Because $X \cdot U^\top = V^\top \cdot U \cdot U^\top + W \cdot U^\top = V^\top \cdot (U \cdot U^\top)$, we have $\|X \cdot U^\top\| = (U \cdot U^\top)\|V\|$ and $V^\top = \frac{X \cdot U^\top}{U \cdot U^\top}$. This leads to $X' = X - (\frac{c}{\|X \cdot U^\top\|} - 1)X \cdot \frac{U^\top \cdot U}{U \cdot U^\top}$.

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Best Neighborhood Matching: An Information Loss Restoration Technique for Block-Based Image Coding Systems

Zhou Wang, Yinglin Yu, and David Zhang

***Abstract*—Imperfect transmission of block-coded images often results in lost blocks. A novel error concealment method called *best neighborhood matching* (BNM) is presented by using a special kind of information redundancy—blockwise similarity within the image. The proposed algorithm can utilize the information of not only neighboring pixels, but also remote regions in the image. Very good restoration results are obtained by experiments.**

***Index Terms*—Error concealment, image coding, image restoration.**

I. INTRODUCTION

Recently, many image coding algorithms have been developed to reduce the bit rate for digital image and video representation and transmission. Among them, block-based techniques have proved to be the most practical and are adopted by most existing image and video compression standards such as the Joint Photographers Expert Group (JPEG) [1], Motion Picture Expert Group (MPEG) [2], and H.261 [3]. Since real-world communication channels are not error free, the coded data transmitted on them may be corrupted. Block-based image coding systems are vulnerable to transmission impairment. Loss of a single bit often results in loss of a whole block and may cause consecutive block losses.

Error concealment is aimed at masking the effect of these missing blocks to create subjectively acceptable images. So far many error concealment methods have been proposed [4]–[10]. Some of them have the capability of detecting damaged blocks before recovering them [7] while the others must be supported by an appropriate transform format and/or an error detection algorithm that helps to identify damaged blocks [6], [8], [9]. Some of them are developed for DCT-based image coding methods [7], [10] while some others can be applied to any block-based approaches [5], [8], [9]. Some of them only deal with still images [8], [9], while the others can be used on image sequences or on both still images and image sequences [4], [7]. In this correspondence, we introduce a new error concealment algorithm that

1) assumes we know which information is received correctly and which is not;
2) can be applied to any block-based image coding system, i.e., independent of block encoding approach;
3) only recovers lost blocks for gray scale still images.

Although there are many variations in previously published methods, they can be categorized into one framework. First, all of them used the information of only neighboring pixels as the source to generate pixel values of the missing blocks. Second, they rely on some predefined constraints on the spectra or structures of the lost blocks. The reconstructed blocks should be smoothly connected with adjacent regions either in spatial or in transform domain. Consequently, a well-recovered lost block is often a smooth block, a sharp edge block, or a stripe block with very consistent directions. The concealment algorithm is a lowpass filter or directionally lowpass filter in nature.

The technique proposed in this correspondence is fundamentally different from the previous framework. It is developed by making use of a special kind of *a priori* information—blockwise similarity within the image. By taking advantage of such kind of information redundancy, the fractal block coding (FBC) technique, a very promising technique for high compression ratios, was developed [11], [12]. The FBC algorithm introduced a practical way to find blockwise self-similarities within natural images. In this correspondence, we will use a similar way to find blockwise similarities. However, our algorithm can not be called *fractal* because the word "fractal" in "fractal image compression" means self-similarity at every different scale while we are trying to utilize blockwise similarities at the same scale.

II. BEST NEIGHBORHOOD MATCHING (BNM) ALGORITHM

Let x_{ij} and x_{ij}^{new} represent the pixel values at position (i, j) in the damaged image and the restored image, respectively. Since we know which blocks in the image are received correctly and which are not, it is easy to give each pixel a binary flag f_{ij} indicating whether it is lost, i.e., $f_{ij} = 1$ means (i, j) is missing and $f_{ij} = 0$ means it is good. The goal of our error concealment algorithm is to give each missing pixel with $f_{ij} = 1$ a new value that can comply with our visual sense. In this section, we first introduce a class of blockwise luminance transformations. The BNM algorithm is then developed by making use of such kind of transformations.

