j) SSIM map of (f) (k) SSIM map of (g) (l) SSIM map of (h)

Fig. 1. Visual and SSIM quality map comparisons of denoising results. Brighter indicates better SSIM value.

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