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Consider an $N \times N$ block in the image with its up-left corner at position (i, j). The block can be represented as $\{x_{i+m,j+n} | 0 \le m \le N-1, 0 \le n \le N-1\}$. A blockwise luminance transformation on the block is defined as

$$x_{i+m,j+n} \rightarrow v(x_{i+m,j+n}) \quad \text{for all } 0 \le m \le N = 1, \; 0 \le n \le N-1 \tag{1}$$

where v is a one-dimensional (1-D) real valued function. Some possible selections of v are listed below: $\forall z \in \mathcal{R}$ ($\mathcal{R}$ denotes the set of real number) and

$$v(z) = z \quad \text{(direct matching)} \tag{2}$$

$$v(z) = a_0 + a_1 z + \cdots + a_p z^p \quad (p\text{-order matching}, p \ge 1). \tag{3}$$

An $N \times N$ image block with its up-left corner at (k, l) in the damaged image may contain both good pixels and missing pixels, so the block can be divided into two parts—the lost part and the good part. In our BNM algorithm, the good part is also called the neighborhood of the lost part or the matching part of the whole block because we will apply a matching test on it. The number of pixels in the matching part can be calculated as

$$n_M = \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} (1 - f_{k+m,l+n}). \tag{4}$$

Such an $N \times N$ block with both lost part and matching part is called a *range block*. To recover the lost part of it, a searching procedure is applied within a large range in the image. The purpose of the searching procedure is to find a good $N \times N$ block (no pixel is missing) in the image that can best approximate the matching part of the range block through an appropriate blockwise luminance transformation. We call the good block a *domain block*. The mean square error (MSE) between the transformed domain block and the range block in the matching part is used as the criterion to evaluate the matching result

$$\text{MSE}_M = \frac{1}{n_M} \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} (1 - f_{k+m,l+n}) \cdot (v(x_{i+m,j+n}) - x_{k+m,l+m})^2. \tag{5}$$

Each of the candidate domain blocks in the image will result in its corresponding MSE_M. The one with the best matching result, i.e., with the minimal MSE_M, is our ultimate choice. When two domain blocks are of the same MSE_M, the one closer to the lost block is the winner. Finally, the lost part of the range block is replaced by the corresponding part of the transformed domain block, while the good part of the range block is kept unchanged, as follows:

$$x_{k+m,l+n}^{(\text{new})} = \begin{cases} x_{k+m,l+n}, & \text{if } f_{k+m,l+n} = 0 \\ v(x_{i+m,j+n}), & \text{if } f_{k+m,l+n} = 1 \end{cases} \quad \text{for all } 0 \le m \le N-1, 0 \le n \le N-1. \tag{6}$$

Briefly speaking, the BNM algorithm for a lost block can be summarized in the following five steps.

1) Extract a lost block from the image together with its neighborhood as a range block.
2) Search for a good candidate domain block with the same size of the range block in a searching range within the image.
3) Try to match the neighborhood of the lost block with the domain block through a blockwise luminance transformation.
4) Compete for the best match (i.e., find the minimal MSE_M) among all the candidate domain blocks.
5) Recover the lost block using the pixel values transformed from the corresponding part of the winning domain block.

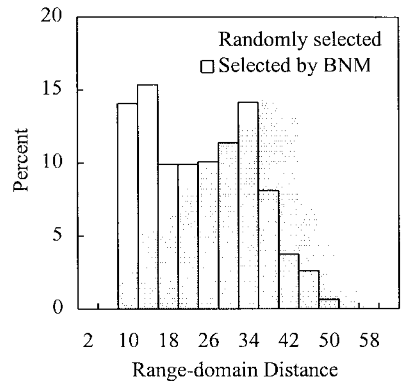

Fig. 1. Statistics on randomly selected and BNM selected range-domain distance. The data are obtained from the test where the Barb image is corrupted with isolated block losses and with a block loss rate of 10%.

III. Implementation

For the real application of the BNM algorithm, there remain some choices, which include the following:

1) the size N of range and domain blocks as well as the composition of the range blocks;
2) the searching range, i.e., given a range block, which part of the image is searched for candidate domain blocks;
3) the choice of the luminance matching function v.

In our simulations, the test images are 8 b/pixel gray-level images. Two cases of damaged images are generated by randomly discarding some 8 × 8 blocks from the original images. In the first case, the lost blocks are restricted to be isolated, i.e., all of their eight surrounding blocks are good blocks; In the other case, no restriction is applied, so that some adjacent missing blocks may be connected into big lost regions.

In our implementations, the sizes of all range and domain blocks are set to be 10× 10, i.e., $N = 10$. We put the 8 × 8 lost block at the center of a range block, thus the matching part of the range block is a one pixel wide boundary around the lost block. This is why we call the matching part the neighborhood of the lost block. When competing for the best match for a certain range block, both a near domain block and a remote domain block may become the winner. Fig. 1 shows a statistic result in percentage on range-domain distance for two cases. In the first case, the domain blocks are randomly selected from the searching range, while in the other case, the domain blocks are found by the BNM algorithm. It appears that near domain blocks are more likely to be chosen by the BNM algorithm, but remote domain blocks (sometimes with a distance of over 40) may also be highly correlative with the range block. The searching range in the image should be neither too small (lose some well matched remote domain block) nor too large (lead to high computation burden or absurd matching). In our implementations, the searching range is an 80 × 80 square region with the range block locating at its center. Fig. 2 illustrates the structure of lost block, range block, domain block, and the searching range block.

The matching function v is the core of the luminance transformation. The direct matching function $v(z) = z$ does not change pixel values and the p-order $(p \ge 1)$ matching function try to match the values of the pixels in the range blocks through polynomial approximations. The parameters $a_0, a_1, \cdots, a_p$ are selected to minimize MSE_M. They can be obtained by solving the equations

$$\begin{aligned} \partial\, \text{MSE}_M / \partial a_0 &= 0 \\ \partial\, \text{MSE}_M / \partial a_1 &= 0 \\ \partial\, \text{MSE}_M / \partial a_p &= 0 \end{aligned} \tag{7}$$

TABLE I
PSNR PERFORMANCE FOR DIFFERENT KINDS OF METHODS WITH THE BLOCK LOSS RATES RANGING FROM 2.5 TO 15.0%

Error Concealment Algorithm	Block Loss Rate					
	2.5 %	5.0 %	7.5 %	10.0 %	12.5 %	15.0 %
without concealment	22.8 dB	19.4 dB	17.7 dB	16.4 dB	15.3 dB	14.6 dB
dc concealment	36.6 dB	32.9 dB	31.4 dB	30.0 dB	29.2 dB	28.0 dB
direct BNM	41.1 dB	39.0 dB	36.3 dB	**35.7 dB**	33.6 dB	32.3 dB
1-order BNM	41.8 dB	39.6 dB	37.6 dB	37.1 dB	35.0 dB	33.2 dB

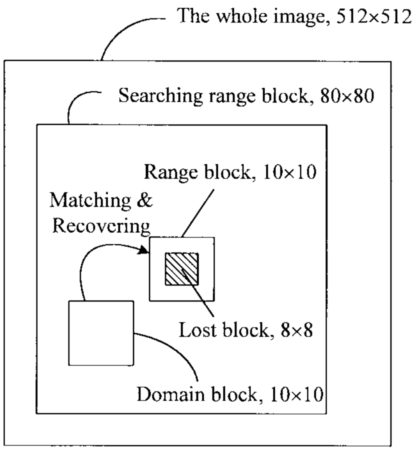

Fig. 2. Structure of lost block, range block, domain block, and searching range block with their default sizes.

For the 1-order case, the solution is (8), shown at the bottom of the page. The direct matching may be more practical in some applications because no parameters need to be calculated. However, its restoration result is not as good as those of 1-order or high-order matching functions. It should be mentioned that the high-order matching function, although it can give smaller MSE_M, can not always get eventual restoration results better than that of the low-order matching function. The reason is that it may falsely lead to absurd recovery. In other words, two blocks that are not so similar may become very similar through a high-order matching function. Actually, the 1-order matching function is always the best in our experiments. We use it in the following simulations.

IV. SIMULATIONS

The peak signal-to-noise ratio (PSNR) is used to give a quantitative evaluation on the quality of the reconstructed image. As mentioned before, two cases of corrupted images are considered in this correspondence. In the case of isolated block losses, we recover the lost blocks one by one using the BNM algorithm. A problem with regard to the usage of the BNM algorithm is that when the lost block is close to the image boundaries, the 80×80 searching range block may exceed the image boundaries. In such a case, we move the searching range block into the image until its boundary reaches the image boundary. For contiguous block losses, the restoration scheme is more complex because we can not assume a completely good boundary (neighborhood) for a missing block. To solve this problem, we perform the following.

1) *Exclude the missing pixels* in the one pixel wide boundary of a missing block from the matching part of the range block, thus the matching test is applied on the remaining good boundary pixels only.
2) *Recover the damaged image* through a progressive procedure that is composed of several steps. The missing blocks recovered in the current step are considered as good blocks in the subsequent steps. By using such a progressive procedure, the more a lost block holds good boundary pixels, the earlier it is recovered.

The proposed restoration algorithm has been tested on some standard gray level images. The sizes of the lost blocks are fixed to be 8×8. A dc concealment algorithm is also developed to recover the lost blocks where we simply cover each lost block with uniform gray value equal to the average gray value of its eight surrounding blocks. In Table I, we present the restoration results for the Barb image using dc concealment, direct BNM and 1-order BNM algorithms, respectively. The dc concealment algorithm can improve PSNR by a magnitude of about 13–14 dB while those for direct BNM and 1-order BNM algorithms are 18–20 and 19–21 dB, respectively. The PSNR performance of the 1-order BNM algorithm for Barb with a block loss rate of 10% is 37.08 dB, which is a significant improvement over 31.2 dB of Sun's scheme [5], 32 dB of HCIE scheme [8] and 34.5 dB of Lee *et al.*'s fuzzy logic scheme [8]. To illustrate the subjective quality of our BNM algorithm, we show some enlarged examples for sharp edge areas, texture areas, very complex areas, stripe areas and small objects in Fig. 3. The visual quality of the recovered blocks are very good even when the areas contain a lot of detail information. In Fig. 4, we demonstrate the progressive procedure for contiguous block losses. Although some missing blocks are connected to large black regions, they are still recovered very well.

V. CONCLUSION AND DISCUSSION

In this correspondence, a novel information loss restoration technique, BNM, is presented. The simulation results indicate that this algorithm provides significant improvement over existing algorithms in terms of both subjective and objective evaluations. It can make use of the information of not only neighboring regions, but also

$$\begin{cases} a_1 = \dfrac{n_M \cdot \displaystyle\sum_{m=0}^{N-1}\sum_{n=0}^{N-1}(1-f_{k+m,l+n})\cdot x_{k+m,l+n}\cdot(x_{i+m,j+n}) - \sum_{m=0}^{N-1}\sum_{n=0}^{N-1}(1-f_{k+m,l+n})\cdot x_{k+m,l+n}\cdot \sum_{m=0}^{N-1}\sum_{n=0}^{N-1}(1-f_{k+m,l+n})\cdot(x_{i+m,j+n})}{n_M\cdot \displaystyle\sum_{m=0}^{N-1}\sum_{n=0}^{N-1}(1-f_{k+m,l+n})\cdot(x_{i+m,j+n})^2 - \left(\sum_{m=0}^{N-1}\sum_{n=0}^{N-1}(1-f_{k+m,l+n})\cdot(x_{i+m,j+n})\right)^2} \\ a_0 = \frac{1}{n_M}\left[\sum_{m=0}^{N-1}\sum_{n=0}^{N-1}(1-f_{k+m,l+n})\cdot x_{k+m,l+n} - a_1\cdot \sum_{m=0}^{N-1}\sum_{n=0}^{N-1}(1-f_{k+m,l+n})\cdot(x_{i+m,j+n})\right] \end{cases} \quad (8)$$

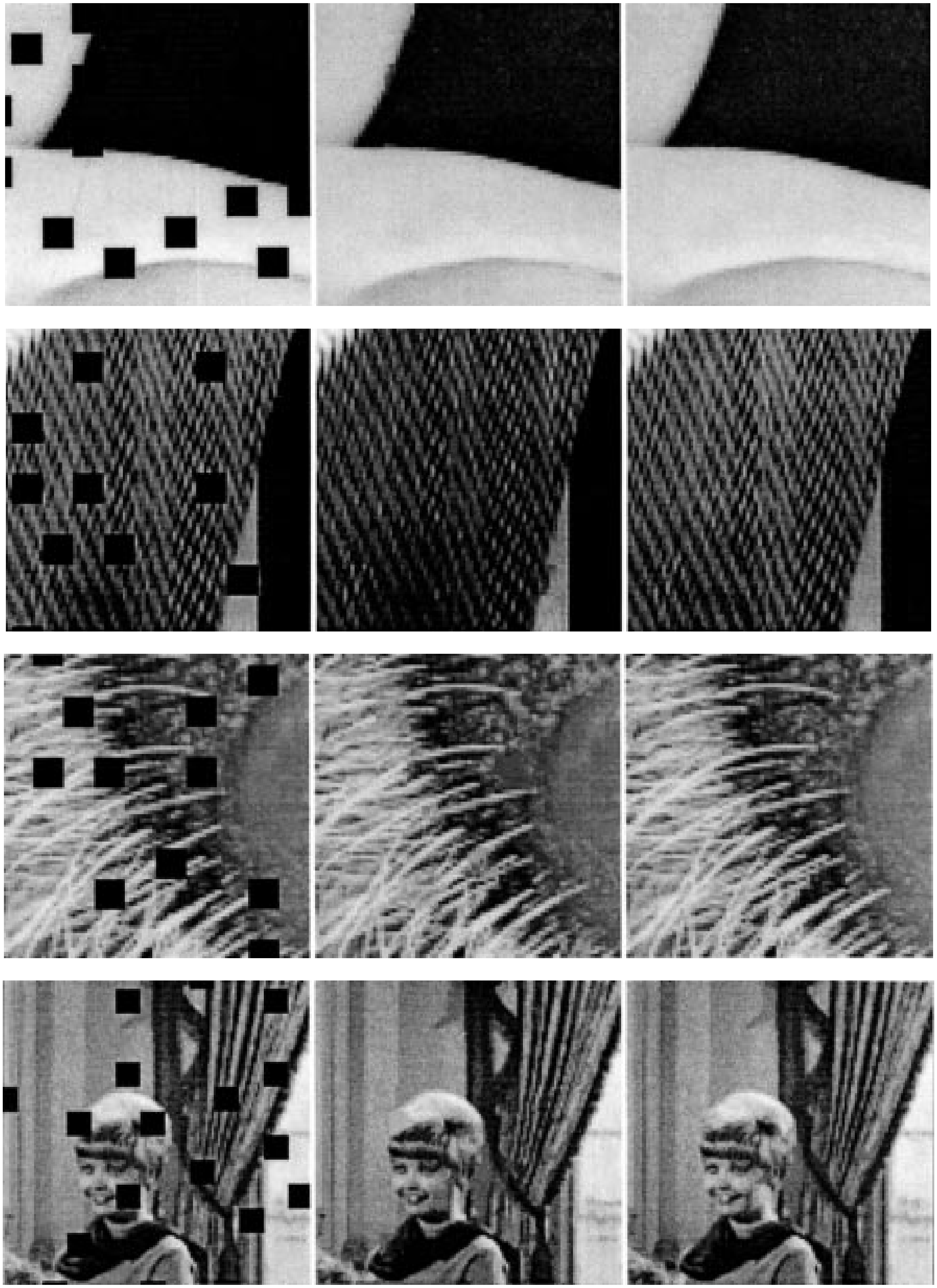

Fig. 3. Examples for various kinds of image details. First row: sharp edge area. Second row: texture area. Third row: very complex area. Fourth row: small object and stripe area. Left column: damaged. Middle column: restored. Right column: original areas extracted from the standard images peppers, Barb, Mandrill, and couple (from top to bottom), respectively.

remote regions within the image. We think the ability to use long-range correlation is the main reason why it can outperform other algorithms. Another advantage is that the BNM algorithm does not apply any constraint on the spectra of local regions. We do not need to classify a missing block into one of several categories due to its neighborhood direction or complexity. Whatever the block looks like,

Fig. 4. (a) Damaged Barb image with contiguous block losses and with block loss rate of 15%, PSNR = 14.74 dB. (b) After Step 1, PSNR = 16.00 dB. (c) After Step 2, PSNR = 17.47 dB. (c) Reconstructed image after Step 4, PSNR = 32.84 dB.

it can be recovered using the same method. In other words, the BNM algorithm is inherently an adaptive algorithm.

The BNM algorithm is not restricted to the particular 8×8 block size. The reason to choose such a size in our simulations is that it is the size frequently used by a lot of proposed image and video coding techniques. Actually, the methodology described in this correspondence can be used for different shapes, sizes, and transforms. Further improvements include developing fast searching algorithms and generalizing the BNM technique for color images, image sequences, or other kinds of signals.

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Correlation-Feedback Technique in Optical Flow Determination

J. N. Pan, Y. Q. Shi, and C. Q. Shu

***Abstract*— In this correspondence, we present a new algorithm to determine optical flow that utilizes a correlation-feedback technique. Several experiments are presented to demonstrate that our method performs generally better than some standard correlation and gradient-based methods in terms of accuracy.**

***Index Terms*—Correlation, estimation of motion vector, feedback, image sequence processing, optical flow computation.**

I. Introduction

Optical flow in image sequences provides important information regarding both motion and object structure and is useful in such diverse fields as robot vision, autonomous navigation and video coding. Although this subject has been studied for more than a decade, reducing the error in the flow estimation remains a difficult problem. A comprehensive review and comparison of various optical flow techniques in terms of accuracy have recently been made [1]. So far, most of the techniques in the optical flow computations use one of the following four basic approaches:

1) gradient-based [2]–[5];
2) correlation-based [6], [7];
3) spatio-temporal energy-based [8], [9];
4) phase-based [10].

Besides these deterministic approaches, stochastic approach is another category [11].

In this work, feedback, a powerful technique widely used in automatic control and many other fields, is applied to a correlation-based algorithm, resulting in a correlation-feedback algorithm to compute optical flow. Our new algorithm is iterative in nature. In each iteration, the estimated optical flow and its several variations are fed back. For each of the varied optical flow vectors, the corresponding displaced frame difference, which is often bilinearly interpolated, is calculated. This useful information is then utilized in a revised version of a correlation-based algorithm [7]. We choose to work with this algorithm because it has several merits and its estimation-theoretic computation framework lends itself to our application of the feedback technique.

As expected, the repeated usage of two given images via the feedback iterative procedure improves the accuracy of optical flow considerably. Several experiments on real image sequences in the laboratory and some synthetic image sequences demonstrate that our correlation-feedback algorithm performs better than some standard gradient-based and correlation-based algorithms in terms of accuracy.

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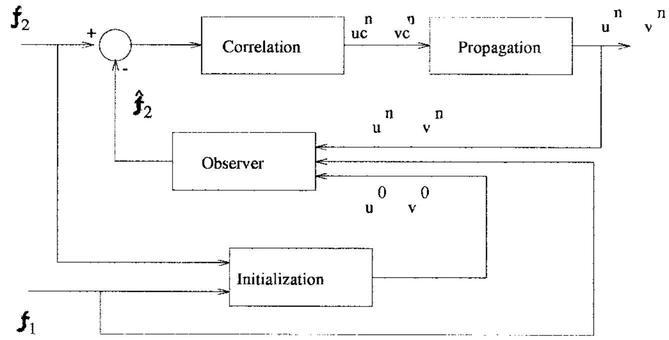

Fig. 1. Schematic diagram.

II. Proposed Framework

The block diagram of the proposed framework is shown in Fig. 1 and described next.

A. Initialization

Although any flow algorithms can be used to generate an initial optical flow field $\vec{u}^o = (u^o, v^o)$ (even a nonzero initial flow field without applying any flow algorithm may work but with a slow speed), we choose to use the Horn and Schunck algorithm [2] (usually five to ten iterations) to provide an appropriate starting point after a preprocessing (involving lowpass filtering) since the algorithm is fast and the problem caused by the smoothness constraint is not serious in the first ten to 20 iterations.

B. Observer

The displaced frame difference at the nth iteration is observed as: $f_k(\vec{x}) - f_{k-1}(\vec{x} - \vec{u}^n)$, where f_k and f_{k-1} denote two consecutive digital images, $\vec{x} = (x, y)$ the coordinates of the pixel under consideration, $\vec{u}^n = (u^n, v^n)$ the optical flow of this pixel estimated at the nth iteration. Demanding the fractional pixel accuracy usually requires interpolation. In our work, the bilinear interpolation is adopted. The bilinearly interpolated image is denoted by $\hat{f}_{k-1}$.

C. Correlation

Once the bilinearly interpolated image is observed, we can select a correlation measure to search for the best match for a given pixel in $f_k(\vec{x})$, in a search-area in the interpolated image $\hat{f}_{k-1}(\vec{x} - \vec{u}^n)$.

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